LIBRARY

THE volumes of the University of Michigan Studies and Publications are published by authority of the Board of Regents under the direction of the Executive Board of the Graduate School. The Contributors are chiefly, but not exclusively, members of the faculties or graduates of the University. The expense is borne in part by gifts, in part by appropriations of the Board of Regents. A list of volumes already published or in preparation is given at the end of this volume.

Volumes may be obtained by ordering from the Librarian, University of Michigan, Ann Arbor, Michigan.

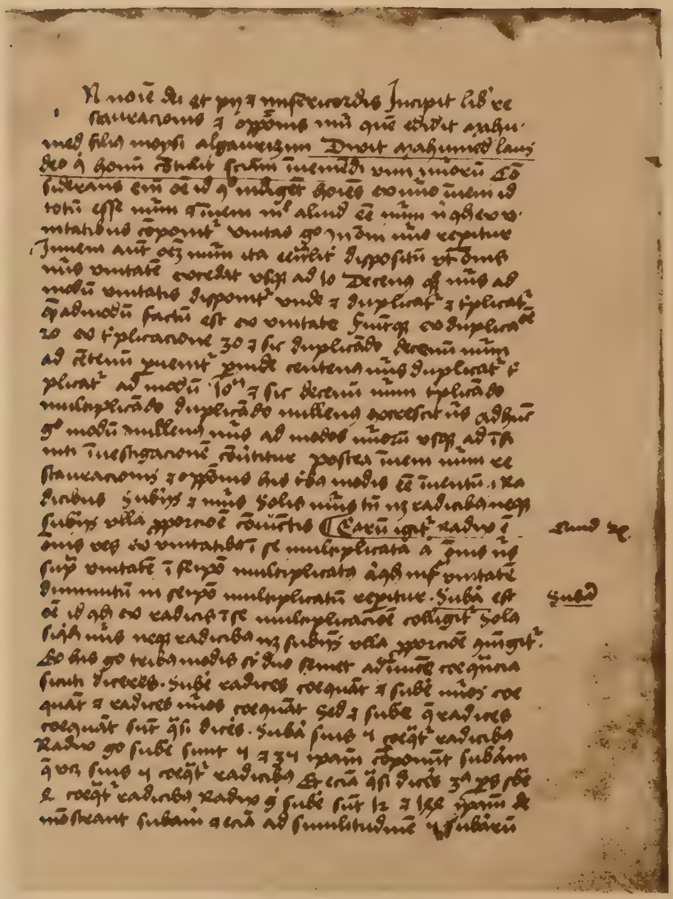
University of Michigan Studies

HUMANISTIC SERIES

VOLUME XI

CONTRIBUTIONS TO THE HISTORY OF SCIENCE

PLATE I.



CODEX VINDOBONENSIS 4770, Fol. 1.a

CONTRIBUTIONS TO THE HISTORY OF SCIENCE

BY
LOUIS CHARLES KARPINSKI
UNIVERSITY OF MICHIGAN

AND

JOHN GARRETT WINTER
UNIVERSITY OF MICHIGAN

ANN ARBOR
UNIVERSITY OF MICHIGAN
1930
All rights reserved

PRINTED IN THE UNITED STATES OF AMERICA

COPYRIGHT, PART I, 1915; PART II, 1916,
BY FRANCIS W. KELSEY, EDITOR

COPYRIGHT, 1930,
GRADUATE SCHOOL, UNIVERSITY OF MICHIGAN

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

LIBRARY

EDITORS' NOTE

The first of the two monographs in this volume was published in 1915; the second, in 1916. It was the original intention of Professor Francis W. Kelsey, who inaugurated the Humanistic Series and continued as editor until his death on May 14, 1927, to add a third monograph on *Vesuvius in Antiquity*. This was announced when Part I of this volume was published, but the constantly increasing demands of his archaeological undertakings left him but little time for writing. Much of Professor Kelsey's time and energy in the last few years of his life was spent in organizing and directing large projects and in gathering materials for the research of others; the monograph on Vesuvius he had set aside for the leisure which never came.

JOHN G. WINTER
EUGENE S. McCARTNEY

PART I

ROBERT OF CHESTER'S LATIN TRANSLATION OF THE ALGEBRA OF AL-KHOWARIZMI

*(The Index to Part I
will be found on pages 159–164.)*

PREFACE

From the point of view of the history of science, no justification is needed for the publication of a mathematical text of the twelfth century, for the available material representing this period is meagre. A wider acquaintance with Robert of Chester's Latin translation of Al-Khowarizmi's Arabic treatise on algebra will perhaps contribute to a more just estimate of the services rendered to science by the Arabs. In the English version I have not attempted to give a literal translation of the Latin, but rather to express the thought in a phraseology which the modern student of mathematics will find easy of comprehension; by consulting the Latin text and footnotes the reader will be able to examine Robert of Chester's own words. For the convenience of readers interested in the text I have added a Latin Glossary in which are noted many variations from the usage of classical writers. In the Introduction I have presented a study of the significance of the treatise in the history of mathematics, and a description of the manuscripts upon which the text is based.

It is a pleasure to express indebtedness to Professor David Eugene Smith for having suggested the work; to Mr. George A. Plimpton for the generous use of his unique mathematical library; to the librarians of the libraries of Vienna and Dresden for photographic reproductions of manuscripts containing this text; to the librarian of Columbia University for the loan of the Scheybl manuscript, and to the librarian of the Cleveland Public Library for the use of works from the John G. White collection. I am much indebted also to colleagues of the University of Michigan, particularly in the Department of Latin and the Department of Mathematics.

I am under special obligation to Mr. William H. Murphy for making possible this publication.

LOUIS C. KARPINSKI.

Ann Arbor, Michigan,
November 1, 1915.

CONTENTS

PAGE

INTRODUCTION:

I. Algebraic Analysis before Al-Khowarizmi 1
II. Al-Khowarizmi and his Treatise on Algebra 13
III. Robert of Chester and Other Translators of Arabic into Latin 23
IV. The Influence of Al-Khowarizmi's Algebra upon the Development of Mathematics 33
V. The Arabic Text and the Translations of Al-Khowarizmi's Algebra 42
VI. Preface and Additions found in the Arabic Text of Al-Khowarizmi's Algebra 45
VII. Manuscripts of Robert of Chester's Translation of Al-Khowarizmi's Algebra 49
ABBREVIATIONS 65

LATIN TEXT WITH CRITICAL NOTES:

Liber Algebrae et Almucabola (Even Pages) . . . . 66 ff.
Regule 6 Capitulis Algabre Correspondentes 126
Addita Quaedam pro Declaratione Algebrae (Even Pages) . 128 ff.

ENGLISH VERSION WITH EXPLANATORY NOTES:

The Book of Algebra and Almucabola (Odd Pages) . . . 67 ff.
Rules Corresponding to the Six Chapters of Algebra . . . 127
Some Additions in Explanation of the Algebra (Odd Pages) . 129 ff.
LATIN GLOSSARY 159

PLATES

I. Codex Vindobonensis, 4770, Folio 1^a . . . . *Frontispiece*
(Text, p. 66, l. 8 *In* . . . p. 68, l. 19 *substantiarum*)
FOLLOWING PAGE
II. Codex Dresdensis, C. 80, Folio 340^b 40
(Text, p. 68, l. 28 *Eodem* . . . p. 72, l. 12 *radicum unius*)
III. Codex Dresdensis, C. 80, Folio 342^a 48
(Text, p. 78, l. 7 *applicantur* . . . p. 80, l. 21 *mul*[*tiplicatione*])
IV. Columbia University Manuscript, X 512, Sch. 2, Q, Page 93 . . . 64
(Text, p. 90, l. 9 [*propo*]*sitae* . . . l. 23 *diminuenda*)

INTRODUCTION

CHAPTER I

Algebraic Analysis before Al-Khowarizmi

Arabic contributions to science have, in the past, been somewhat neglected by historians. More recent studies are recognizing our indebtedness to Mohammedan scholars, who kept the embers of learning aglow while Europe was in the darkness of the Middle Ages. Much of our knowledge of Greek mathematics comes to us from Arabic sources; the early Latin versions were frequently based upon Arabic texts rather than the Greek originals. Similarly, Hindu arithmetic and astronomy were transmitted to Europe by Islam. The services of the Arabs to science were not limited to the preservation and transmission of the learning of other nations. They made independent contributions in many fields.

Among these achievements is the Arabic algebra of Al-Khowarizmi, which for centuries enjoyed wide popularity in the original, and for further centuries extended its popularity through translations and adaptations. A study of the content of this work is an excursion into mediæval thought. By a study of the text in a form as nearly like the original as possible, we discover the reason for its long-continued appeal to the Occidental as well as the Oriental mind, its interest for the Englishman, the German, and the Italian, as well as for the Arab. Even to-day teachers of elementary mathematics may find this book fruitful in suggestion: the geometric solutions of quadratic equations presented by the Arabic writer more than a thousand years ago may be used with profit in our classrooms.

Simple equations of the first degree in one unknown, of the type $ax=b$, are found in the oldest mathematical text-book which we possess, the Ahmes papyrus of about 1700 B.C., which was published with a German translation by Eisenlohr.[1] This Egyp-

---

[1] A. Eisenlohr, *Ein mathematisches Handbuch der alten Aegypter*, Leipzig, 1877; *Facsimile of the Rhind Mathematical Papyrus in the British Museum*, with preface by E. A. Wallis Budge, London, 1898.

LIBRARY
LOS ANGELES COUNTY MUSEUM

tian document presents not only first degree equations together with symbols for the unknown quantity and for the operations of addition and subtraction, but also shows traces of a study of simultaneous linear equations some two thousand years before the Christian Era. Later, but still before the golden age of Greek mathematics, the quadratic equation appears in Egypt. The problems found involve simultaneous quadratic equations, thus:[1]

"Another example of the distribution of a given area into squares. If you are told to distribute 100 square ells (units of area) over two squares so that the side of one shall be $\frac{3}{4}$ of the side of the other: please give me each of the unknowns." The solution follows by assuming the side of one square to be unity, and the other $\frac{3}{4}$. The sum of these areas is $\frac{25}{16}$, of which the root is $\frac{5}{4}$. The root of 100 is 10; 10, then, is to the required side as $\frac{5}{4}$ is to 1, whence one side is 8 and the other 6. The algebraical equivalent of this geometrical problem is, evidently,

$$x^2+y^2=100,$$
$$y=\tfrac{3}{4}x.$$

Noteworthy also is the fact that a symbol for square root occurs in the discussion of these problems.

The solution above leads to the number relation, $6^2+8^2=10^2$, which connects directly with the simpler form, $3^2+4^2=5^2$, and to the same relation other problems of this kind reduce.[2] This makes connection, of course, with the so-called Pythagorean theorem that the sum of the squares on the sides of a right triangle equals the square on the hypotenuse. Even though the Egyptians had no logical proof for this proposition, their familiarity with it is well established. In the time of Plato, and for some centuries afterwards, the Egyptians were famed as surveyors, and the principle stated seems to have been applied by them in laying out right angles by means of a long rope knotted at equal intervals. Two pegs situated three units apart are set out along the line to which it is desired to draw a perpendicular. From one peg an arc is swung with a radius of four units, while from the other end an arc is swung with a radius of five units. The intersection is connected with the peg from which the shorter arc is swung, forming thus a right angle with the desired line, for in any triangle with sides in the ratio three to four to five, a right angle lies opposite the longest side.

---

[1] M. Cantor, *Vorlesungen über Geschichte der Mathematik*, Vol. I, third edition (Leipzig, 1907), pp. 95–96. To this work we shall refer as Cantor, I (3), and to the other volumes similarly.

See also Max Simon, *Geschichte der Mathematik im Altertum* (Berlin, 1909), pp. 41–42; Schack-Schackenburg, *Zeitschrift für Aegyptische Sprache*, Vol. XXXVIII (1900), pp. 135–140, and Vol. XL, pp. 65–66.

[2] Cantor, I (3), p. 96.

The Pythagorean theorem was applied also in India, before the time of Pythagoras, in the construction of altars. With this theorem as developed in the Apastamba Sulba Sutras,[1] the rules for altar construction, are associated careful approximations of square root, pure quadratic equations, and even, as Milhaud has shown,[2] the possible solution of the complete quadratic equation,

$$ax^2+bx=c.$$

The ancient Babylonians, furthermore, constructed tables of squares and cubes. Such tables are found upon the famous tablets of Senkereh,[3] which are contemporary with the Ahmes papyrus. Application of these quadratic numbers to problems similar to those of Egypt already mentioned has not been discovered, but the fact is evident that such tables were a step toward the study of quadratic equations. Cantor[4] shows that the ancient Hebrews were probably familiar with the 3, 4, 5 right triangle. In China, too, students mathematically inclined had come upon this number relation,[5] and evidently were studying quadratic numbers.

Familiarity of Greek mathematicians with the geometrical solution of quadratic equations in the time of Pythagoras is now well established.[6] Hippocrates (fifth century B.C.) writing on the quadrature of the lunes, in an attempt to square the circle, assumes a construction which is equivalent to the solution of the equation,[7]

$$x^2+\sqrt{\frac{3}{2}}\,ax=a^2.$$

---

[1] Bürk, *Das Āpastamba-Śulba-Sutra*, *Zeitschrift der deutschen Morgenländischen Gesellschaft*, Vol. LV (1901), pp. 543–591, and Vol. LVI (1902), pp. 327–391.

[2] G. Milhaud, *La Géométrie d'Apastamba*, *Revue générale des Sciences*, Vol. XXI (1910), pp. 512–520; see also T. L. Heath, *The thirteen books of Euclid's Elements* (3 vols., Cambridge, 1908), Vol. I, pp. 352–364. We shall refer to this latter work as Heath's *Euclid*.

[3] Cantor, I (3), pp. 25–31.

[4] Cantor, I (3), p. 49.

[5] Cantor, I (3), pp. 181 and 679–680.

[6] Heath's *Euclid*, Vol. I, pp. 386–387, 403.

[7] T. L. Heath, *Diophantus of Alexandria, A study in the history of Greek algebra*, second edition (Cambridge, 1910), p. 63; more detailed in Rudio, *Der Bericht des Simplicius über die Quadraturen des Antiphon und des Hippokrates* (1907), p. 58, and same author in the *Bibliotheca Mathematica*, Vol. III, third series (1902), pp. 7–42.

Several propositions of Euclid present geometrical equivalents of the solution of various types of quadratic equations, not involving negative coefficients, and further study of similar problems appears in Euclid's *Data*. Of this nature are the fifth, sixth, and eleventh propositions of the second book of the *Elements* and the twenty-seventh, twenty-eighth, and twenty-ninth of the sixth book, and problems 84, 85, 86, and others of the *Data*.[1] Problem 84, for example, reads:

"If two straight lines include a given area in a given angle, and the excess of the greater over the less is given, then each of them is given."

This corresponds to the equations:

$$xy = k^2$$
$$x - y = a.$$

The two following problems (85 and 86) correspond to the simultaneous quadratic equations:

$$xy = k^2,$$
$$x + y = a,$$

and

$$xy = k^2,$$
$$x^2 - y^2 = a^2.$$

The eleventh proposition of the second book of the *Elements* furnishes the solution of the equation

$$x^2 + ax = a^2$$

or even more general,

$$x^2 + ax = b^2.$$

As this so well illustrates the geometrical solution, it is given in full, following Heath's *Euclid*.

BOOK II OF THE *Elements of Euclid*, PROPOSITION 11

"To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

"Let AB be the given straight line; thus it is required to cut AB so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

---

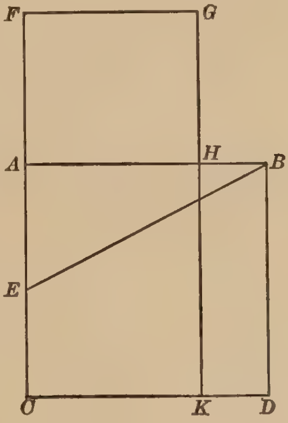
[1] References and citations from the *Elements* are to Heath's *Euclid* and the *Data* (Greek and Latin) edited by H. Menge, Leipzig, 1896, being Vol. VI of *Euclidis opera omnia*, ed. Heiberg et Menge. An English translation of the *Data* is found in the numerous editions of *The Elements of Euclid* by Simson. The numbering of the problems is slightly different in the two versions.

"For let the square *ABDC* be described on *AB* (I. 46); let *AC* be bisected at the point *E*, and let *BE* be joined; let *CA* be drawn through to *F*, and let *EF* be made equal to *BE*; let the square *FH* be described on *AF*, and let *GH* be drawn through to *K*.

"I say that *AB* has been cut at *H* so as to make the rectangle contained by *AB*, *BH* equal to the square on *AH*.

"For, since the straight line *AC* has been bisected at *E*, and *FA* is added to it, the rectangle contained by *CF*, *FA* together with the square on *AE* is equal to the square on *EF*. (II. 6.)

"But *EF* is equal to *EB*; therefore the rectangle *CF*, *FA* together with the square on *AE* is equal to the square on *EB*.



"But the squares on *BA*, *AE* are equal to the square on *EB*, for the angle at *A* is right (I. 47); therefore the rectangle *CF*, *FA* together with the square on *AE* is equal to the squares on *BA*, *AE*.

"Let the square on *AE* be subtracted from each; therefore the rectangle *CF*, *FA* which remains is equal to the square on *AB*.

"Now the rectangle *CF*, *FA* is *FK*, for *AF* is equal to *FG*; and the square on *AB* is *AD*; therefore *FK* is equal to *AD*.

"Let *AK* be subtracted from each; therefore *FH* which remains is equal to *HD*.

"And *HD* is the rectangle *AB*, *BH*, for *AB* is equal to *BD*; and *FH* is the square on *AH*; therefore the rectangle contained by *AB*, *BH* is equal to the square on *HA*.

"Therefore the given straight line *AB* has been cut at *H* so as to make the rectangle contained by *AB*, *BH* equal to the square on *HA*. Q. E. F."

The ordinary algebraical solution of the corresponding equation

$$x^2 + ax = a^2, \qquad \text{from } a(a-x) = x^2,$$

parallels this geometrical demonstration.

To complete the square in the left-hand member, $\frac{a^2}{4}$ is added to both members. This corresponds to marking the point *E* on the figure, for then the square on *BE* equals $a^2 + \frac{a^2}{4}$ or $AB^2 + AE^2$.

Extracting the square root of both members, we have, algebraically,

$$x + \frac{a}{2} = +\sqrt{\frac{5\,a^2}{4}},$$

the negative sign being disregarded. The right-hand member corresponds to the line *BE* and the left-hand member to *EF*, which is equal to *BE*.

Algebraically we proceed by subtracting $\frac{a}{2}$ from both members, giving

$$x=\sqrt{\frac{5\,a^2}{4}}-\frac{a}{2}.$$

This corresponds to the line AF in the figure which is $BE-AE$.

Analytical solution of the quadratic equation appears quite definitely in the works of Heron of Alexandria, who flourished about the beginning of the Christian Era. Heron states in effect that given the sum of two line segments and their product then each of the segments is known.[1] However, he goes farther than any work of Euclid in applying this to a numerical example,

$$144\,x(14-x)=6720.$$

Without putting this into the form of an equation, Heron states that the approximate value of x is $8\frac{1}{2}$, and this evidently indicates an analytical solution. The geometrical garb is absolutely discarded in a problem in the *Geometry* doubtfully attributed to Heron.[2] The problem is to compute the diameter of a circle given the sum of the area, the circumference, and the diameter, summing an area and lengths, entirely contrary to Greek usage. The form of the result, practically

$$x=\frac{\sqrt{(154\times 212+841)}-29}{11},$$

indicates that the equation

$$\tfrac{11}{14}x^2+\tfrac{29}{7}x=212$$

was put in the form

$$121\,x^2+638\,x=(212)(154).$$

Somewhat similar problems[3] in which lines and areas are summed appear in Greece in the period between Heron and Diophantus (about 250 A.D.) as well as in the works of the latter. One of these problems is to find a square whose area and perimeter together equal 896 ($x^2+4\,x=896$). The solution proceeds in the ordinary manner by adding to 896 the square of half the coefficient of x and then taking the square root of this sum. From this is subtracted one-half the coefficient of x, giving the side 28. Four other problems of this series deal with right triangles having rational sides and hypotenuse, in which the sum of the area and perimeter is to equal a given number. If a, b are the sides, c the hypotenuse, S the area, r radius of the inscribed circle, and $s=\frac{1}{2}(a+b+c)$, then the solution depends upon the following formulæ:

---

[1] Heron, *Metrica*, ed. Schöne (Leipzig, 1903), pp. 148–151.

[2] Cantor, I (3), p. 405; Heron, *Geometria*, ed. Hultsch (Berlin, 1864), p. 133; *Heronis Opera*, ed. Heiberg, Vol. IV, *Geometria*, p. 381; Heath, *Diophantus*, pp. 63–64.

[3] Heiberg and Zeuthen, *Ueber einige Aufgaben der unbestimmten Analytik*, *Bibliotheca Mathematica*, Vol. VIII, third series, pp. 118–134. The date is conjectural. See Heath, *Diophantus*, pp. 118–121.

$$S=rs=\tfrac{1}{2}ab,\ r+s=a+b,\ c=s-r.$$

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = \frac{r+s\pm\sqrt{(r+s)^2-8rs}.}{2}$$

In the sixth book of the *Arithmetica* Diophantus treats rational right triangles in which the area plus or minus one side is a given number, or the area plus or minus the sum of two sides or one side and the hypotenuse, is a given number. Again, such a problem appears in an algebraic work by Shojā ben Aslam, Abū Kāmil, an Arabic writer of the tenth century.[1]

In respect to analysis Diophantus is the greatest name among the Greeks. Recently it has been established that he flourished in the third century of the Christian Era, when Greek supremacy in mathematics was waning. No doubt whatever exists that this Alexandrian was familiar with the analytical solution of the various forms of quadratic equations, neglecting negative roots and, of course, imaginary roots, which did not receive serious treatment for more than a millennium after Diophantus. The three types of complete quadratic equations, involving only positive coefficients, are the following:

$$ax^2+bx=c,$$
$$ax^2+c=bx,$$
$$ax^2=bx+c.$$

All three types appear in the *Arithmetica* of Diophantus, not systematically treated but solved as incidental to the solution of other problems. In fact, after dealing with the solution of equations of the form

$$ax^m=bx^n,$$

---

[1] Suter, *Die Abhandlung des Abū Kāmil Shoǵā' b. Aslam "über das Fünfeck und Zehneck,"* *Bibliotheca Mathematica*, Vol. X, third series (1910–11), pp. 15–42.

Diophantus makes the following explicit statement regarding his intention of writing a systematic treatise on the quadratic equation:[1]

> "This should be the object aimed at in framing the hypotheses of propositions, that is to say, to reduce the equations, if possible, until one term is left equal to one term; but I will show you later how, in the case also where two terms are left equal to one term, such a problem is solved."

So far as we know this promise was never fulfilled.

An equation of the first type is presented by the sixth problem of the sixth book, and this we reproduce from Heath:[2]

> "6. To find a right-angled triangle such that the area added to one of the perpendiculars makes a given number.
>
> "Given number 7, triangle $(3x, 4x, 5x)$.
>
> "Therefore $6x^2 + 3x = 7$."
>
> "In order that this might be solved it would be necessary that (half coefficient of $x)^2$ + product of coefficient of x^2 and absolute term should be a square: but $(1\frac{1}{2})^2 + 6 \cdot 7$ is not a square. Hence we must find, to replace (3, 4, 5), a right-angled triangle such that
>
> "($\frac{1}{2}$ one perpendicular)^2 + 7 times area = a square;"

and the subsequent work leads to the equation, $84x^2 + 7x = 7$, $x = \frac{1}{4}$; and the solution $(6, \frac{7}{4}, \frac{25}{4})$.

In the following problem (VI. 7) the value of x is given as $\frac{1}{3}$ for the equation

$$84x^2 - 7x = 7,$$

which equation is of the third type when the negative term is transposed after the manner of Diophantus. The equations

$$630x^2 + 73x = 6, \qquad x = \tfrac{1}{18},$$
$$630x^2 - 73x = 6, \qquad x = \tfrac{6}{35},$$
$$630x^2 + 81x = 4, \qquad x = \tfrac{4}{105},$$

and

$$630x^2 - 81x = 4, \qquad x = \tfrac{1}{6},$$

occur in the next four problems (VI. 8–11). Another problem of the third kind (IV. 39) is of especial interest because the rule is given for solving this type:[3]

> "To find three numbers such that the difference of the greatest and the middle has to the difference of the middle and the least a given ratio, and also the sum of any two is a square."

---

[1] Heath, *Diophantus*, p. 131.

[2] Heath, *Diophantus*, pp. 228–229.

[3] Heath, *Diophantus*, pp. 197–198.

The discussion leads to the inequality, $2m^2 > 6m + 18$, which, since only integral solutions are desired, explains the use of 7 as the approximate square root of 45, in the following paragraph:

> "When we solve such an equation, we multiply half the coefficient of x into itself,—this gives 9,—then multiply the coefficient of x^2 into the units,—$2 \cdot 18 = 36$,—add this last number to the 9, making 45, and take the side [square root] of 45, which is not less than 7; add half the coefficient of x,—making a number not less than 10,—and divide the result by the coefficient of x^2; the result is not less than 5."

Of the second type, $ax^2 + c = bx$, Diophantus gives several illustrations, requiring frequently only the approximate value of the root. Problems of this kind are the following:

$72m > 17m^2 + 17$, m not greater than $\frac{67}{17}$, (V. 10)
$19m^2 + 19 > 72m$, m not less than $\frac{66}{19}$, (V. 10)
$m^2 + 60 > 22m$, m not less than 19, (V. 30)
$m^2 + 60 < 24m$, m not greater than 21, (V. 30)
and $172x = 336x^2 + 24$, (VI. 22)

of which the statement is made that the root is not rational, and in the same problem

$$78848x^2 - 8432x + 225 = 0,$$

which has the rational root $\frac{25}{448}$.

Commentaries on the *Arithmetica* began to appear very early. Probably the most interesting commentary from the modern point of view was the one written in the late fourth or early fifth century by Hypatia, the daughter of Theon of Alexandria. Unfortunately her writings are all lost, although there is ground for the belief[1] that some remarks made by Michael Psellus (eleventh century) concerning Egyptian arithmetic and algebra were based on her commentary. She came naturally by her mathematical ability; her father Theon wrote a commentary on Ptolemy's Almagest and makes in this the earliest known reference to Diophantus.

Cossali,[2] writing in 1797 on the history of algebra, conjectures that the step from the geometrical to the analytical form of solution took place in the period between Euclid and Diophantus.

---

[1] *Diophantus*, ed. P. Tannery (Leipzig, 1893), Vol. II, pp. 37–38; Heath, *Diophantus*, pp. 18, 41.

[2] *Origine, trasporto in Italia, primi progressi in essa dell' algebra* (Parma, 1797), Vol. I, pp. 87–91.

Now the Arabic Book of Chronicles[1] (987 A.D.) states that the astronomer Hipparchus (second century B.C.) wrote a treatise on algebra, and Cantor[2] inclines to the belief that there actually was such a work. No trace, however, has been found of it, and the probability is that Hipparchus did not write any systematic treatise on algebra or on quadratic equations. The word "algebra" indeed is Arabic in its origin and the use of it as a title goes back only to the time of our author, Mohammed ibn Musa. Nevertheless it is possible that Greek mathematicians of the time of Hipparchus did occupy themselves with problems of the kind in question, because this was a natural development out of the consideration of rational right triangles, as given by Pythagoras and Plato, in connection with the geometrical treatment of quadratic equations as given by Euclid. Quadratic equations connect even more directly with the application of areas, of Pythagorean origin, which is extensively treated by Euclid.

Some two centuries after the period of Diophantus, Āryabhaṭa, one of the earliest Hindu mathematicians of prominence, was born (476 A.D.). In the work of Āryabhaṭa as presented by Rodet[3] we find the solution of a quadratic equation assumed in the rule for finding the number of terms of an arithmetic series when the sum, difference, and first term are given. Nor does Āryabhata in India stand alone in the study of analysis, as Diophantus does in Greece. Brahmagupta of Ujjain, the centre of Indian learning, wrote on algebra in the early part of the seventh century and gave a rule[4] for the solution of quadratic equations:

> "To the absolute number multiplied by the [coefficient of the] square, add the square of half the [coefficient of the] unknown, the square root of the sum, less half the [coefficient of the] unknown, being divided by the [coefficient of the] square, is the unknown."

In formula this corresponds to the solution

$$x=\frac{\sqrt{(b/2)^2+ac}-b/2}{a}$$

---

[1] *Das Mathematiker-Verzeichniss im Fihrist des Ibn Abi Ja'kub an-Nadim*, translated by H. Suter, *Abhandl. z. Geschichte der Mathematik*, Vol. 6 (Leipzig, 1892), p. 22, and note, pp. 54–55. Suter holds that there is some error in the text, and this seems probable.

[2] *Geschichte*, Vol. I (3), pp. 362–363.

[3] *Leçons de Calcul d'Aryabhata, Journal Asiatique*, seventh series, Vol. XIII (1879), pp. 393–434.

[4] Colebrooke, *Algebra, with Arithmetic and Mensuration, from the Sanskrit of Brahmegupta and Bháscara* (London, 1817), p. 347; Cantor, I (3), p. 625.

of the equation,

$$ax^2 + bx = c.$$

Contemporary with Al-Khowarizmi is the Hindu writer Mahaviracarya, whose arithmetical and algebraical work has been translated into English by M. Raṅgacarya.[1] Rules are given in the sections devoted to algebra for the three types of complete quadratic equations. A peculiarity of the treatment is that the unknown quantity and the square root of the unknown appear, rather than the unknown and its square. The significance of the work is that it shows a persistence of interest in algebra in India from the time of Āryabhaṭa. Three centuries later Bhaskara (b. 1114 A.D.), another Hindu mathematician, made important contributions to the advance of the science.

The brief survey which we have given of the study of algebra before the time of Mohammed ibn Musa does not at all purpose to present the sources from which the great Arab drew his inspiration. Greece undoubtedly took mathematical ideas from Egypt, as Rodet[2] some years ago pointed out with reference to algebra. Even more definite evidence is presented by the Greek use of unit fractions as well as by the references to Egyptian mathematics which were made by Plato and Herodotus, and much later by Michael Psellus. Babylon and Greece were constantly exchanging ideas;[3] a striking proof of this is the Greek use of sexagesimal fractions. India, too, was not out of touch with these neighbors to her west. Especially in the fields of religion and the closely associated astrology we have abundant evidence not only of interchange of ideas between the East and the West but also of the recurrence in mediæval times of ideas advanced by more ancient civilizations. Yet we need to notice that we are dealing with the independent appearances of algebraic ideas, and that the mathematics of Egypt, Babylon, China, Greece, and India was developing from within. Algebra is not, as often assumed, an artificial effort of human ingenuity, but rather the natural expression of man's interest in the numerical side of the universe of thought. Tables of square and cubic numbers in Babylon; geometric progressions, involving the idea of powers, together with linear and quadratic equations in Egypt; the so-called Pythagorean theorem in India, and possibly in China, before the time of Pythagoras; and the geometrical solution of quadratic equations even before Euclid in Greece, are not isolated facts of the history of mathematics. While they do indeed mark stages in the development of pure mathematics, this is only a small part of their significance. More vital is the implication that the algebraical side of mathematics has an intrinsic interest for the human mind not conditioned upon time or place, but dependent simply upon the development of the reasoning faculty. We may say that the study of powers of numbers, and the related study of quadratic equations, were an evolution out of a natural interest in numbers; the facts which we have presented are traces of the process of this evolution.

---

[1] M. Raṅgācārya, *The Gaṇita-Sāra-Sangraha of Mahāvīrācārya* (Madras, government press, 1912). Professor D. E. Smith gave a brief preliminary report of the work in the *Bibliotheca Mathematica*, Vol. IX, third series, pp. 106–110.

[2] L. Rodet, *Sur les notations numériques et algébriques antérieurement au XVI^e siècle* (Paris, 1881), pp. 43–51.

[3] F. Cumont, *Babylon und die griechische Astronomie*, *Neue Jahrbücher f. das klassische Altertum . . .*, Vol. 27 (1911), pp. 1–10; *The Oriental Religions in Roman Paganism* (Chicago, 1911); and *Astrology and Religion among the Greeks and Romans* (New York, 1912).

CHAPTER II

AL-KHOWARIZMI AND HIS TREATISE ON ALGEBRA

THE activity of the great Arabic mathematician Abu ʿAbdallah Mohammed ibn Musa al-Khowarizmi marks the beginning of that period of mathematical history in which analysis assumed a place on a level with geometry; and his algebra gave a definite form to the ideas which we have been setting forth. The arithmetic of Al-Khowarizmi made known to the Arabs and, through an early twelfth-century translation, to Europeans also, the Hindu art of reckoning. Any consideration of the difficulties attending arithmetical operations with the Greek letter numerals,[1] or even with the Roman numerals, shows how essential the adoption of numerals with place value was for the development of the analytical side of mathematics; the way was being prepared also for the appearance of decimal fractions, many centuries later, and for logarithms, both indispensable tools of modern science. Quite as important as the arithmetic for the development of mathematics was the systematic treatise on algebra[2] which Mohammed ibn Musa gave to the world. This is the work of which we present the Latin translation made by Robert of Chester while living in Segovia in 1183 of the Spanish Era (1145 A.D.).[3]

Our chief source of information in regard to the life and the writings of our Arabic author is the *Book of Chronicles*[4] (*Kitab*

---

[1] Nine letters represent the units from 1 to 9, nine further letters the tens from 10 to 90, and nine others the hundreds. The thousands up to 900,000 are represented by the same letters with a kind of accent mark.

[2] The Arabic text as given in the MS. Hunt, 214, of the Bodleian library, a unique copy, was published with English translation by Frederic Rosen, *The Algebra of Mohammed ben Musa* (London, 1831), being one of the volumes published for the Oriental Translation Fund; a French translation of the chapter on Mensuration was published by Marre, based on Rosen's Arabic text, *Nouvelles Annales de Mathématiques*, Vol. 5 (1846), pp. 557–581, and also later revised in *Annali di Matemat.*, Vol. VII (1866), pp. 268–280.

[3] A preliminary note by the author concerning this translation appeared in the *Bibliotheca Mathematica*, third series, Vol. XI (1911), pp. 125–131, with the title, *Robert of Chester's Translation of the Algebra of Al-Khowarizmi.*

[4] The *Kitab al-Fihrist* was edited with notes by G. Flügel and published after Professor Flügel's death by J. Roediger and A. Mueller (Leipzig, 1871–1872). I quote from the German translation by H. Suter, *Das Mathematiker-Verzeichniss im Fihrist*, *Abhandlungen zur Geschichte der Mathematik*, Vol. VI, Leipzig, 1892.

al-Fiḥrist) by Ibn Abi Yaʿqub al-Nadim. This work, which was completed about 987 A.D., gives biographies of learned men of all nations, together with lists of such of their works as were known to Al-Nadim. We quote the passage relating to our author, regarding whose life and activity we have only meagre information.

> "AL-KHOWARIZMI"
>
> "Mohammed ibn Musa, born in Khowarizm (modern Khiva), worked in the library of the caliphs under Al-Mamun. During his lifetime, and afterward, where observations were made, people were accustomed to rely upon his tables, which were known by the name *Sind-Hind* (Hindu *Siddhanta*). He wrote: *The book of astronomical tables* in two editions, the first and the second; *On the sun-dial; On the use of the astrolabe; On the construction of the astrolabe; The Book of Chronology.*"

Neither the date of the birth nor the date of the death of Al-Khowarizmi has been definitely established. However, the fact, mentioned by Al-Nadim in the *Fihrist*, that he worked in the library of the caliph Al-Mamun, who reigned from 813 to 833 A.D., indicates the period of his literary activity. The introduction to the algebra, which is not found in the extant Latin translations, is here given in English (p. 45), following Rosen. This brings further evidence of the acquaintance with Al-Mamun, for Al-Khowarizmi states that the interest of the caliph in science encouraged him to write the treatise. The probability is that early in the reign of Al-Mamun our author began work upon the Hindu astronomical tables which, as the Fihrist account implies, brought him almost immediate fame. This stimulated him to undertake the work upon algebra, and the success of the second work induced him to write the treatise on arithmetic in which reference is made to the algebra. The height of his literary activity may reasonably be placed about 825 A.D.

The bibliography of Al-Nadim does not include four works from the hand of Al-Khowarizmi which have come to us. These are his arithmetic, his algebra, a work on the quadrivium, and an adaptation of Ptolemy's geography. To Sened ibn ʿAli, the Jew, whose biography immediately follows that of Mohammed ibn Musa, are ascribed works entitled, *On Increasing and Decreasing* (algebraical), *The Book of Algebra*, and *On the Hindu Art of Reckoning*. The probability is, as Suter points out,[1] that an interchange has taken place here, although this must have been relatively early since Ibn al-Qifti,[1] who died in 1248 A.D., in his *Chronicles of the Learned*, gives the same account of Al-Khowarizmi as Al-Nadim. Furthermore, the author of the Fihrist knew of the algebra, for he mentions no less than three men as commentators on the algebra of Mohammed ibn Musa; Sinan ibn al-Fath of Harran, ʿAbdallah ibn al-Hasan al-Saidanani, and Abu'l-Wefa al-Buzjani are credited with such commentaries.[2]

---

[1] *Fihrist, loc. cit.*, pp. 62–63.

The arithmetic of Al-Khowarizmi has come down to us only in a Latin translation, and this survives in a unique copy belonging to the library of the University of Cambridge, published in 1857 by Prince Baldassare Boncompagni.[3] Several references[4] in this work to the writer's other book on arithmetic make it evident that the Al-Khowarizmi in question is the author of the algebra. Our word *algorism*, as well as the obsolete form *augrim*,[5] used by

---

[1] Suter, *loc. cit.*, pp. 62–63; Casiri, *Bibliotheca Arabico-Hispana Escurialensis* (Madrid, 1760–1770), I, pp. 427–428. Ibn al-Qifti mentions the arithmetic.

[2] Suter, *loc. cit.*, pp. 37, 36, and 39 respectively.

[3] *Trattati d' aritmetica* (Rome, 1857), I. *Algoritmi de numero indorum*, pp. 1–23, which is the arithmetic in question, and bears internal evidence that it is a translation from the Arabic; II. *Joannis Hispalensis liber algorismi de pratica arismetrice*, evidently an adaptation and enlargement of the preceding. Some other manuscripts of the second treatise are anonymous, while others ascribe it to Gerard of Cremona.

[4] One passage is in *Trattati*, I, pp. 2–3; Etiam patefeci in libro algebre et almucabalah, idest restaurationis et oppositionis, quod uniuersus numerus sit compositus, et quod uniuersus numerus componatur super unum. Vnum ergo inuenitur in uniuerso numero; et hoc est quod in alio libro arithmetice dicitur. Quia unum est radix uniuersi numeri, et est extra numerum. Radix numeri est, quare per eum inuenitur omnis numerus. . . . Reliquus autem numerus sine uno inueniri non potest. . . . Reliquus autem numerus indiget . . . uno: quare non potes dicere duo uel tria, nisi precedat unum. Nichil aliud est ergo numerus, nisi unitatum collectio. . . .

Inueni, inquit algorizmi, omne quod potest dici ex numero, et esse quicquid excedit unum usque in IX., id est quod est inter. IX. et unum, id est duplicatur unum et fiunt duo; et triplicatur idem unum, fiuntque tria, et sic in ceteris usque in IX. De inde ponuntur, X. in loco unius, et duplicantur, X. ac triplicantur, quemadmodum factum est de uno, fiuntque ex eorum duplicatione, XX, et triplicatione, XXX, et ita usque ad XC. Post hec redeunt C in loco unius, et duplicantur ibi atque triplicantur, quemadmodum factum est de uno et X; efficiunturque ex eis CC, et CCC, et cetera usque in DCCCC. Rursum ponuntur mille in loco unius; et duplicando et triplicando, ut diximus, fiunt ex eis II mila, et III et cetera usque in infinitum numerum, secundum hunc modum.

Compare with this our text, p. 66, lines 10–21.

Another passage is *Trattati*, I, p. 10. Etiam patefeci in libro, quod necesse est omni numero qui multiplicatur in aliquo quolibet numero, ut duplicetur unus ex eis secundum unitates alterius.

Compare with this our text, p. 90, lines 6–8.

[5] See my paper on *Augrim-stones*, *Modern Language Notes*, Vol. XXVII (1912), pp. 206–209; compare also the use of the term *Algaurizin* in our text, p. 102.

Chaucer, is derived from the use of the name Al-Khowarizmi in the opening sentence of the arithmetic, which reads, *Dixit algoritmi* or 'Algorithm says'; the word 'algebra' is derived from its use as a title by Al-Khowarizmi in the work which we are presenting.[1] Up to the eighteenth century the common name for the new arithmetic with the ten figures of India, 1 2 3 4 5 6 7 8 9 0, was algorism, or in Latin *algorismus*. Interesting also is the fact that a Spanish transliteration of Khowarizm, *guarismo*, is used for 'numerals,' corresponding to our ordinary use of 'ciphers.' Aside from Al-Khowarizmi's arithmetic in Latin translation, which was never widely used, the works which served to introduce the numerals into Europe were the *Carmen de Algorismo*,[2] in verse, written by Alexander de Villa Dei (about 1220), and the *Algorismus vulgaris*[3] by John of Halifax, commonly known as Sacrobosco (about 1250). Both of these works were somewhat dependent upon Mohammed ibn Musa's arithmetic, and both continued in wide use for centuries. Many manuscript copies of the *Carmen* are found in European libraries, and rather more of the *Algorismus vulgaris*. Even after the invention of printing Sacrobosco's Algorism was widely used for university instruction in arithmetic, many editions appearing in the fifteenth and sixteenth centuries.[4] Detailed and extended commentary upon the work was given by Petrus de Dacia in 1291, in his lectures, evidently, and probably in a similar manner by many other university lecturers.

Another arithmetical treatise ascribed to Al-Khowarizmi is found in the *Liber ysagogarum Alchorismi in artem astronomicam a magistro A. compositus*.[5] The principles of arithmetic, geometry, music, and astronomy are explained in five books, or chapters, and in two manuscripts there follow three books on astronomy. Three of the books which deal particularly with arithmetic have been published,[1] but no study has yet been made of the three books on astronomy. The writer A. is supposed to be Adelard of Bath, who was active at the time that this work was written; he translated the tables of Al-Khowarizmi. So far as these five books of the introduction are concerned, the work may well be a summary of the elementary teachings of Al-Khowarizmi according to the unknown writer's conception of them.

---

[1] See my note on *Algebra*, *Modern Language Notes*, Vol. XXVIII (1913), p. 93; compare also the use of the term in our text, p. 2.

[2] Published by J. O. Halliwell, *Rara Mathematica* (London, 1839).

[3] There were many early editions; see Curtze, *Petri Philomeni de Dacia in Algorismum vulgarem Johannis de Sacrobosco commentarius, una cum Algorismo ipso* (Ed. M. Curtze, Copenhagen, 1897.)

[4] See Smith, *Rara Arithmetica* (Boston, 1908), pp. 31–33; see also my article, *Jordanus Nemorarius and John of Halifax*, *American Mathematical Monthly*, Vol. XVII (1910), pp. 108–113.

[5] Nagl, in *Zeitschrift f. Math. u. Physik, Hist.-litt. Abth.*, XXXIV, pp. 129–146, 161–170; the first three books complete, and summary of the other two, by Curtze, *Ueber eine Algorismus-schrift des XII. Jahrhunderts* in *Abhandl. z. Gesch. d. math. Wissen.*, VIII, pp. 1–27; Haskins, in *The English Historical Review*, XXVI, p. 494.

Al-Mas'udi (885–956 A.D.) in his *Meadows of Gold*[2] mentions Mohammed ibn Musa among the historians and chroniclers, basing his reference doubtless on the *Book of Chronology* above mentioned. Al-Biruni (973–1048 A.D.), whose work on India[3] has recently (1910) appeared in a second English edition, refers to the tables and the astronomical work of our author. No less than three works by Al-Biruni[4] are explanatory of works written by the distinguished mathematician and astronomer who was his fellow-countryman, both being from Khowarizm (or Khiva). Not only in the fields of astronomy, chronology, and mathematics did Mohammed ibn Musa achieve fame, but also as a geographer. His contribution in this field has been set forth by Nallino, who states, by way of conclusion to his article on *Al-Khowarizmi and his reconstruction of the geography of Ptolemy*,[5] that this geography is not a servile imitation of the Greek model, but an elaboration of Ptolemaic material made with more independence and ability than is displayed by any European writer of that period. The trigonometric tables, in Latin translation by Adelard of Bath, appear to be the earliest of Al-Khowarizmi's works to be known in Europe, and indeed one of the earliest mathematical treatises taken from the Arabic; it was translated in 1126 A.D. A study of these tables, with extracts from the Latin text, was made by A. A. Björnbo[1] and the completed work, edited by Suter, has recently been published. Suter has found[2] that this translation is not of the original work by Al-Khowarizmi, but is based on a revision of that work made by Maslama al-Majriti (about 1000 A.D.). The work is of considerable importance for the history of trigonometry, for even though the tangent function which appears therein may be the addition of Maslama, yet the introduction of the sine function is certainly carried back as far as Al-Khowarizmi.

---

[1] Curtze, *loc. cit.;* Haskins, *loc. cit.*, gives the incipit and explicit of the astronomical work: Quoniam cuiusque actionis quantitatem temporis spacium metitur, celestium motuum doctrinam querentibus eius primum ratio occurrit investiganda. . . . Divide quoque arcum diei per 12 et quod fuerit erunt partes horarum eius, si deus inveniri consenserit.

[2] *Les prairies d'or*, translated by Barbier de Meynard and Pavet de Courteille, I–IX (Paris, 1861–1877), I, 11.

[3] *Alberuni's India*, translation by C. E. Sachau (London, 1888, and second edition, London, 1910).

[4] Suter, *Der Verfasser des Buches Gründe der Tafeln des Chowārezmī*, *Bibliotheca Mathematica*, third series, Vol. IV, 1903, pp. 127–129.

[5] *Al-Ḫuwarizmi e il suo rifacimento della Geografia di Tolomeo*, *Atti della R. Accademia dei Lincei*, fifth series, *Memorie*, *Classe di scienze morali, storiche e filologiche*, Vol. II (1896), pp. 11–53.

The combination of mathematical, geographical, and astronomical interests exhibited by Al-Khowarizmi renders plausible the hypothesis advanced by Suter,[3] on chronological grounds, that this Mohammed ibn Musa took part in the measurement of a degree of the earth's circumference which was made at the request of the caliph Al-Mamun. Some early Arabic chroniclers join the three sons of Moses, the Beni Musa, in this task. The oldest of these brothers was Mohammed and so he was called after the Arabic custom, Mohammed ibn Musa, meaning Mohammed, the son of Moses. Abu Ja'far is the prefix to his name, and this has been incorrectly given by numerous modern dictionaries[4] as the name of our author.

Concerning Mohammed ibn Musa's fame among the Arabs as an algebraist abundant evidence exists. Not only the commentaries cited bear witness to this fame but also the recurrent appearance for centuries of the numerical examples, $x^2 + 10x = 39$, $x^2 + 21 = 10x$, $3x + 4 = x^2$ and many others, which Al-Khowarizmi used. Some of the later authors, like Abu Kamil Shoja' ibn Aslam[5] (about 925 A.D.), explicitly acknowledge their indebtedness to our author. Others, like the poet Omar ibn Ibrahim al-Khayyami (about 1045–1123 A.D.), familiar to English readers as Omar Khayyam, and Mohammed ibn al-Ḥasan al-Karkhi (died about 1029), do not consider it necessary to state the source of these problems and the proofs, which had become classic. Thus the equation

---

[1] Björnbo, *Al-Chwarizmi's trigonometriske Tavler*, in *Festskrift til H. G. Zeuthen*, Copenhagen (1909), pp. 1–26; the sudden death of this brilliant student of mediæval mathematics is a great loss, for his systematic studies in this field were beginning to bear fruit in numerous and important publications.

[2] Suter, *Mémoires de l'Acad. Royale des Sciences et des Lettres de Danemark*, Series 7 (Lettres), Vol. 3, pt. 1 (Copenhagen, 1914). The dependence upon Maslama's version was noted by C. H. Haskins, *Adelard of Bath*, *The English Historical Review*, Vol. XXVI (1911), pp. 491–498.

[3] Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke*, in *Abhandl. z. Gesch. d. math. Wissenschaften*, Vol. X (Leipzig, 1900), p. 20.

[4] *Century*, *Webster's*, *The New English Dictionary*, *Encyclopædia Britannica*.

[5] L. C. Karpinski, *The algebra of Abu Kamil Shoja' ben Aslam*, *Bibliotheca Mathematica*, Vol. XII, third series (1912), pp. 40–55; *The Algebra of Abu Kamil*, *American Mathematical Monthly*, Vol. XXI (1914), pp. 37–48.

$$x^2 + 10x = 39$$

runs like a thread of gold through the algebras for several centuries, appearing in the algebras of the three writers mentioned, Abu Kamil, Al-Karkhi[1] and ʿOmar al-Khayyami,[2] and frequently in the works of Christian writers, centuries later, as we shall have occasion to note below.

Recognition of the fame of Al-Khowarizmi is to be found in the explicit statement by Ibn Khaldun (1332–1406) in his encylopædic work[3]: "The first who wrote upon this branch (algebra) was Abu ʿAbdallah al-Khowarizmi, after whom came Abu Kamil Shojaʿ ibn Aslam." Haji Khalfa, some two hundred years later, makes a similar statement. The *Chronicles of the Learned* by Ibn al-Qifti (d. 1282 A.D.) speak very highly of his ability as an arithmetician,[4] and a contemporary of Al-Qifti, Zakariya ibn Moh. ibn Mahmud al-Qazwini, refers to him as translating the art of algebra for the Mohammedans.[5]

I have recently made a study[6] of the algebra of Abu Kamil, whose name we have seen associated by Arabic historians with that of Al-Khowarizmi. I have shown that he drew extensively upon the work of his predecessor, and further that Leonard of Pisa (1202 A.D.) drew, in turn, even more extensively from Abu Kamil. Thus the influence of the Khowarizmian was carried over to Italy as well as to the Arabic commentators and the European translators of Abu Kamil's algebra. Two of these commentaries on Abu Kamil's work appeared in the tenth century and are listed in the Fihrist. The Persian Al-Istakhri,[1] known as The Reckoner, was the author of one, and ʿAli ibn Ahmed al-ʿImrani[2] wrote the other. Neither commentary has come down to us. The author of the Latin translation upon which my study is based is not known, but the probability is that the translation was made about the time of Gerard of Cremona. Another commentary of uncertain date in Arabic by a Mohammedan Spaniard, Al-Khoreshi, is probably of later origin. In the fifteenth century a Hebrew translation was prepared by Mordechai Finzi of Mantua (about 1475); this is of especial interest since it bears internal evidence, in the terminology employed, that it was based upon a Spanish original. The author of the Spanish treatise, doubtless a Christian, is unknown. Ibn Khaldun mentions the commentary written by Al-Khoreshi, but no trace has yet been found of either Spanish translation or commentary. A unique copy of the Latin translation of the algebra is in Paris (Mss. Lat. 7377 A) and copies of the Hebrew translation are found in Paris and Munich.[3]

---

[1] F. Woepcke, *Extrait du Fakhri* (Paris, 1853); A. Hochheim, *Die Arithmetik des Abu Bekr Muhammed ben Alhusein Alkarkhi* (Programm, Magdeburg, 1878), and *Kāfī fīl Hisāb* (*Genügendes über Arithmetik*) (Halle, 1878–1880). The *Kāfi fīl Hisāb* is an earlier work by Al-Karkhi which includes a treatment of algebra.

[2] F. Woepcke, *L'algèbre d'Omar Alkhayyāmī* (Paris, 1851).

[3] MacGuckin de Slane, *Prolegomènes historiques d'Ibn Khaldoun*, *Notices et Extraits des Manuscrits de la Bibliothèque Imperiale et autres Bibliothèques* (Text, Vol. XIX and XX, Translation, Vol. XXI, 1868), Vol. XXI, p. 136.

[4] Casiri, *Bibliotheca Arabico-Hispana Escurialensis*, p. 427.

[5] Casiri, *loc. cit.*, p. 427; Cossali, *Origine, trasporto in Italia, primi progressi in essa dell' algebra* (Parma, 1797), Vol. I, p. 178.

[6] *The Algebra of Abu Kamil*, *loc. cit.*

The widespread interest among Arabic scientists in the study of algebra is attested by the number of works upon the subject. Even as early as the tenth century besides Al-Khowarizmi there were three other writers[4] of sufficient prominence to warrant their appearance in the Fihrist. Abu ʿOtman Sahl ibn Bishr. ibn Habib ibn Hani, a Jew, wrote an algebra which Al-Nadim states was praised by the Romans, and Aḥmed ibn Dā'ūd, Abū Ḥanifa, al-Dīnawarī (d. 895 A.D.) and Abu Yusuf al-Missisi also wrote treatises on the subject. These works are preserved only in title. In the thirteenth century Ibn al-Banna,[5] including in his arithmetic a brief exposition of algebra, employs the title, "algebra and almucabala," as given by Al-Khowarizmi, and follows the same peculiar order of the six types of quadratic equations. Al-Banna adds no numerical illustrations but states simple general rules for the solution of equations. A separate work on the same subject by Ibn al-Banna is included among the Arabic manuscripts in Cairo. In modern times Arabs have used lithograph copies of an arithmetic and algebra by Mohammed ibn al-Hosein, Behā al-dīn al-ʿĀmilī (Behā ed-dīn), who died in 1622. Two editions[1] with a translation into the Persian language and commentary were published in the nineteenth century. Beha ed-din continues the use of some of Al-Khowarizmi's problems as well as the six types of quadratics.

---

[1] Suter, *Die Mathematiker und Astronomen der Araber*, p. 51.

[2] Suter, *loc. cit.*, pp. 56–57.

[3] Steinschneider, *Die Hebraeischen Uebersetzungen des Mittelalters* (Berlin, 1893), pp. 584–588; *Sitzungsber. d. Akad. d. Wissen.* (Vienna), *Phil.-hist. Klasse*, Vol. 149 (1905), p. 31.

[4] Suter, *Die Mathematiker*, pp. 15, 31, and 66 respectively.

[5] Suter, *Die Mathematiker*, pp. 162–163; A. Marre, *Le Talkhys d' Ibn Albanna, Atti dell' accad. pontif. dei nuovi Lincei*, Vol. XVII (1864), pp. 289–319.

Greek influence on Arabic geometry is revealed by the order of the letters employed on the geometrical figures. These letters follow the natural Greek order and not the Arabic order. The same is true of the order of the letters in the geometrical figures used by Al-Khowarizmi for verification of his solutions of quadratic equations. However, the somewhat ingenious hypothesis put forth by Cantor[2] that this fact shows that these demonstrations are from Greek sources is hardly tenable. The Arabs were much more familiar with and grounded in Euclid than are mathematicians to-day, and it was entirely natural in constructing new figures that they should follow the order of lettering to which they had grown accustomed in their study of Euclid. Until further and more definite historical evidence to the contrary is brought to light we must regard Al-Khowarizmi as the first to bring out sharply the parallelism between the analytical and geometrical solutions of quadratic equations.

The Arabic students of algebra included poets, philosophers and perhaps even kings. Omar Khayyam, whom we have mentioned among the writers on the subject, was too excellent a mathematician and too true a poet to woo the muse of algebra in verse. But in the library of the Escurial at Madrid there is preserved a poem treating of algebra, written by a native of Granada, Mohammed al-Qasim. Needless to say, the content, adapted to the exigencies of verse, does not compare with the prose of Omar Khayyam. The King of Saragossa, Jusuf al-Mutamin (reigned 1081–1085), was a devoted student of the mathematical sciences. The title of one of his works suggests the possibility that it was algebraical. For the study of law a knowledge of algebra seems to have been necessary, as various questions of inheritance were treated by this science, and even to-day in the great Mohammedan schools at Cairo and Mecca the study *aljebr w'al-muqabala* is considered of peculiar value to the prospective lawyer.

---

[1] Calcutta, 1812, and Constantinople, 1851–1852; Arabic with German translation by Nesselmann, Berlin, 1843; French translation, A. Marre, Paris, 1864.

[2] Cantor, I (3), pp. 724–725; Simon has noted the weakness of Cantor's argument in the article, *Zu Hwarizmi's hisāb al ǵabr wal muqābala*, *Archiv der Mathematik und Physik*, third series, XVIII (1911), pp. 202–203; see also Björnbo and Vogl, *Alkindi, Tideus und Pseudo-Euklid*, *Abhandl. z. Geschichte d. math. Wissen.*, XXXVI (Leipzig, 1912), p. 156, and the review of the same by A. Birkenmajer, *Bibliotheca Mathematica*, Vol. XIII, third series (1913), pp. 273–280.

CHAPTER III

ROBERT OF CHESTER AND OTHER TRANSLATORS OF ARABIC INTO LATIN

WHEN towards the beginning of the twelfth century European scholars turned to Islam for light, the works of Mohammed ibn Musa came to occupy a prominent place in their studies. One of the first of these students was styled John of Seville,[1] or of Luna, or of Spain, whose name is attached to some manuscript copies of an adaptation of Al-Khowarizmi's arithmetic. Thus, the version, mentioned above, published by Prince Boncompagni, bears the title, *Joannis Hispalensis liber Algorismi de pratica arismetrice*, and in a sub-head the editing of the arithmetic is ascribed to John of Spain. This work contains also a very brief treatment of algebra, entitled *Exceptiones de libro, qui dicitur gleba mutabilia*.[2] *Res* is employed for the square of the unknown, and *radix* for the unknown, the usage being, in this respect, unique. The problems,

$$x^2 + 10x = 39,$$
$$3x + 4 = x^2,$$

occur as in Al-Khowarizmi, with a variation in the second type,

$$x^2 + 9 = 6x.$$

The authorship is in question, since some manuscripts ascribe the work to Gerard of Cremona, some to John of Spain, and others are anonymous. However, no doubt now exists that John of Spain was familiar with the arithmetic of Al-Khowarizmi, for Dominicus Gundisallinus, co-laborer with John in translating from the Arabic, mentions the *Liber algorismi* in the chapter on arithmetic of the *De divisione philosophiae*.[3] About 1133 A.D. Bishop Raimund of Toledo commissioned John of Spain to work with Gundisallinus on translations from the Arabic. John made the translation into Spanish, and this was put into Latin by Gundisallinus. The probability is that the *De divisione philosophiae* is their joint production.[1] A somewhat similar method was pursued at first by Gerard of Cremona, collaborating with an Arab named Galippus or Ġalib.[2]

---

[1] Steinschneider, *Die Hebr. Uebers.*, p. 981, and note 82, p. 380.

[2] Unintelligible Latin forms for "algebra w' almucabala."

[3] Edited by L. Baur, *Beiträge z. Gesch. d. Philos. d. Mittelalters* (Vol. IV, Münster, 1903), p. 91.

Adelard of Bath translated the astronomical tables which we have mentioned and possibly another astronomical work by Al-Khowarizmi.[3] His life is typical of the life of learned men of that period. Although born in England, he evidently went to France at an early age. There he studied at Tours and delivered lectures in Laon. At least seven years of his life were spent in study and travel in the East. Tarsus, Antioch, and Salerno are mentioned by him as cities which he visited. While no direct evidence is known that he studied in Spain, yet many of his works are based on Arabic documents transmitted to Europe through the Spanish schools at Toledo and Segovia. Learning was quite as international in that time as to-day.

Gerard of Cremona, too, desiring to find the works of Ptolemy, journeyed to Spain and there took up the study of the Arabic language in order to understand the Arabic version of Ptolemy, with the result that he devoted his life to translations from the Arabic. Included in an early list[4] of his translations is the algebra of Al-Khowarizmi, and it seems probable that the Latin version published by Libri[5] is from his hand. However, Boncompagni in his discussion of the life and works of Gerard of Cremona has published another mediæval adaptation of the algebra which is ascribed to Gerard. The words *res* and *census* for the unknown and its square, and also the title *aliabre et almuchabala*, are used by Gerard in his translation of Ababucri's *Book of the measurement of the earth and of solids*, as yet in manuscript.[6] Plato of Tivoli was also doubtless familiar with Mohammed ibn

---

[1] Steinschneider, *Die Hebr. Uebers.*, p. 981 and note 82, p. 380.

[2] Valentin Rose, *Ptolemaeus und die Schule von Toledo, Hermes*, Vol. VIII, pp. 332 ff.

[3] C. H. Haskins, *Adelard of Bath, The English Historical Review*, Vol. XXXVI (1911), pp. 491–498, and *Adelard of Bath and Henry Plantagenet*, *ibid.*, Vol. XXXVIII (1913), pp. 515–516.

[4] Boncompagni, *Della vita e delle opere di Gherardo Cremonese*, etc., *Atti dell' Accademia pontificia de' nuovi Lincei*, Vol. IV (1851), pp. 378–493.

[5] Libri, *Histoire des sciences mathématiques en Italie*, Vol. I (Paris, 1838), pp. 253–297; Björnbo, *Gerhard von Cremona's Uebersetzung von Alkwarizmi's Algebra und von Euklid's Elementen, Bibliotheca Mathematica*, Vol. VI (1905), third series, pp. 239–248.

[6] My statements are based upon the Paris MS. Latin 9335 and the Cambridge University Library MS. Mm. 2, 18, both of which contain the work in question.

Musa's works, for he mentions him as one of the Arabic mathematicians.[1] Contemporary with these men was Robert of Chester.

The assertion was made by Curtze[2] that the *Liber embadorum*, *Book of Measures*, in Hebrew, by Abraham bar Chiyya Ha Nasi, known as Savasorda, was the first work to appear in Latin showing to the Western world how the solution of quadratic equations is accomplished. This statement is made on the basis of the date DX of the Hegira for the translation of the work made by Plato of Tivoli. It has recently been shown by Haskins,[3] on the basis of astronomical data in the work, that this date is undoubtedly a scribe's error for DXL, corresponding to 1145 A.D. Savasorda was approximately contemporary with his translator. Of the early Jewish writers many were familiar with the works of Al-Khowarizmi. Thus Abu Masar[4] cites the tables of our author while Abraham ben Esra[5] refers frequently to the same tables.

Three other Englishmen besides Adelard of Bath are known to have been students of Arabic mathematical science as taught in the schools of Spain in the twelfth century. The names and dates, as given by Wallis,[6] are: Adelard of Bath in 1130, Robertus Retinensis in 1140, William Shelley (de Conchis) in 1145, and Daniel Morley (Merlac) in 1180. This Robertus Retinensis[7] was also known as Robertus Ketenensis, de Ketene, Ostiensis,

---

[1] Favaro, *Intorno alla vita ed alle opere di Prosdocimo de' Beldomandi*, *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, Vol. XII (Rome, 1879), pp. 1–74, 115–251. See pp. 122–123.

[2] M. Curtze, *Der Liber Embadorum des Savasorda in der Uebersetzung des Plato von Tivoli*, *Abhandl. z. Geschichte d. math. Wissen.*, Vol. XII (1902), p. 7.

[3] C. H. Haskins, *The Romanic Review*, Vol. II (1911), p. 2, note 5; *The English Historical Review*, Vol. XXVI (1911), p. 491, note 1.

[4] Steinschneider, *Zum Speculum des Albertus Magnus*, *Zeitschrift f. Mathematik und Physik*, Vol. XVI (1871), p. 376.

[5] Steinschneider, *Zur Geschichte der Uebersetzungen aus dem Indischen in's Arabische und ihres Einflusses auf die arabische Literatur*, *Zeitschrift der deutschen morgenländischen Gesellschaft*, Vol. XXIV (1870), pp. 339, 355 *et al.*

[6] Wallis, *A Treatise on Algebra, both historical and practical* (London, 1685), pp. 10–12.

[7] The *Dictionary of National Biography* includes two accounts of the life of Robert of Chester; Vol. X (London, 1887), p. 203, *Chester, Robert*, by A. M. Clerke; Vol. XLVIII (New York, 1896), pp. 362–364, *Robert the Englishman*, *Robert de Ketene*, or *Robert de Retines*, by T. A. Archer. Wüstenfeld, *Die Uebersetzungen arabischer Werke in das Lateinische*, *Abhandl. d. Königlichen Gesellschaft der Wissenschaften zu Göttingen*, Vol. XXII (1877), pp. 44–47; L. Leclerc, *Histoire de la médecine arabe* (Paris, 1876), Vol. II, pp. 380–387; Jourdain, *Recherches sur les anciennes traductions Latines d'Aristote* (Paris, 1843, revised edition), pp. 100–104; Thomas Wright, *Biographia Britannica Literaria*, *Anglo-Norman Period* (London, 1846), pp. 116–119.

Astensis, or Cestrensis, the final form being the most common. Retinensis (Retenensis) has been somewhat doubtfully referred to Reading, England, while Cestrensis certainly refers to Chester. Similar peculiarities in the dual or multiple designation of writers have been noted in connection with the name of John of Spain; such variations seem to have been common in this period. Robert of Chester, known to fame chiefly as the first translator of the Qoran,[1] was doubtless educated in the well-known school located at Chester. Of the more personal side of Robert's life we have but scattered facts. His nationality is established not only by his name and his return to England in 1150, but also by the direct statement made by Peter the Venerable in a letter[2] of 1143 concerning the Qoran, addressed to Bernard of Clairvaux. Peter states in this letter that Robert was then archdeacon of Pampeluna, in northern Spain. Hermann the Dalmatian, commonly known as Hermannus secundus, but also spoken of as Scholasticus, Sclavus, or Chaldæus, refers to Robert as his "special and inseparable comrade, his peerless partner in every deed and art." In the year 1141 Robert and Hermann were living in Spain near the Ebro, studying the arts of astrology. There in that year Peter the Venerable found them and "by entreaty and a good price" induced them to take up studies in Mohammedan religion and law, and also to translate the Qoran.[3]

---

[1] *Machumetis Saracenorum principis, eiusque successorum vitae, doctrina, ac ipse Alcoran . . . cum doctiss. uiri Philippi Melancthonis praemonitione. . . .* Haec omnia in unum uolumen redacta sunt, opera et studio Theodori Bibliandri (1550, place of publication, Basle, not given in book itself), Vol. I, pp. 213–223; a different edition was edited by Wallis (Basle, 1643); possibly the first edition was printed at Basle, 1543. I have used a copy of the edition of 1550, loaned to me from the John G. White Collection, Cleveland Public Library, by the courtesy of the librarian.

[2] *Qoran*, 1550 edition, pp. 1–2; Migne, *Patrologia Latina*, Vol. 189 (Paris, 1890), col. 649–652; see also col. 1073–1076, and col. 339.

[3] As there is some question as to the real translator of the Qoran it seems desirable to add from the 1550 edition of the Qoran the evidence that the translation is due to Robert, pp. 1–2: Epistola Domini Petri Abbatis, ad Dominum Bernhardum Claraeuallis Abbatem, de translatione sua, qua fecit transferri ex Arabico in Latinum, sectam, siue haeresim, Saracenorum. . . . Mitto uobis, charissime, nouam translationem nostram, contra pessimum nequam Machumet haeresim disputantem. Quae nuper dum in Hispaniis morarer meo studio de Arabico uersa est in Latinam. Feci autem eam transferri a perito utriusque linguae uiro magistro Petro Toletano. Sed quia lingua Latina non ei adeo familiaris uel nota erat, ut Arabica, dedi ei coadiutorem doctum uirum dilectum filium et fratrem Petrum notarium nostrum, reuerentiae uestrae, ut extimo, bene cognitum. Qui uerba Latina impolite uel confuse plerumque ab eo prolata poliens et ordinans, epistolam, imo libellum multis, ut credo, propter ignotarum rerum notitiam perutilem futurum perfecit. Sed et

In the prologue to his treatise[1] against the "sect" of the Saracens, Peter says: "Contuli ergo me ad peritos linguae Arabicae, . . . eis ad transferendum de lingua Arabica in Latinam perditi hominis originem, vitam, doctrinam, legemque ipsam quae Alchoran vocatur, tam prece quam pretio, persuasi. Et ut translationi fides plenissima non deesset, nec quidquam fraude aliqua nostrorum notitiae subtrahi posset, Christianis interpretibus etiam Sarracenum adjunxi. Christianorum interpretum nomina: Robertus Kecenensis, Armannus Dalmata, Petrus Toletanus; Saraceni Mahumeth nomen erat. Qui intima ipsa barbarae gentis armaria perscrutantes, volumen non parvum ex praedicta materia Latinis lectoribus ediderunt. Hoc anno illo factum est quo Hispanias adii, . . . qui annus fuit ab Incarnatione Domini 1141."

The letter to Bernard seems to be of date 1143, and that is the date, evidently, of the completion of Robert's translation, and so it is given at the end of the Qoran: "Illustri gloriosoque viro Petro cluniacensi Abbate praecipiente, suus Angligena, Robertus Retenensis librum istum transtulit Anno domini MCXLIII, anno Alexandri MCCCCIII, anno Alhigere DXXXVII, anno Persarum quingentesimo undecimo."

Robert and Hermann appear to have been associated in translating scientific works particularly along the lines of astrology from Arabic into Latin, as well as the Qoran.[2] In connection with these astrological treatises a mediæval reference,[3] probably contemporary, mentions Robert as "a man most learned in astrology." Peter the Venerable refers to Robert and Hermann as most acute and well-trained scholars, while Peter of Poitiers, in a letter[4] of unknown date addressed to Peter the Venerable, cites Robert

---

totam impiam sectam, uitamque nefarii hominis, ac legem, quam Alcoran, id est, collectaneum praeceptorum appellauit, sibique ab angelo Gabriele de coelo callatam miserrimis hominibus persuasit, nihilominus ex Arabico ad Latinitatem perduxi, interpretantibus scilicet uiris utriusque linguae peritis, Roberto Retenensi de Anglia, qui nunc Papilonensis ecclesiae archidiaconus est, Hermanno quoque Dalmata acutissimi et literati ingenii scholastico. Quos in Hispania circa Hiberum Astrologicae arti studentes inueni, eosque ad haec faciendum multo precio conduxi.

[1] *Patrologia Latina*, Vol. 189, col. 671.

[2] See the article by Björnbo, *Hermannus Dalmata als Uebersetzer astronomischer Arbeiten*, *Bibliotheca Mathematica*, Vol. VI, third series (1905), pp. 130–133 and p. 28 below.

[3] Steinschneider, *Die Europäischen Uebersetzungen aus dem Arabischen*, *Sitzungsberichte der Philosophisch-historischen Klasse der kaiserlichen Akademie der Wissenschaften*, Vol. 149 (Vienna, 1905), p. 72.

[4] *Patrologia Latina*, Vol. 189, col. 661.

as an authority on Mohammedan customs. A further manuscript reference[1] seems to indicate that in the year 1136 Robert was studying in Barcelona with Plato of Tivoli, while Fabricius[2] states, but upon what authority does not appear, that Robert travelled in Italy, Greece, and Spain. The time and the place of Robert's death are equally uncertain.

To Peter the Venerable, Abbot of Cluny, who induced Robert to undertake the translation of the Qoran, the latter addressed a Saracenic chronicle, *Chronica mendosa et ridiculosa Saracenorum*,[3] which was published in 1550 with the Qoran. Other names have been connected with this translation of the Qoran, but a study of Peter's letters, and the introduction to Peter's treatise, "Against the Sect of the Saracens,"[4] shows that Robert and Hermann were definitely requested to undertake the translation, which it appears from Robert's preface was finally completed by Robert alone. Possibly Peter of Toledo made an earlier,[5] unsatisfactory translation of the same work. A Mohammedan by the name of Mohammed was engaged by Peter the Venerable to scrutinize the various treatises with a view to correcting errors due to mistranslation.

Of Robert's works the version of the Qoran was completed in 1143. In a prefatory letter he states that this task was regarded by him only as a digression from his principal studies of astronomy and geometry,[6] but posterity knew of him through the earlier work rather than through the sciences to which he dedicated himself in this letter. Even the mathematician John Wallis, editor of one edition of this translation, did not connect the translator with the Robertus Retinensis who was mentioned, as we have seen, by Wallis in his *Algebra*. An unpublished letter written by Robert, beginning *Cum jubendi religio*, is preserved in the Selden manuscript, sup. 31, in the Bodleian library.

---

[1] Archer, *Dictionary of National Biography*, Vol. XLVIII (New York, 1896), p. 362; Professor Haskins examined the MS. but found no evidence for this statement.

[2] *Bibliotheca med. et infim. Lat.* (Florence, 1858–1859), Vol. VI, p. 107.

[3] *Qoran*, 1550 edition, pp. 213–223.

[4] Migne, *Patrologia Latina*, Vol. 189, col. 659–720, Petri Venerabilis, Abbatis Cluniacensis noni, adversus nefandam sectam Saracenorum libri duo.

[5] Since Peter the Venerable expressly refers to a new translation by the labors of Robert and Hermann. In the letter to Bernhard of Clairvaux Peter says: "Mitto vobis, charissime, novam translationem nostram," which may mean, possibly, another document on Arabic customs.

[6] *Machumetis Saracenorum principis, eiusque successorum vitae, doctrina, ac ipse Alcoran* (Basle, 1550), pp. 7–8: Praefatio Roberti translatoris ad Dominum Petrum Abbatem Cluniacensem, in libro legis Saracenorum, quem Alcoran uocant, id est, Collectionem praeceptorum quae Machumet pseudopropheta per angelum Gabrielem quasi de coelo sibi missa confinxerit. Begins, Domino suo Petro diuino instinctu Cluniacensi abbati, Robertus Retenensis suorum minimus in Deo perfecté gaudere; and ends, Istud quidem tuam minimé latuit sapientiam, quae me compulit interim astronomiae geometriaeque studium meum principale praetermittere. Sed ne proemium fastidium generet, ipsi finem impono, tibique coelesti, coelum omne penetranti, coeleste munus uoueo: quod integritatem in se scientiae complectitur. Quae secundum numerum, & proportionem

Several sets of astronomical tables are included among the products of Robert's literary activity. The *Canones in motibus coelestium corporum ad meridiem urbis Londoniarum in duos partes, prior 1149 ad fidem tabularum toletanarum Arzachelis, altera pro anno 1150 iuxta Albatem Haracensis*[1] evidently includes two sets of tables. This reference to the methods of Al-Battani may account for the fact that Robert is sometimes credited with a translation of Al-Battani's tables.[2] Wallis asserts that the tables accommodated to the meridian of London were adjusted to the year 1150 and in fact the text in MS. Savile 21, fol. 86ʳ, states the date as March first, 1150. Reference is made also to a preceding work as accommodated to the meridian of Toledo for the year 1148 or 1149 and based on tables by Rabbi Abraham ibn Esra.[3] Extracts, apparently from these tables of Robert, are found under the title, *De diuersitate annorum ex Roberto Cestrensi super tabulas toletanas*,[4] and this contains a reference to a sexagesimal multiplication table, evidently constructed by Robert, in which appear all products from 1 × 1 up to 60 × 60.

---

atque mensuram coelestes, circulos omnes, & eorum quantitates & ordines & habitudines, demum stellarum motus omnimodos, & earum effectus atque naturas, & huiusmodi caetera diligentissime diligentibus aperit, nunc probabilibus, nonnunquam necessariis argumentis innitens.

[1] Steinschneider, *Sitzungsber.*, *loc. cit.*; MS. Savile 21, 63–95ᵛ. Fol. 63ʳ, *Incipiunt canones in motibus coelestium corporum*, which begins, Quoniam cuiusque accionis quantitatem metitur celestium spacium, and proceeds with a discussion of various chronological systems. Fol. 86ʳ, Incipit pars altera huius operis que videlicet ad meridiem urbis Londoniarum iuxta Albatem Haracensis sententiam per Robertum Cestrensem contexitur: Begins, Premissa uero electe facultatis . . .; ends, Ergo iuxta hanc scienciam planetarum loca figura ponenda sunt. See above, p. 17, note 1.

[2] See Nallino, *Al-Battānī opus astronomicum*, *Pubbl. del reale Osservatorio di Brera* (Milan), Vol. XL (1903), Introduction.

[3] In MS. Savile: *ebenza* (?)

[4] MS. Digby 17. Beg. fol. 156, Diuersi astronomi secundum diuersos annos tabulas faciunt et quidam secundum annos Alexandri seu grecorum, alii secundum ierdaguth (Yezdegerd) seu persarum.

In 1144 Robert completed the translation, entitled *De compositione alchemiae*, or *De re metallica*, by one Morienus Romanus. This was published in Paris in 1546, and later by Manget.[1] Another of the products of his literary activity, dated 1185 of the Spanish Era (*circa* 1150 A.D.), was the work, *De compositione astrolabii*,[2] which is ascribed to Ptolemy. The place of composition is given in one manuscript copy as London.[3] Like Adelard of Bath, Robert seems also to have written a chemical work, dealing with pigments and other associated topics, entitled *Liber metricus qui dicitur Mappa claviculae*. This version, Greek in its origin, is in verse.[4]

An astrological work, entitled *Judicia Alkindi astrologi* or *De judiciis astrorum*, which was written by the great Arabic philosopher Yaʿqub ibn Ishaq ibn Sabbah al-Kindi (died about 874 A.D.), was translated by Robert of Chester and not by Robertus Anglicus in 1272 as usually[1] stated. This treatise in forty-five chapters is preceded by an introduction[2] which definitely establishes the authorship of the translation. In these prefatory remarks the writer addresses himself to "my friend Hermann, second to no astronomer of our time, of those who speak Latin," and further makes reference to the translation as being made at the request or wish of Hermann. As some manuscripts ascribe this work to Robert of Chester who was associated with Hermann the Dalmatian, the reference to Hermann would be almost conclusive, and the terminology of the introduction by its similarity to that of the Qoran introductory letter establishes the fact beyond question.

---

[1] J. J. Manget, *Bibliotheca Chemica Curiosa* (Geneva, 1702), Vol. I, pp. 509–519, Liber De compositione Alchemiae, quem edidit Morienus Romanus, Calid Regi Aegyptorum ; quem Robertus Castrensis de Arabico in Latinum transtulit.

The prologue begins : Praefatio Castrensis. Legimus in Historiis veterum divinorum, tres fuisse Philosophos, quorum unusquisque Hermes vocabatur. . . . Sed nos, licet in nobis juvene sit ingenium, et latinitas permodica : hoc tamen tantum ac tam magnum opus ad transferendum de Arabico in Latinum suscepimus. Vnde et de singulari gratia nobis a Deo inter modernos collata, summas illi Deo vivo, qui trinus extat et unus, grates referimus. Nomen autem meum in principio Prologi taceri non placuit, ne aliquis hunc nostrum laborem sibi assumeret, et etiam ejus laudem et meritum sibi quasi proprium vendicaret. Quid amplius ? humiliter omnes rogo et obsecro, ne quis nostrorum erga meum nomen mentis livore (quod saepe a multis fieri consuevit) tabescat. Scit namque Deus omnium, cui suam conferat gratiam : et spiritus ex gratia procedit, qui quos vult inspirat. Merito igitur gaudere debemus quum omnium Creator et conditor omnibus quasi particularem suam monstret divinitatem.

The treatise itself begins : Morieni Romani, Eremitae Hierosolymitani, Sermo. Omnes Philosophiae partes, mens Hermetis divina plenarie attigit . . . and ends, Quod si quando Alchymia confecta fuerit, ejus una pars inter novem partes argenti ponatur, quoniam totum in aurum purissimum convertetur. Sic ergo Deus benedictus Amen, per cuncta seculorum secula.

Explicit Liber Alchymiae de Arabico in Latinum translatus, anno millesimo centesimo octuagesimo secundo, in mense Februarii et in ejus die undecimo.

[2] Steinschneider, *Zeitschrift f. Mathematik und Physik*, Vol. XVI, p. 393, with reference to the Vienna MS. 5311^{8}, *De compositione astrolabii universalis liber a Roberto Castrensi translatus*, which begins : *Quoniam in mundi spera*, and ends : *ad altitudines accipiendas ;* also to the Oxford MS. Cod. Canonic. Misc. 61^{6}, *Liber de officio astrolabii secundum mag. Rob. Cestrensem*, in thirty-five chapters, which begins : 1. *De gradu solis per diem et diei*, and ends : *et cetere ceteris per diametrum ut jam dictum est opponuntur*. Revised after 1150, as it cites the tables of that year (Bodleian MS. Canon Misc. 61, f. 22^{2}; examined by Professor Haskins).

[3] Steinschneider, *ibid.*, p. 393, refers to the Gonville and Caius College, Cambridge, MS. 35^{14}, *Ptholomaei de compositione astrolabii, translatus a Rob. Cestiensi in civitate London ;* also MS. Digby 40, dates this "*Aera* 1185 *in civitate London*."

[4] Steinschneider, *loc. cit.*, p. 72.

A rearrangement of the Al-Khowarizmian tables as translated by Adelard of Bath was made by Robert of Chester.[3] The basis of this revision was doubtless a translation of the same tables by Hermann, who refers to his own translation in his astrological work, *Introductorium in astronomiam Albumasaris abalachi, octo continens libros partiales*.[1] This work is dedicated to Robert; the similarity of the phraseology with that of the prefaces by Robert bears witness to the intimacy of their literary labors. Also, in the prologue to Hermann's translation of Ptolemy's *Planisphere*,[2] Robert is mentioned as an associate of Hermann and again in the text. This work on the Planisphere has been incorrectly ascribed in the printed edition of 1537 and elsewhere to Hermann's pupil, Rudolph of Bruges.[3]

---

[1] Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke, loc. cit.*, p. 26; Steinschneider, *Die europäischen Uebersetzungen aus dem Arabischen, loc. cit.*, p. 66, *et al.*

[2] Introduction, for which I am indebted to the courtesy of Professor C. H. Haskins, following MS. Ashmole, 369, fol. 85^a. The heading and *explicit* are from the Cotton MS. App. vi. Numerous other manuscripts of the same are found in European libraries.

Incipiunt iudicia Alkindi astrologi, Rodberti de Ketene translatio.

Quamquam post Euclidem Theodosii cosmometrie libroque proportionum libentius insudarem, unde commodior ad almaiesti quo praecipuum nostrum aspirat studium pateret accessus, tamen ne per meam segnitiem nostra surdesceret amicitia, vestris nutibus nil preter equum postulantibus, mi Hermanne, nulli latinorum huius nostri temporis astronomico sedere (sedem?) penitus parare paratus, eum quem commodissimum et veracissimum inter astrologos indicem vestra quam sepe notauit diligencia, voto vestro seruiens transtuli non minus amicitie quam pericie facultatibus innisus. In quo tum nobis tum ceteris huius scientie studiosis placere plurimum studens, enodato verborum vultu rerum seriem et effectum atque summam stellarium effectuum pronosticationisque quorumlibet eventuum latine brevitati diligenter inclusi. Cuius examen vestram manum postremo postulans non indigne vobis laudis meritum si quod adsit communiter autem fructus pariat, mihique non segne res arduas aggrediendi calcar adhibeat, si nostri laboris munus amplexu fauoris elucescat. Sed ne proemium lectori tedium lectionique moram faciat vel afferat, illius prolixitate supersedendo, rem propositam secundum nature tramitem a toto generalique natis exordiis texamus, (P)rius tamen libri totius capitulis enunciatis ad rerum evidentiam suorumque locorum repertum facilem. *Explicit prohemium. Incipiunt libri capitula.*

Primum igitur capitulum, zodiaci diuisiones, earumque proprietates, tam naturales quam accidentales generaliter complectitur. . . .

The text proper begins (Ashmole MS.): Circulus itaque spericus cuius atque terre centrum est . . . and ends: sequitur in proximo.

[3] Haskins, *The English Historical Review*, XXVI, footnote on p. 498: Madrid MS. (no. 10016) of the translation of the Al-Khowarizmian tables, which has this heading: *Incipit liber ezeig id est chanonum Alghoarizmi per Adelardum bathoniensem ex arabico sumptus et per Rodbertum cestrensem ordine digestus!* Suter, *Die astron. Tafeln des al-Khwarizmi in der Bearbeitung des Maslama ibn Aḥmed al-Madjriṭi und der lat. Uebers. des Athelard von Bath auf Grund der Vorarbeiten von A. Björnbo † und R. Besthorn*, loc. cit.

---

[1] H. Suter, in a letter (Aug. 13, 1914) to the author.

The *Introductorium* was printed at Augsburg in 1489, and other editions followed. I am indebted to Professor Haskins for a transcription of the preface, from which the following passage is taken: Que cum ego prolixitatis exosus et quasi minus contentia: cum et hunc morem latinis cognoscerem preterire volens animo ipso potius tractatum exordiri pararem. Tu mihi studiorum olim specialis atque inseperabilis comes, rerumque et actuum per omnia consors unice, mi Rodberte, si memores obviasti dicens: Quanquam equidem nec tibi pro amore tuo, mi Hermanne, nec ulli consulto aliene lingue interpreti in rerum translationibus. . . .

Steinschneider, *Ueber die Mondstationen* (*Naxatra*) *und das Buch Arcandum*, *Zeitschrift d. deutschen morgenländischen Gesellschaft*, Vol. XVIII (1864), pp. 170–172, and *Die Hebräischen Uebersetzungen*, pp. 568–570, and in *Die europäischen Uebersetzungen*, *loc. cit.*, p. 34, demonstrated from the introduction that Hermann was the author. This view was accepted by Björnbo, *Hermannus Dalmata als Uebersetzer astronomischer Arbeiten*, *Bibliotheca Mathematica*, third series, Vol. IV (1903), pp. 130–133.

[2] J. L. Heiberg, *Claudii Ptolemaei Opera quae exstant omnia*, Vol. II, *Opera astronomica minora*, pp. clxxxiii–clxxxvi: "tuam itaque uirtutem quasi propositum intuentes speculum ego et unicus atque illustris socius Rebertus Ketenensis nequitie dispicere, licet plurimum possit, perpetuum habemus propositum, cum, ut Tullius meminit, misera sit fortuna, cui nemo inuideat."

In the text proper Hermann inserts the following note: "quem locum a Ptolomeo minus diligenter perspectum cum Albateni miratur et Alchoarismus, quorum hunc quidem ope nostra Latium habet, illius uero comodissima translatione studiosissimi Roberti mei industria Latine orationis thesaurum accumulat, nos discutiendi ueri in libro nostro de circulis rationem damus."

[3] After this chapter was in type a noteworthy article, *The reception of Arabic science in England*, by Professor C. H. Haskins, appeared in *The English Historical Review*, Vol. XXX (1915), pp. 56–69. The works of Robert of Chester are carefully considered. Fortunately, through the courtesy of Professor Haskins, I was enabled to incorporate in advance the results which bear directly upon our discussion.

CHAPTER IV

THE INFLUENCE OF AL-KHOWARIZMI'S ALGEBRA UPON THE DEVELOPMENT OF MATHEMATICS

By the translators of Arabic lore we are brought from Islam to Christendom. Mathematical science in Europe was more vitally influenced by Mohammed ibn Musa than by any other writer from the time of the Greeks to Regiomontanus (1436–1476). Through his arithmetic, presenting the Hindu art of reckoning, he revolutionized the common processes of calculation and through his algebra he laid the foundation for modern analysis. Evidence of the influence of the great Arab is presented by the relatively large number of translations and adaptations of his various mathematical works which appeared before the invention of printing. Undoubtedly the earliest translation of the Arabic algebra, although not the most widely used, was that made by Robert of Chester. Probably the version published by Libri appeared shortly afterwards for, as we have mentioned, Gerard of Cremona employs the terminology of that version in algebraic work. Roger Bacon (1214–1294), too, has occasion[1] to mention the algebra, as well as the arithmetic, and uses terms not found in Robert of Chester's version. Bacon shows that he had but superficial familiarity with the subject, for he made incorrect statements about the fundamental elements of the algebra. Similarly, Vincent de Beauvais (about 1275) in his encyclopædic work, *Speculum Principale*,[2] refers under arithmetic to the book, *qui apud Arabes mahalehe dicitur*. Albertus Magnus (1193–1280) mentions the tables of Al-Khowarizmi.[3]

Even earlier than Roger Bacon is Leonard of Pisa, whose monumental *Liber abaci* contains a chapter involving the title, *Aljebra*

---

[1] In his unfinished *Scriptum principale*. See J. H. Bridges, *The "Opus Majus" of Roger Bacon* (London, 1900), Vol. I, p. lvii; D. E. Smith, *The place of Robert Bacon in the history of mathematics*, in *Roger Bacon, Essays*, collected and edited by A. G. Little (Oxford, 1914), p, 177.

[2] Liber xviii, Cap. V, *De Arithmetica;* Liber xviii, Cap. ix, is entitled, *De Computo et algorismo*, and takes up representation of numbers by the Hindu numerals.

[3] Steinschneider, *Zum Speculum astronomicum des Albertus Magnus, Zeitschrift f. Mathematik und Physik*, Vol. XVI, pp. 375–376, and in *Zeitschrift d. deutschen morgenl. Gesellschaft*, Vol. XXV, pp. 404–405.

et almuchabala.[1] The first draft of this work was written in 1202, and in 1228 a revised and enlarged version appeared, dedicated to Michael Scotus. Woepcke has shown that Leonard drew many of his problems from Al-Khowarizmi,[2] but some of these may have come indirectly through Abu Kamil, from whom, as I have shown,[3] Leonard took many of his algebraic problems. In the manuscripts of the Italian's treatise the only mention of Al-Khowarizmi is in the margin, simply *Maumet*, at the beginning of the section dealing with algebra; but the term *algorismus* occurs for arithmetic.[4]

In the century following Leonard of Pisa, another Italian mathematician, William of Luna, is reputed to have put Al-Khowarizmi's algebra into the Italian language. Raffaello di Giovanni Canacci, a Florentine citizen of the fifteenth century, states in an Italian work on algebra,[5] as yet in manuscript, that William had translated the rules of algebra out of Arabic into "our language." Reference to his work is also made by at least three writers of the sixteenth century, the Florentine Francesco Ghaligai, the Spaniard Marco Aurel, and another Spaniard Antich Rocha of Gerona.[6] An Italian manuscript of 1464 in the library of George A. Plimpton, Esq., of New York, does contain an Italian version of the Algebra of Al-Khowarizmi in which reference is made to William of Luna as a translator of algebra. The possibility is that we have here the version of William. The writer of the manuscript is not known, but he explicitly states that he bases his treatise on the labours of numerous predecessors in this field. One chapter, as I have shown in a recent study of this manuscript,[7] deals with the algebra of an unknown Maestro Biagio (died 1340) and to Leonard of Pisa another section is devoted. The writer purposed also to deal with the works of a "subtle Maestro Antonio," doubtless Antonio Mazzinghi da Peretola, who wrote a treatise on algebra called *il fioretto*, and a "Maestro

---

[1] *Scritti di Leonardo Pisano* (Vol. I, Rome, 1857; Vol. II, Rome, 1862), Vol. I, p. 406.

[2] *Extrait du Fakhri*, p. 29.

[3] *The Algebra of Abu Kamil*, loc. cit.

[4] *Scritti*, Vol. I, p. 1.

[5] Codex Palat. 567, Biblioteca Nazionale, Florence.

[6] Ghaligai, *Pratica d'arithmetica* (Florence, 1552); Aurel, *Libro primero, de arithmetica algebratica* (Valencia, 1552); Rocha, *Arithmetica* (Barcelona, 1565).

[7] *An Italian Algebra of the fifteenth century*, *Bibliotheca Mathematica*, third series, Vol. XI, pp. 209-219.

Giouanni," but either this manuscript is incomplete or the plan was not carried out. Prominent also in the discussions is an Augustinian monk, Gratia de Castellani (about 1340), famed as a theologian. The relatively large number of names of men who had evidently attained something more than local repute in the study of algebra shows the place which it had reached in instruction.

Another prominent writer of the fourteenth century, Johannes de Muris, included a discussion of algebra in the third book of his popular *Quadripartitum numerorum*.[1] Of this section of the work of John of Meurs I have recently made a study,[2] showing that he drew extensively from Leonard of Pisa and from Al-Khowarizmi, thus continuing the Arabic influence. Regiomontanus included the work in a list of important early works on mathematics, and further Regiomontanus refers to algebra as the *ars rei et census*. This corresponds to a line of the *Quadripartitum*:

> "Que tamen ars minor est quam sit de censibus et rei."

Later the expression, *Arte magiore*,[3] or *Ars mayor*,[4] or *Ars magna*,[5] was used for algebra, tracing back to this passage here given, in which *ars minor* refers to arithmetic as opposed to algebra. Adam Riese presents the problem,[6] $x^2 + 21 = 10x$, as being found in the eleventh chapter of the third book of the *Quadripartitum*; we have mentioned that this problem is one of the type problems found in Al-Khowarizmi's algebra. Another French author who gives an adaptation in Latin of the Arabic algebra is Rollandus, Canon of St. Chapelle. At the command of John, Duke of Lancaster, Rollandus wrote in the year 1424 a compendium of mathematics; the labor of composition was considerably lightened by making large extracts from the *Quadripartitum*, including most of the arithmetic and algebra. A summary of the contents of the manuscript is given in the *Rara Arithmetica*,[1] but the somewhat extensive treatment of algebra is not mentioned.

---

[1] Two chapters of the second book, dealing with arithmetic, were published by A. Nagl, *Das Quadripartitum etc.*, *Abhandl. z. Geschichte der mathem. Wissenschaften*, Vol. V (Leipzig, 1890), pp. 137–146.

[2] *The "Quadripartitum numerorum" of John of Meurs*, *Bibliotheca Mathematica*, third series, Vol. XIII, pp. 99–114.

[3] Paciuolo, *Summa d'arithmetica* (Venice, 1494).

[4] In Aurel's *Libro primero, de arithmetica algebratica* (Valencia, 1552).

[5] Cardan, *Ars Magna* (Nuremberg, 1545).

[6] B. Berlet, *Adam Riese, sein Leben, seine Rechenbücher und seine Art zu rechnen*, *Die Coss von Adam Riese* (Leipzig, 1892), p. 37.

The first work in the German language on algebra was an excerpt from Al-Khowarizmi which begins:[2] "Mohammed in the book of algebra and almucabala has spoken these words 'census, radix (root), and number.'" This is followed by two problems from the text. The manuscript which contains this brief discussion, of date 1461, is now in Munich, having been moved from the Benedictine Abbey of St. Emmeran. The first treatment in the English language appears to be that by Robert Recorde, *The Whetstone of Witte*, which was published in 1557. This work which, as I have elsewhere shown,[3] does not display any marked originality on the part of Recorde, introduced our present symbol of equality, =, and contributed to the study of algebra in England by presenting the material in the mother tongue.

Regiomontanus seems to have been familiar with Al-Khowarizmi's work, for he not only refers to the art of thing and square (*ars rei et census*), but also uses certain technical expressions, *restaurare defectus*, for example, similar to those in the algebra. A manuscript copy[4] of Mohammed ibn Musa's algebra in Mr. Plimpton's collection shows astonishing similarity to the handwriting and abbreviations of Regiomontanus as well as to the form of equation used by the great German. Furthermore, some of the problems given in this manuscript, which are not part of Al-Khowarizmi's text, are discussed by Regiomontanus in his correspondence with Cardinal Blanchinus.[5] We must suppose him to have been familiar with this text if not actually, as we suspect, the transcriber of this copy. Regiomontanus was twenty years of age when this manuscript was written (1456), and we know that he did transcribe numerous mathematical and astronomical works of historical importance. Algebra was employed by Regiomontanus in his trigonometry[1] in the solution of problems.

---

[1] *Rara*, 446–447.

[2] Gerhardt, *Zur Geschichte der Algebra in Deutschland, Monatsbericht der Königl. Akad. d. Wissensch. zu Berlin*, 1870, pp. 141–153.

[3] Karpinski, *The whetstone of witte, Bibliotheca Mathematica*, third series, Vol. XIII, pp. 223–228.

[4] *Rara Arithmetica*, 454–456.

[5] M. Curtze, *Der Briefwechsel Regiomontan's mit Giovanni Bianchini, Jacob von Speier und Christian Roder, Abhandl. z. Gesch. d. math. Wissen.*, Vol. XII (Leipzig, 1902), problem 6, p. 219.

Regiomontanus has been cited by Nesselmann[2] as an illustration of one who employed rhetorical algebra as opposed to syncopated or symbolical. Later writers have followed Nesselmann in the assertion that Regiomontanus used rhetorical algebra, but, whereas the statement in Nesselmann is correct in so far as the illustration which he gives is concerned, the assumption that this was the general practice of the great Teuton is an error. In fact, his correspondence with Cardinal Blanchinus shows that he had a form of equation little inferior to ours. The + sign which he uses is a ligature for *et*, the minus sign a ligature for *minus*, and for an equality sign he uses a single straight line. Further he has separate symbols for the various powers of the unknown up to the cube, so that Regiomontanus approached modern forms more closely than most mathematicians even of the sixteenth century. This attempted division of the history of algebra into rhetorical, syncopated, and symbolic periods is an excellent illustration of a plausible and taking theory, in historical matters, which lacks only the first essential for such a theory; namely, historical evidence. Development in mathematics, as in art and literature, does not proceed in a logical manner, but rather in waves advancing and receding, and yet withal constantly advancing.

We have mentioned the Hebrew translation of the algebra of Abu Kamil, which was made by Mordechai Finzi (about 1475 A.D.) of Mantua. Another treatise on algebra, in Hebrew, dedicated to Finzi, was written by Simon Motot.[3] As the words *cosa* and *censo* are mentioned by Motot as being found in the works of Christian authors with which he was familiar the Italian source of his information is established, although the particular writers in question are not known. As we have above indicated, the Italians were sufficiently active in this science, so that many Italian works on algebra were available, in manuscript, at this time.

In the summer of 1486 Johann Widmann of Eger is known to have lectured on algebra in the university at Leipzig, and the fee for the course was set extraordinarily high, being two florins.[1] Widmann was in possession of the Dresden manuscript,[2] which contains Robert of Chester's version of the algebra of Al-Khowarizmi, and himself added certain algebraic problems to another part of the same manuscript dealing with algebra. The Arab Al-Kalasadi,[3] contemporary with Widmann, wrote also on similar topics, and although he does not cite Al-Khowarizmi, yet he continues the old order of the six types of quadratic equations.

---

[1] Regiomontanus, *De triangulis libri quinque* (Nuremberg, 1533), problem 12, p. 51.

[2] Nesselmann, *Die Algebra der Griechen* (Berlin, 1842), p. 303.

[3] G. Sacerdote, *Le livre de l'algèbre et le problème des asymptotes de Simon Motot*, *Revue des études juives*, 1893–1894.

Adam Riese wrote in 1524 a work on algebra entitled, *Die Coss*, which contains, as we have noted, the problem

$$x^2 + 21 = 10\,x.$$

Riese refers to "that most celebrated Arabic master Algebra, learned in number, whose like in computation there never was, and hardly will any one exceed him." He refers also to the book, "named gebra and almucabola," by this mythical Algebra. A reference to Algum is also doubtless to Al-Khowarizmi. Several contemporaries, students of the Coss, are mentioned[4] by Adam Riese, and included among these is Grammateus,[5] also known as Schreiber or Scriptor, to whom is credited the first algebra in print in the German language. Interesting is Riese's note that Hans Conrad, to whom he frequently refers, paid the mathematician Andreas Alexander one florin in gold, to be taught how to solve certain types of problems by the Coss. This title is from the Italian *cosa* (Latin *res*, Arabic *shaī*) and connects with the use by Al-Khowarizmi, and subsequent Arabs, of the word *shaī*, meaning *thing*, for the first power of the unknown. For centuries the title continued in circulation in Germany, and even in English appeared in the form, "the arte of cosslike nombers."[6]

Luca Paciuolo, otherwise Luca de Burgo San Sepulchro, to whom is credited the first printed work on algebra, was evidently influenced by Al-Khowarizmi. Paciuolo gives[1] the equation

---

[1] Wappler, *Zur Geschichte der deutschen Algebra in* 15. *Jahrhundert*, in *Programm*, Zwickau, 1887, pp. 9–10; also in *Zeitschrift f. Math. und Physik, Hist.-lit. Abtheil.*, Vol. 45, 1900.

[2] Codex Dresden C. 80.

[3] Woepcke, *Atti dell' accad. pont. de' nuovi Lincei*, Vol. XII (Rome, 1859), 230–275, 399–438.

[4] Berlet, *loc. cit.*, pp. 33, 34, 36, and 62 for references.

[5] . . . *Rechenbüchlin*, including *Etlichen Regeln Cosse*, written 1518 and published 1521.

[6] Recorde, *loc. cit.*

$$x^2 + 10x = 39$$

and presents the geometrical explanation as given by the Arab. In certain others of the early printed algebras the fundamental or type equations as given by Al-Khowarizmi do not appear. Of such are the works by Grammateus appearing in 1518, by Christian Rudolph in 1525,[2] and by Estienne de la Roche in 1520,[3] whose work is known to have been a plagiarism of the *Triparty* by Nicolas Chuquet (1484).[4] However, many other writers did continue the type problems of the first systematic treatise. Thus Elia Misrachi[5] in an arithmetic which appeared in Constantinople in 1534, eight years after the author's death, devotes a section to algebra, and this is to a large extent an adaptation of the *Algebra and Almucabala.* In the work by Perez de Moya (1562)[6] and in the arithmetic of 1539 by Cardan we come upon the type equations. Mennher de Kempten, a Dutch mathematician, states that Algorithmo was the first writer on algebra. Ghaligai, the Italian, and the Spaniard Pedro Nuñez follow the peculiar order of equations found in Al-Khowarizmi. In these and other ways we might trace through the centuries the persistent influence, direct and indirect, of our Arabic author, but that is beside our present purpose.

Among the writers who made a serious study of Robert of Chester's translation we must place Johann Scheybl (1494–1570), who was professor of mathematics at Tübingen from about 1550 to the time of his death. He was the author of an algebra which appeared in two editions in Paris, 1551 and 1552. This treatment of algebra was first published in 1550 by Scheybl, prefixed to his Greek and Latin edition of the first six books of Euclid. Scheybl prepared for publication the Latin version of Al-Khowarizmi's algebra as translated by Robert of Chester. His manuscript copy is now in the Columbia University library. The title page[1] reads, in translation: ' A brief and clear exposition of the rules of algebra by Johann Scheybl, Professor of Euclid in the famous University of Tübingen. To this is added the work, *On given numbers*, by that most excellent mathematician Jordanus. Furthermore there is presented the book containing the demonstrations of the rules of algebraic equations, written some time ago in Arabic. All of these are now published for the first time by the above-mentioned Scheybl. These are corrected as far as possible and illustrated by appropriate and useful examples.'

---

[1] *Summa de arithmetica* (Venice, 1494), fol. 146 rec.

[2] I have not seen a copy of this edition. My remark is based upon *Die Coss Christoffs Rudolffs* (Königsberg, 1553) by Stifel. From certain notes about the history of the terminology and the words *dragma*, *res* and *substantia*, and the like, it appears that Stifel had seen a copy of Robert of Chester's version.

[3] Smith, *Rara Arithmetica*, p. 128.

[4] A. Marre, *Notice sur Nicolas Chuquet et son Triparty en la science des nombres*, Boncompagni's *Bulletino di bibliografia e di storia delle scienze matematiche e fisiche*, Vol. XIII (1880), pp. 555–592; the text of the *Triparty*, same volume, pp. 593–659 and pp. 693–814; the third section from 736–814 deals with quadratic equations and problems.

[5] G. Wertheim, *Die Arithmetic des Elia Misrachi* (Braunschweig, 1896), pp. 54–59.

[6] *Arithmetica practica, y speculativa* (Salamanca, 1562).

The algebra contained in this manuscript is not the same as the published work mentioned above. However, the method of treatment is not materially different. The work by Jordanus Nemorarius, entitled *De numeris datis*, dates from the early part of the thirteenth century. The importance of the work, chiefly with respect to the development of algebra, is well attested by the fact that Regiomontanus and Maurolycus[2] both planned to publish the work, although neither carried the plan to completion. The work was published in 1879 by Treutlein.[3] The Scheybl version contains the complete list of 113 propositions to which Chasles made reference in 1841. These are divided into four books containing respectively 30, 26, 22 and 35 propositions. Scheybl adds solutions by the rules of algebra[4] in which he employs the same algebraic notation as in his published algebra, but he does not give the complete text of the work of Jordanus. The other two works in the manuscript are presented in this monograph. The manuscript was carefully prepared, but, for some reason which we do not know, the publication was not accomplished.

---

[1] Breuis ac dilucida regularum Algebrae descriptio, autore Joanne Scheubelio, in inclyta Tubingensi academia Euclidis professore ordinario.

Huic accedit liber consumatissimi mathematici Jordani, de datis.

Liber praeterea, continens demonstrationes aequationum regularum Algebrae, Arabice olim conscriptus.

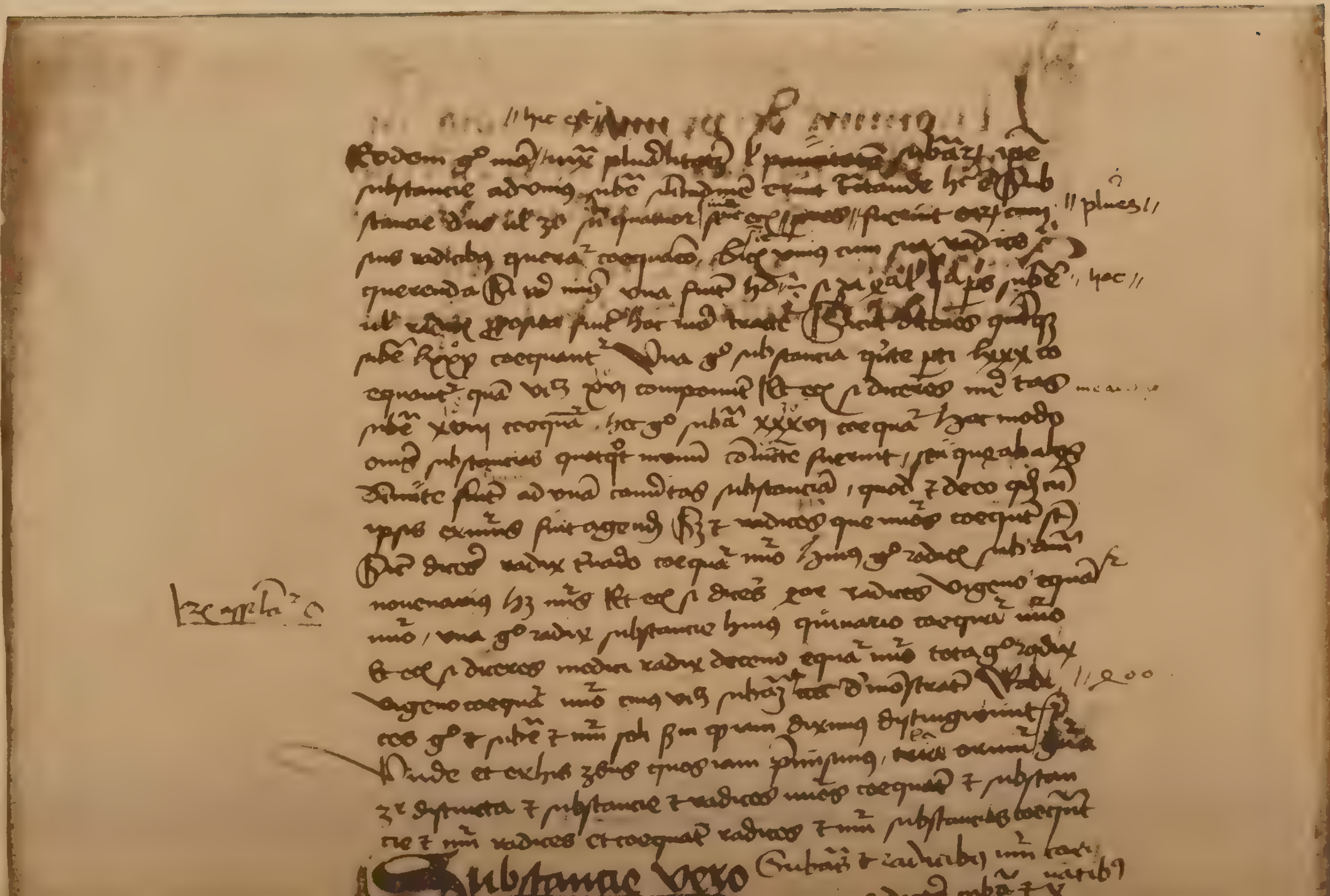
Quae (*corr.* Qui) ambo ab eodem Scheubelio nunc primum, quantum fieri potuit, emendato (*corr.* emendati) in lucem aedita, et aptissimis atque utilibus exemplis illustrata sunt.

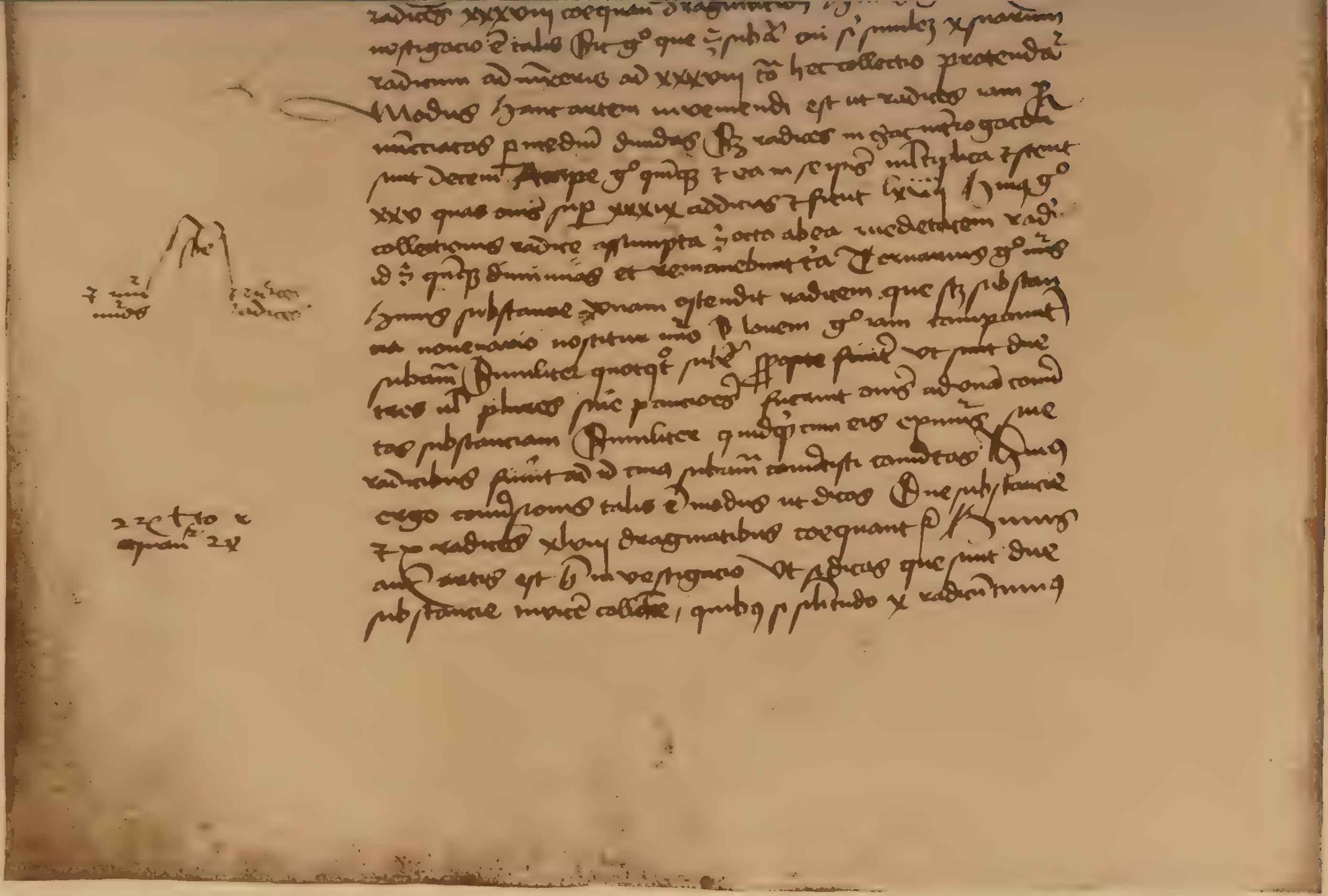
[2] Treutlein, *Abhandlungen zur Gesch. d. Math.*, Vol. II (Leipzig, 1879), pp. 127–128.

[3] Treutlein, *loc. cit.*, pp. 127–166; corrected by Curtze, *Zeitschrift f. Mathematik und Physik*, Vol. XXXVI (1891), pp. 1–23, 41–63, 81–95, and 121–138.

[4] Frequently adding, *Sequitur solutio ex regula Algebrae.*

PLATE II.





Codex Dresdensis C. 80, Fol. 340.^{b}

A few words about the life of Scheybl[1] may be of interest. His student days include an early stay at the University of Vienna, made famous in mathematical studies by Peurbach and Regiomontanus. In 1532 Scheybl matriculated at Tübingen, which was a stronghold of Protestantism, and in 1535 he was a student there. In 1540 he was Magister in Tübingen, and four years later Docent in mathematics. After another period of about five years we find him Professor (*ordinarius*) of Euclid, and in 1555 Professor of Euclid and Arithmetic. How little some aspects of university life have changed during four centuries is shown by the fact that Scheybl twice, in 1551 and 1562, requested of the university authorities an increase of salary in order that he might pay his debts and obtain the necessaries of life. In addition to the treatises on geometry and algebra which have been mentioned, Scheybl published other works on geometry and arithmetic.

As late as the end of the sixteenth century an able mathematician, Adrien Romain, deemed the algebra of Al-Khowarizmi sufficiently worthy of serious study to justify him in publishing a commentary on the work. The version upon which Romain based his study is that of Robert of Chester. In the course of his commentary he gives small portions of this translation. At the time of writing this, according to Bosmans[2] in 1598 or 1599, Romain was teaching in Würzburg. His student and teaching life covered periods of residence in Germany, Italy, and Poland, besides his native Louvain where he studied and taught.

---

[1] H. Staigmüller, *Johannes Scheubel, ein deutscher Algebraiker des XVI Jahrhunderts Abhand. zur Gesch. d. math. Wissenschaften*, Vol. IX, pp. 431–469.

[2] H. Bosmans, S.J., *Le fragment du commentaire d'Adrien Romain sur l'algèbre de Mahumed ben Musa el-Chowarezmi, Annales de la Société scientifique de Bruxelles*, Vol. XXX (1906).

CHAPTER V

The Arabic Text and the Translations of Al-Khowarizmi's Algebra

The Arabic text of Al-Khowarizmi's algebra, together with an English translation, was published by Frederick Rosen in 1831. This excellent[1] work is unfortunately out of print, and so is not available to most students of mathematics. The translation is made with care and intelligence, but not literally. Thus the Arabic invocation to the Deity is frequently omitted, just as it often is by modern translators.

The translation of the Algebra into Latin was made not only by Robert of Chester, but also, as we have indicated, by some other student of Arabic science who lived about the same time as Robert. This Latin version, as found in two Paris manuscripts, was published by Libri;[2] Björnbo believed that he had established[3] this to be the work of Gerard of Cremona, which indeed is probable. A list of the numerous translations due to Gerard was made soon after Gerard's death by some friend and admirer, and the list was published by Boncompagni.[4] Included among the titles is the algebra, *Liber alchoarismi de iebra et almucabula .tractatus I.* However, the question is somewhat complicated by the fact that a mediæval adaptation of the algebra which was published by Boncompagni[5] bears the name of Gerard of Cremona. The text of this version does not follow the Arabic at all closely, and there is little reason for considering it as a direct translation. Probably the meaning of the title[6] is that the text of this version is based upon Gerard's translation.

---

[1] Portions of the Arabic text and translation have been examined by Professor W. H. Worrell, to whose courtesy I am indebted for the information about the character of the translation.

[2] *Histoire des sciences mathématiques en Italie* (Paris, 1838), Vol. I, pp. 253–297.

[3] *Gerhard von Cremona's Übersetzung von Alkwarizmi's Algebra und von Euklid's Elementen*, *Bibliotheca Mathematica*, third series, Vol. VI (1905), pp. 239–248.

[4] *Della vita e delle opere di Gherardo Cremonese etc.*, *Atti dell' Accademia de' nuovi Lincei*, Vol. IV (1851), pp. 4–7.

[5] *Loc. cit.*, pp. 28–51.

[6] *Loc. cit.*, p. 28: Incipit liber qui secundum Arabes vocatur algebra et almucabala, et apud nos liber restauracionis nominatur, et fuit translatus a magistro Giurardo cremonense in toleto de arabico in latinum.

The Libri text varies essentially in phraseology and construction from that by Robert. The Arabic is closely followed up to the long list of problems, "Various Questions." Even here all the problems with the exception of two[1] are given in the Latin by Gerard, but not absolutely in the order in which they occur in the Arabic. The slight changes in the sequence of problems may well have been the fault of the particular Arabic manuscript which Gerard used, if it is not due to some transcriber of Gerard's work. One problem which is omitted is not very clear in the Arabic, but the second omission is a problem of the same type as others which are given. Some other slight omissions are made in the Latin text, and the longest of these corresponds to the passage in our text p. 84, line 25 to p. 86, line 2. Another omission in the Libri text corresponds to our text, page 74, line 25, *quod . . . reperies.* The Libri text also frequently omits the common invocation to the Deity which is so often interjected by Arabic writers.

The Latin translation by Robert of Chester is not as faithful nor as correct as the text ascribed to Gerard of Cremona, published by Libri. Omissions, transpositions, and additions to the text are so numerous that it does not seem desirable to list them all. No evidence exists, however, that Robert's text is based upon another Arabic original than that of the Libri text. The text proper, as opposed to the illustrative problems, follows the general lines of the Arabic original. The longest omission is the section dealing largely with the operations upon the square root of 200, which is illustrated, in the Arabic and in the Libri text, by geometrical figures with corresponding demonstrations.[2]

A sentence is left out on page 98 of our text, line 6, after the word *aequiparatur*. This sentence Rosen translates, 'Compute in this manner every multiplication of the roots, whether the multiplication be more or less than two.' Lines 9–11, *Natura . . .*

---

[1] Rosen's translation, p. 48, line 15 to p. 50, line 5, and p. 53, lines 12–20. Neither of these problems is given by Robert of Chester, nor does either appear in the Boncompagni version. The first problem reads: "If some one say: 'I have purchased two measures of wheat or barley, each of them at a certain price; I afterwards added the expenses, and the sum was equal to the difference of the two prices, added to the difference of the measures.'" The second reads: "Three-fourths of the fifth of a square are equal to four-fifths of its root."

[2] Rosen, *loc. cit.*, p. 27, lines 5–18, and p. 31, line 11, to the bottom of p. 34; Libri, *loc. cit.*, p. 269, lines 2–12, and p. 271, line 16–p. 274, line 14. The Libri version omits the statement of one problem, as stated by Rosen, p. 27, lines 14–16, but the geometrical explanation is complete.

fractionibus, on the same page of our text, seems to be an addition by Robert. The introduction of the passage, 'On Mercantile Transactions,' pp. 120–124, is not at all carefully translated by Robert, who retains poor transliterations of four technical expressions used in the Arabic. The four expressions in question refer to the four terms of a proportion in which when three are given the fourth is determined. If a given quantity of goods is sold at a fixed or set price, then the price of any other quantity of the same goods, or the amount of goods to be obtained for a given sum of money, is determined by a proportion in which the three given quantities enter. The unit of measure, or quantity sold at a fixed price, is termed by Robert *Almusarar*, instead of *al-musa-ʿir*, and the fixed price *Alszarar*, instead of *al-siʿr;* the quantity of goods desired is *Almuthemen*, instead of *al-muthamman* and the amount to be expended for goods is termed *Althemen*, instead of *al-thaman*. *Magul*, which is used by Robert for the unknown term in a proportion, would be in modern transliteration *al-maqul.*

Robert of Chester does not present the complete list of problems which occur in the Arabic text of Al-Khowarizmi's algebra, but only a selection of about one-half of the total number. Upon what basis this selection was made does not appear, except that typical problems are chosen, and the repetitions which are found in the Arabic and the Libri text are eliminated. In the footnotes to our English version we have indicated the problems which have been omitted by our author.

The translation of the text and solutions of the problems which are given present peculiarities entirely similar to those which have been noted in the preceding discussion of the Latin text by Robert. A noteworthy omission is made both by Robert of Chester and by the translator of the version published by Libri. This concerns the fifth problem of the set of six which illustrate in order each of the six types of quadratic equations. After the solution of the problem to the point to which our text[1] carries the problem, the Arabic, as translated by Rosen, adds: 'Or, if you please, you may add the root of four to the moiety of the roots; the sum is seven, which is likewise one of the parts. This is one of the problems which may be solved by addition and subtraction.'

---

[1] Page 108, lines 1–13.

CHAPTER VI

PREFACE AND ADDITIONS FOUND IN THE ARABIC TEXT OF AL-KHOWARIZMI'S ALGEBRA

THE Arabic text of Al-Khowarizmi's algebra published by Rosen contains an author's preface which is not found either in the translation published by Libri, or in that by Robert of Chester. As this reveals his conception of the purpose of the algebra, as well as some of the causes which led him to undertake the work, we present it here in the translation by Rosen.[1] Such prefaces in Arabic works usually, just as this one, contained invocations to the Deity and to Mohammed his prophet: in consequence the Christian translators, who were commonly connected with the Church, were wont to leave them out. A summary of the sections in the Arabic text which appear in neither of the Latin translations is also given since the Arabic-English work by Rosen is not widely available, and since these additions show that Al-Khowarizmi had grasped the possibility of the application of the algebra to geometry and trigonometry. This application is frequently neglected to-day by teachers of elementary algebra.

THE AUTHOR'S PREFACE

"In the Name of God, gracious and merciful!"

"This work was written by Mohammed ben Musa, of Khowarezm. He commences it thus:

"Praised be God for his bounty towards those who deserve it by their virtuous acts: in performing which, as by him prescribed to his adoring creatures, we express our thanks, and render ourselves worthy of the continuance (of his mercy), and preserve ourselves from change: acknowledging his might, bending before his power, and revering his greatness! He sent Mohammed (on whom may the blessing of God repose!) with the mission of a prophet, long after any messenger from above had appeared, when justice had fallen into neglect, and when the true way of life was sought for in vain. Through him he cured of blindness, and saved through him from perdition, and increased through him what before was small, and collected through him what before was scattered. Praised be God our Lord! and may his glory increase, and may all his names be hallowed — besides whom there is no God; and may his benediction rest on Mohammed the prophet and on his descendants!

---

[1] Rosen, *The Algebra of Mohammed ben Musa*, pp. 1–4.

"The learned in times which have passed away, and among nations which have ceased to exist, were constantly employed in writing books on the several departments of science and on the various branches of knowledge, bearing in mind those that were to come after them, and hoping for a reward proportionate to their ability, and trusting that their endeavors would meet with acknowledgement, attention, and remembrance — content as they were even with a small degree of praise; small, if compared with the pains which they had undergone, and the difficulties which they had encountered in revealing the secrets and obscurities of science.

"Some applied themselves to obtain information which was not known before them, and left it to posterity; others commented upon the difficulties in the works left by their predecessors, and defined the best method (of study), or rendered the access (to science) easier or placed it more within reach; others again discovered mistakes in preceding works, and arranged that which was confused, or adjusted what was irregular, and corrected the faults of their fellow-laborers, without arrogance towards them, or taking pride in what they did themselves.

"That fondness for science, by which God has distinguished the Imam al Mamun, the Commander of the Faithful (besides the caliphat which He has vouchsafed unto him by lawful succession, in the robe of which He has invested him, and with the honours of which He has adorned him), that affability and condescension which he shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties, — has encouraged[1] me to compose a short work on Calculating by (the rules of) Completion and Reduction, confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, law-suits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computation, and other objects of various sorts and kinds are concerned — relying on the goodness of my intention therein, and hoping that the learned will reward it, by obtaining (for me) through their prayers the excellence of the Divine mercy: in requital of which, may the choicest blessings and the abundant bounty of God be theirs! My confidence rests with God, in this as in everything, and in Him I put my trust. He is the Lord of the Sublime Throne. May His blessing descend upon all the prophets and heavenly messengers!"

The Arabic version differs from the Latin translations which have come down to us, in giving an extended discussion of inheritance problems and also in discussing geometrical measurements. In the English translation by Rosen the inheritance problems, involving largely legal questions rather than algebraical ones, occupy 79 pages as opposed to 70 for the algebra proper. The mensuration problems take some sixteen pages of text. The formulas are given for the area of a square and triangle. Three formulas are given for the circumference of a circle and the writer evidently recognizes them all as approximations. The formulas are:

---

[1] Several writers have asserted that the work of Al-Khowarizmi was written at the request of the caliph. The text shows that this is not Al-Khowarizmi's statement of the case.

See Woepcke, *Extrait du Fakhri*, p. 2; A. Marre, *Le Messâhat de Mohammed ben Moussa al Khârezmî, extrait de son algèbre*, in *Annali di matematica*, Vol. VII, first series, Rome, 1865, pp. 269–280.

(1) $c = 3\frac{1}{7} d,$

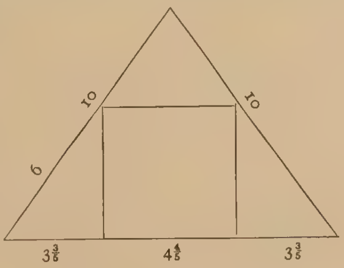
(2) $c = \sqrt{10 d^2}$

(3) $c = \frac{62832\, d}{20000}$

The area of a circle is given as $A = d^2 - \frac{1}{7} d^2 - \frac{1}{2}$ of $\frac{1}{7} d^2$. Other simple areas and volumes are discussed. Application of the algebra is found in two problems. One of these deals with finding the altitude of a triangle of which the sides are given; the other with inscribing a square in a given triangle.

As the problems, on finding the altitude of a triangle, being given the lengths of the sides, and on inscribing in an isosceles triangle a square, show that Al-Khowarizmi had an appreciation of the possibilities of the algebra, I present one of the problems, following Rosen's translation.

> "If some one says: 'There is a triangular piece of land, two of its sides having 10 yards each, and the basis 12; what must be the length of one side of a quadrate situated within such a triangle?' the solution is this. At first you ascertain the height of the triangle, by multiplying the moiety of the basis, (which is six) by itself, and subtracting the product, which is thirty-six, from one of the two short sides multiplied by itself, which is one-hundred; the remainder is sixty-four; take the root from this; it is eight. This is the height of the triangle. Its area is, therefore, forty-eight yards: such being the product of the height multiplied by the moiety of the basis, which is six. Now we assume that one side of the quadrate inquired for is thing. We multiply it by itself; thus it becomes a square, which we keep in mind. We know that there must remain two triangles on the two sides of the quadrate, and one above it. The two triangles on both sides of it are equal to each other: both having the same height and being rectangular. You find their area by multiplying thing by six less half a thing, which gives six things less half a square. This is the area of both the triangles on the two sides of the quadrate together. The area of the upper triangle will be found by multiplying eight less thing, which is the height, by half one thing. The product is four things less half a square. This altogether is equal to the area of the quadrate plus that of the three triangles: or, ten things are equal to forty-eight, which is the area of the great triangle. One thing from this is four yards and four-fifths of a yard; and this is the length of any side of the quadrate. Here is the figure:"



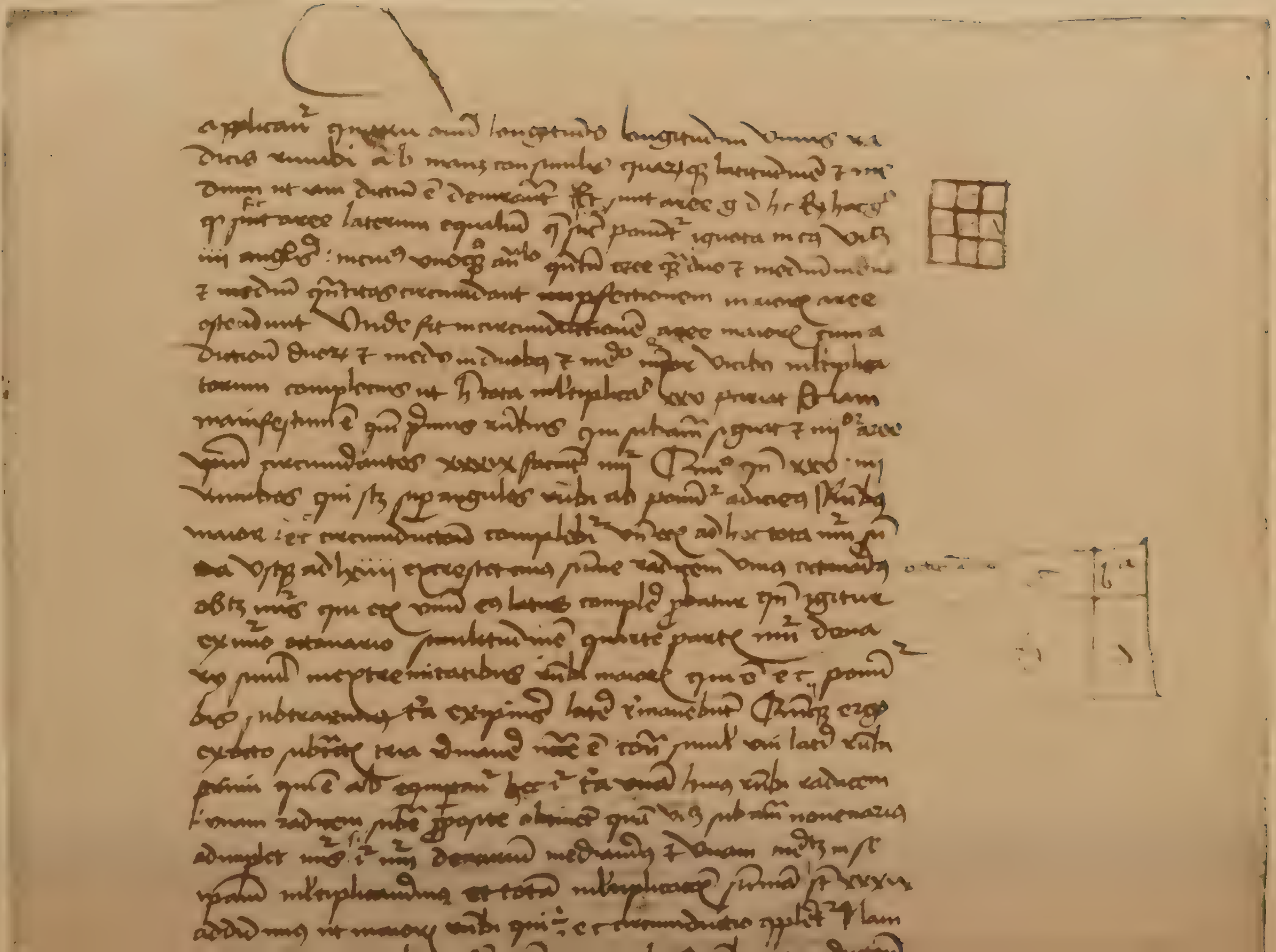
The inheritance problems occupy a large part of the original work; the inclusion of one of these problems here will perhaps not be amiss. Only the first of the problems is given since the following problems are of the same general nature, involving other legal peculiarities.

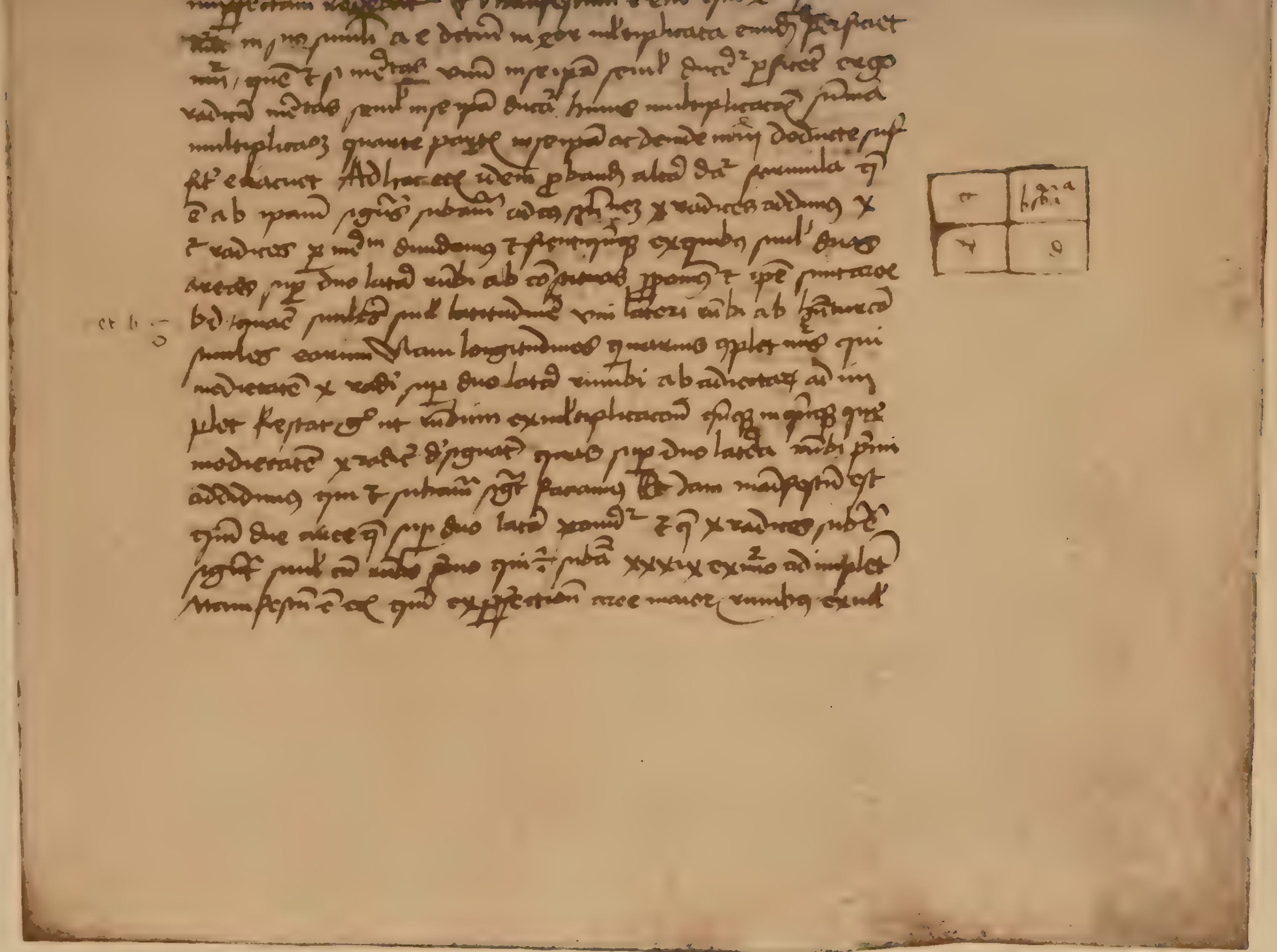
> "A man dies, leaving two sons behind him, and bequeathing one-third of his capital to a stranger. He leaves ten dirhems of property and a claim of ten dirhems upon one of the sons.
>
> "Computation: You call the sum which is taken out of the debt thing. Add this to the capital which is ten dirhems. The sum is ten and thing. Subtract one-third of this, since he has bequeathed one-third of his property, that is, three dirhems and one-third of thing. The remainder is six dirhems (and two-thirds) and two-thirds of thing. Divide this between the two sons. The portion of each of them is three dirhems and one-third plus one-third of thing. This is equal to the thing which was sought for. Reduce it, by removing one-third from thing, on account of the other third of thing. There remain two-thirds of thing, equal to three dirhems and one-third. It is then only required that you complete the thing, by adding to it as much as one-half of the same; accordingly, you add to three and one-third as much as one-half of them: This gives five dirhems, which is the thing that is taken out of the debts."

The legal point involved in the problem given is that a son who owes to the estate of his father an amount greater than the son's portion of the estate, retains, in any event, the whole sum which he owes. Part is regarded as his share of the estate, and the remainder as a gift from the father. The above problem would have given exactly the same numerical results for any debt from five dirhems up; however, if there were a claim of four dirhems against one of the sons, instead of ten, the debtor son would have received in cash $\frac{2}{3}$ of one dirhem, the other son four and $\frac{2}{3}$ dirhems, and the stranger four and $\frac{2}{3}$ dirhems.

In algebraical symbolism, the equation is $\frac{2}{3}(10+x) = 2x$ whence $x = 5$; $10 + x$ is the total estate left, and x is the share of each son.

PLATE III.





Codex Dresdensis C. 80, Fol. 342.a

CHAPTER VII

MANUSCRIPTS OF ROBERT OF CHESTER'S TRANSLATION OF AL-KHOWARIZMI'S ALGEBRA

I. THE EXTANT MANUSCRIPTS

STEINSCHNEIDER[1] was the first in recent times to call attention to the translation of Al-Khowarizmi's algebra made by Robert of Chester. He suggested the desirability of publishing this text, referring to the manuscript in Vienna. To this same manuscript Curtze[2] later, and independently of Steinschneider, directed attention and also suggested the desirability of the publication of the work. Wappler,[3] in 1887, found a second copy of the algebra in a manuscript in Dresden, while some time later David Eugene Smith acquired for the Columbia University Library a manuscript from the hand of Johann Scheybl which contains a third transcription of the algebra.

In addition to these manuscript copies of the text, a fragment of the translation was published by Adrien Romain of Louvain in 1599, in a work bearing the title *Commentaire sur l'algèbre de Mahumed ben Musa el Chowarezmi*. Unfortunately but a fragment of this published work had been preserved to modern times, and that precious fragment was doubtless destroyed with other and rarer books and manuscripts in the recent destruction of the University at Louvain. This work of Romain's was mentioned in a work of 1643, published at Louvain, as being found in the Library there. Henri Bosmans, S. J., of Brussels has given a description[4] of the work, mentioning the fact noted by Romain that the latter had obtained an excellent manuscript copy of the algebra from his friend Thaddeus Hagec of Prague. I am indeed fortunate, through the courtesy of Professor Bosmans and of Professor B. Lefebvre of Louvain, to be able to include this fragment in my collation. Another fragment in Ms. was found after the textual notes were in type; this brief portion from Codex Dresdensis C. 80$^{\mathrm{m}}$, of the fifteenth or sixteenth century, is included later in this chapter.

---

[1] Steinschneider, *Zeitschrift d. deutsehen morgenländ. Gesellschaft*, Vol. XXV (1871), p. 104; *Zeitschrift f. Mathematik*, Vol. XVI (1871), pp. 392–393; *Bibliotheca Mathematica*, third series, Vol. I (1900), pp. 273–274; *Sitzungsbericht d. Akad. d. Wissenschaften in Wien, Phil. hist. Kl.*, Vol. CXLIX (1904), p. 72.

[2] Curtze, *Centralblatt fur Bibliothekswesen*, Vol. XVI (1899), p. 289.

[3] Wappler, *Zur Geschichte der deutschen Algebra im 15. Jahrhundert*, Programm (Zwickau, 1887), pp. 1–2.

[4] Bosmans, *Le fragment du commentaire d'Adrien Romain sur l'algèbre de Mahumed ben Musa el-Chowarezmi*, *Annales de la Société scientifique de Bruxelles*, Vol. XXX, part II (1906).

Yet another fragment of the algebra was published by Wappler,[1] who did not, however, ascribe the passage to Robert of Chester. The failure to connect it directly with the algebra in the same manuscript elsewhere mentioned by Wappler was due in part to the fact that the section in question, which is found p. 120, l. 21 to p. 124, l. 16 of our text, is not in its proper place in the Dresden manuscript. The same paragraphs are found also in a manuscript of the University Library at Leipzig, Codex Lips. 1470. I am indebted for information concerning this manuscript to the courtesy of Director Boysen and Dr. Helssig, of the University Library in Leipzig. It appears that this manuscript is almost entirely from the hand of Magister Virgilius Wellendorfer of Leipzig, written during his student days, between the years 1481 and 1487. The passage referred to was in all probability copied from the Dresden Codex C. 80. Apparently Wellendorfer had the intention of copying the algebra entire, for on folio 478$^{\mathrm{a}}$ we find the title, *Textus algabre edidit Mahume Mosi filius*. To the title he added a brief note, Sed utitur aliis nominibus . . . substantia et dragma, radicis . . . algorithmi. These words clearly indicate some acquaintance with our text, but the text itself is not found in the manuscript.

The Scheybl Ms. (C.), written in 1550, was evidently intended for publication. In printing Robert of Chester's text I have thought it best to follow the Ms. which was prepared by Scheybl. Although it contains some errors, and slight additions by Scheybl, these are quite easily distinguished. The advantage of following Scheybl's careful revision seemed obvious; particularly the chapter divisions and sub-titles, many of which he supplied, make the text easier to follow. That the reader may have before him all the evidence in regard to the text, the readings of the other Mss. are recorded in the critical notes. These may seem needlessly full on account of the inclusion of many apparently unimportant variants; yet in any attempt to determine the parentage of the Mss. such evidence frequently possesses a significance which is not at first sight apparent.

---

[1] Wappler, *Zur Geschichte der Mathematik, Zeitschrift f. Math. und Physik*, Vol. XLV (1900), *Hist. lit. Abth.*, pp. 55–56.

The Dresden Codex C. 80^m, of which I have photographic copies through the courtesy of the director of the Dresden library, contains a fragment of Robert of Chester's translation of the algebra of Al-Khowarizmi. The passage is introductory to a work on arithmetic; the latter contains portions of the algorism by Sacrobosco, and of the algorism in verse by Alexander de Villa Dei, and also parts of the commentary on the former by Petrus de Dacia (1291), together with other comment and further exposition. The "Rules corresponding to the rules of algebra," which we have reproduced on page 126 from the Vienna Ms. are also included with other algebraic material in this mathematical Ms. Here these rules[1] follow closely the Vienna text, whereas similar rules[2] in Codex Dresden C. 80, fol. 351^a, do not.

The fragment from Codex Dresden C. 80^m follows:

> "Incipit liber restauracionis numeri quem edidit machumed filius moysi algauriszmi quare dixit machumued.
>
> "Laus deo creatori qui homini contulit scientiam inveniendi vim numerorum. Considerans enim omne id quo indigent homines ex numero, inveni id totum esse numerum. Et nil aliud esse numerum nisi quod ex vnitatibus componitur. Vnitas ergo est qua vnaquaque res dicitur vna et vnitas in omni numero reperitur. Inveni autem omnem numerum essentialiter ita dispositum vt omnis numerus vnitatem excedat vsque ad 10. Decenus quoque numerus ad modum vnitatis disponitur, vnde et duplicatur et triplicatur quemadmodum factum est ex vnitate. Fiuntque ex eius duplicatione 20, ex eius triplicatione 30. Et sic multiplicando decenum numerum ad centenum peruenitur. Proinde centenus numerus duplicatur et triplicatur, etc. ad modum numeri deceni. Et sic centenum numerum duplicando et triplicando, etc. Millenus excrescat numerus. Ad hunc ergo modum millenus numerus ad modos numerorum, vsque ad infinitam numeri investigationem conuertitur."

In this passage the sentence, "Vnitas ergo est qua vnaquaque res dicitur vna," is interjected from Sacrobosco's algorism; it is a translation of Euclid's definition of a unit. The work which immediately follows concerns arithmetic proper, and the remainder of the material in the Ms. is also mathematical.

---

[1] Published by Wappler, *Programm* (Zwickau, 1887), note 1, page 14.

[2] Published by Wappler, *loc. cit.*, pp. 13–14.

The Vienna Ms. (V) is assigned in the catalogue of Mss. of the Vienna library to the fourteenth century, and the character of the writing agrees with this dating. In the fifteenth century Peurbach and Regiomontanus, diligent students of Arabic mathematics, were connected with the University of Vienna, which was then a mathematical centre. This copy itself may have been acquired by Regiomontanus for the library at Vienna; other Mss. from the hand of Regiomontanus are preserved in Vienna.

The Dresden Codex C. 80 (D) was written in the latter part of the fifteenth century. Wappler[1] states that Johann Widmann of Eger, whose activity at the University of Leipzig falls at the end of the fifteenth century, transcribed certain portions of the Dresden Codex, but the writer of our text is not known. Adam Riese, in the beginning of the sixteenth century, also used this Ms. The algebra of our text begins on folio 340^a and terminates in the middle of folio 348^b; the section relating to commercial problems is not found in its proper place, but appears in the Ms. on folio 301^a. For the collation of this latter section I have used the printed text, mentioned above, by Wappler.

II. THE VIENNA MANUSCRIPT (V)

Codex Vindobonensis 4770 (Rec. 3246) XIV. 339.8° (V).

Fol. 1^a–12^b, *Liber restaurationis et oppositionis numeri;* our text.

Fol. 13^a–40^a, M. Jordanus Nemorarius, *De numeris datis*, incorrectly designated in the catalogue as the *Tractatus arithmeticus* by the same author. Many manuscript copies are extant, and the text was published by Treutlein in Vol. II, *Abhandlungen zur Geschichte der Mathematik* (Leipzig, 1879), pp. 135–166, but from an incomplete manuscript of the work. The necessary additions are given by Curtze, *Commentar zu dem "Tractatus de Numeris Datis" des Jordanus Nemorarius*, *Zeitschrift für Mathematik und Physik*, *Hist.-lit.-Abth.*, Vol. XXXVI, pp. 1–23, 41–63, 81–95, and 121–138.

Fol. 40^b–44^b, blank.

---

[1] Wappler, *loc. cit.*, *Programm*, p. 9.

Fol. 45^{a}–50^{b}, *Tractatus geometricus cum figuris*. Begins, "Punctum est cuius pars non est . . ." and ends, "equaliter distantes fuerint constituti."

Fol. 51^{a}–173^{b}, blank.

Fol. 174^{a}–324^{b}, *Carmen quadripartitum de matematica cum commentario subnexo*. This is the *Quadripartitum numerorum* by Johannis de Muris, of which many manuscript copies are extant. The work as a whole has never been published. Two chapters of the second book which relate to practical arithmetic have been published by Nagl, *Abhandlungen zur Geschichte der Mathematik*, Vol. V (Leipzig, 188), pp. 135–146. I have made a study of the third book and I have given selections from the metrical portion of the work, as well as a few passages from the third book and from other parts of the work, *Bibliotheca Mathematica*, third series, Vol. XIII (1913), pp. 99–114.

Fol. 325^{a}–327^{b}, blank.

Fol. 328^{a}–337^{a}, *Tractatus de ponderibus* (in fine mutilus). This treatise begins, "Marcha est limitata ponderis . . ." and ends, "ad tertium altare et deinceps." This is also the work of Jordanus Nemorarius, and has not been edited.

Fol. 337^{b}–338^{b}, blank.

Fol. 339^{a}, *Notabile de algorismo proportionum.*

III. THE DRESDEN MANUSCRIPT (D)

Codex Dresdensis C. 80. Fifteenth century. 416 pages, numbered 1–225, 227–417. Paper. By various hands. Folio, bound in parchment (D).

Page 1, Vnum dat finger brucke duo. . . . etc. Evidently on finger-reckoning.

Page 1^{b}–5^{b}. Large portions of the algorism of Sacrobosco.

Page 6^{a}. *Divisio numeri.*

Page 6^{b}–9^{b}, *Schachirica mercatorum computatio*, etc. Various fragments.

Pages 10, 20–23, 84–128, 145–153, 173–176, 186–190, 198–200, 220–233, 246–257, 269–279, 327–339, and 381–384 are blank.

Page 11–19, Arithmetic of Johannis de Muris. This is a fragment of the *Arithmetica communis* of de Muris which was printed in 1515 at Vienna, and again, as *Arithmeticae speculativae Libri duo*, at Mainz in 1538.

Page 24–71^{b}, Arithmetic of Boethius, of which there are numerous editions, with a critical edition by Friedlein (Leipzig, 1867).

Pages 72–83. Excerpts from the arithmetic of Boethius.

Pages 129–134. *Regule de Alegorismo.* The algorism (of integers) published by Boncompagni (*Trattati d'arithmetica*, Rome, 1857, pp. 25–49).

Pages 135–142^{b}. *Sequitur de phisicis.* Probably the discussion of fractions in the preceding algorism.

Pages 142^{b}–144. *Carmen de ponderibus.*

Begins, Pondera postremis Veterum memorata libellis; ends, Argentum argento liquidis cum mergitur Vndis.

Pages 154–157. *Incipit liber de Sarracenico et de Limitibus et cetera Orelibacio de Abacis. Latino. Arabico. Sarracenico et de Limitibus.*

Begins, Quoniam ad Raciones quasdam presentis libelli.

Pages 157^{b}–166. *De diuersitate fraccionum capitulum primum. De additione et duplatione. . . . De radicum extractione capitulum* 14.

Begins, Quoniam in precedentibus frequenter contingit; ends, Et haec de computacione fraccionum sufficiant.

Pages 167–172. *Incipit Canon Magistri Johannis de Muris super tabula tabularum que dicitur proporcionum.*

Canon by John of Meurs, edited by John of Gmunden in 1433.

Pages 177–185^{b}. *Jordanus Nemorarius de minuciis libri duo.*

See Eneström, *Das Bruchrechnen des Jordanus Nemorarius*, *Bibliotheca Mathematica*, third series, Vol. XIV, pp. 41–54.

Pages 191–197^{b}. *Quid sit proporcio capitulum primum. De* 3^{ci} *Manerie proporcionis capitulum secundum . . . De diuisione proporcionum cap.* 10. *Quantitatem aliquam mensurare.*

Pages 201^{a}–206^{a}, *Algorithmus proportionum*, by Nicholas Oresme. Published by Curtze, *Programm* (Thorn, 1868).

Pages 206^{b}–219^{b}, 234–245^{b}. (*De proporcionibus.*)

Pages 258–268^{b}. *Regule super rithmachiam.* On the ancient game called *Rythmomachia* of which there are three standard treatises published; see Smith, *Rara Arithmetica*, p. 271, and a description of the game by Smith and Eaton, American Mathematical Monthly, Vol. XVIII (1911), pp. 73–80.

Pages 280^{a}–285^{b}, *Algorithmus minuciarum* by Johannes de

Lineriis (fourteenth century). Printed at Padua, 1483, and Venice, 1540.

Pages 286^a–291^a, 292^b–300^b, 303^b–305^a, 306^a–315^b.

Various fragments, including portions of the *Quadripartitum numerorum* by John of Meurs; see Wappler, *Programm*, pp. 7, 31–32.

Pages 291^b–292^a, *Algorithmus de duplici differencia.*

Page 301^a. The passage of our text relating to commercial transactions, in the handwriting of Johann Widmann of Eger.

Pages 301^b–303^a, Mathematical lecture by Gottfried de Wolack, written 1467 or 1468.

Pages 316^a–323^b. *De numeris datis*, by Jordanus Nemorarius.

Pages 331^a–334^b. *Pro regularum Algabre.*

Pages 340^a–348^b. Our text.

Pages 349^a–365^b. Latin algebra, published by Wappler, *Programm* (Zwickau, 1887), pp. 11–30.

Pages 366–367^b. *Cautelae Magistri Campani ex libro de Algebra siue de Cossa et Censu.*

Pages 368–378^b. An algebra in German, somewhat similar to the preceding Latin algebra; see Wappler, *Programm*, and *Abhandl. z. Geschichte d. Math.*, Vol. IX, pp. 539–540, where it is discussed.

Pages 379–380, various.

Pages 385–397^b. *De mensuratione terrarum et corporum*, translated by Gerard of Cremona. Not published. Includes algebraic problems.

Pages 397^b–406. *Liber augmenti et diminutionis;* published by Libri, *Histoire des sciences mathématiques en Italie*, Vol. I, (Paris, 1838), pp. 304–371.

Pages 406^b–407, 409–417. Various.

The above description is based upon the *Katalog der Handschriften der Königl. Öffentlichen Bibliothek zu Dresden* (Leipzig, 1882) by Schnorr von Carolsfeld, and upon the articles by Wappler, as cited above.

IV. THE COLUMBIA UNIVERSITY MANUSCRIPT (C)

Codex Universitatis Columbiae; Columbia University Library Manuscript, X 512, Sch. 2, Q. 308 pages. (C.)

Pages 1–68, *Breuis ac dilucida regularum Algebrae descriptio* *autore Joanne Scheubelio, in inclyta Tubingensi academia Euclidis professore ordinario.* This is a briefer treatment of the algebra than that published by Scheybl in 1550, as a preface to the first six books of Euclid, *Euclidis Megarensis, Philosophi Mathematici excellentissimi, sex libri priores. . . . Algebrae porro Regulae, propter numerorum exempla, passim propositionibus adiecta, his libris praemissae sunt, eaedemque demonstratae* (Basle, 1550). This algebra was published, separate from the Euclid, in Paris, in 1551. The text of the algebra in the Columbia manuscript has not been published.

Pages 69–70, blank.

Pages 71–122, *Liber Algebrae et Almucabola, continens demonstrationes aequationum regularum Algebrae.* Our text.

Pages 123–157, *Addita quaedam pro declaratione Algebrae*, by Scheybl. This is explanatory of the preceding. It is printed on pages 128 to 156 of this book.

Page 158, blank.

Pages 159–308, *Liber Jordani Nemorarii de datis in quatuor partes digestus.* This is not the complete text of the *De numeris datis*, but contains the statement of the problems and their solutions according to the rules of algebra. In these solutions Scheybl introduces the use of N for number, *co.* for *cosa*, for the first power of the unknown, and a symbol which is very similar to the square root sign for the second power of the unknown, or *substantia*.

V. THE RELATIONS OF THE MANUSCRIPTS

The determination of the relation between the Vienna and Dresden Mss., as well as the relation of the Scheybl Ms. and the two fragments to each of these, is based not only upon a study of particular words and phrases but in large measure upon the omissions made by the various scribes in copying. The main test used is, appropriately enough for these mathematical Mss., an arithmetical one.[1] In connection with the omissions we may observe, as noted by Havet,[2] that a copyist passes easily by error from a given ending or word to a similar ending or word which occurs later in the contiguous text; this type of error is the most frequent one made in copying Mss. The "jump from like to like" is particularly prone to occur when the parallel passages recur in similar parts of neighboring lines; in this event the omission approximates the length of a line. Another frequent error in copying is to omit entirely one or more lines or to repeat a whole line. An examination of the lengths of all such passages leads to a fairly definite notion of the length of line of the parent manuscript. It need hardly be stated that any omission is more readily made when the appearance, at least, of sense is preserved after the omission. In our mathematical text the recurrence of similar words and phrases is frequent, and in many places the omission of a line is possible with the preservation of a measure of meaning.

---

[1] A. C. Clark, *The Primitive Text of the Gospel and Acts* (Oxford, 1914), pp. i–vii and 1–10.

[2] L. Havet, *Manuel de critique verbale appliquée aux textes latins* (Paris, 1911), pp. 130, 200.

In Mss. which antedate the tenth century, the length of any omission is determined with comparative ease as a certain number of letters, for few abbreviations, and those of a standard type, were used. However, in Mss. of the type of the Dresden and Vienna Mss., with which we are primarily concerned, the length in letters of any omission is by no means a fixed and definite quantity. The abbreviations used by the copyists of the twelfth to the fifteenth century commonly varied, not only from page to page, but even from line to line. On the first page of the Vienna Ms. (Plate I), where the conscious effort would be made, probably, to be uniform in notation, the copyist wrote *t'bus* in line 18, and *tribus* in line 26, while on the following page he wrote *tbꝰ*, and elsewhere he writes 3^{b}ꝰ. Another possible form is *tribꝰ* although this does not appear in our Ms. This word then could count either for 3, 4, 5, or 6 letters. Similarly on this same first page (line 11) *duplicatione* is written *duplica*^{oe}, while *triplicatione*, which immediately follows, is written *t'plicatione*. In the Dresden Ms. there are entirely similar variations. Thus in line 5 of fol. 340^{b} (Plate II) 5^{a} appears for *quinta*, in line 7 *q'nte* for *quintae*, *quinario* in line 15, and *qūiq3* and *qūq3* in lines 29 and 32 for *quinque;* in line 37 *substanciam* appears in full and in the next line as *subâm*.

In counting the number of letters in any omission I have assumed the common abbreviations used in the contiguous passages of the Vienna Ms.; for as that is quite certainly the oldest of the Mss. which we are examining, it is in all probability more nearly like the parent Ms. whose existence is established with considerable certainty by our study. On the first page of the Vienna Ms. (Plate I) the line varies in length from 35 to 48 letters, with an average of $40\frac{1}{4}$ letters, while on the first half of fol. 340^b of the Dresden Ms. the line varies from 40 to 52 letters with an average of $45\frac{1}{3}$ letters. These facts give an indication of the latitude in variation which we may assume in the line length of the parent Ms.

The omissions of the Vienna Ms. will first be investigated as that is the oldest of our texts. In the text of page 84, lines 7–9, the parent Ms. evidently read:

Sed linea b h similis est linee g d. Nam quoniam linea g l et linea h e in quantitate habentur consimiles quoniam linea g l similis est linee d e.

The Vienna scribe passed from the first *similis est linee* to the second. In the text of the same page, lines 12–13:

Area igitur quam linee *t e*, *e a* circumdant similis est aree quam linee *m l*, *l d* circumdant. Area igitur . . .

The scribe dropped from the one *Area igitur* to the other.

In the text given in lines 8–9 of the footnote to lines 1–12, page 88:

. . . similis est aree quam *c t*, *t l* similis quam *t e* circumdant. Area ergo *n z* similis est aree . . . the scribe of the Vienna Ms. passed from the first *similis est aree* to the second. On page 112, text of lines 1–4:

. . . absque xx rebus. Rem quoque in re multiplicata, et erit substantia. Hec insimul iunge et erunt c et due substantie absque xx rebus. The omission can be conceived of here as of the text between *absque* and *absque*, or xx and xx, or between *rebus* and *rebus*, and similarly in the preceding illustrations. So in line 18, page 116 (footnote): in duo et 4^a et erunt xx et 4^{ta}, et multiplica v radices in duo et 4^{ta} et erunt 11 res et 4^{ta}. . . . the text is omitted between any word of the first *in duo et 4^a et erunt* and the same word where this phrase recurs.

These five passages are in length, roughly, 45, 43, 38, 54 to 60, and 41 letters, respectively. They indicate a line length of about 40 letters in the parent Ms. One omission which is not from like to like appears to be that of a complete line in the text of page 104, lines 22–24. The length of this omission is 45 letters. On page 110, in line 27, there is an omission by the Vienna scribe of some 36 letters, but as the passage is not at all clear in the Dresden Ms., having *unamquamque* where *antequam unamquamque* should have been written, this may have been a deliberate omission. The omission in the text of page 100, line 19, which is short (about 20 letters) may also have been deliberate as the meaning is not clear. Another short omission is found in the text to page 78, line 12.

As these omissions in the Vienna Ms. correspond to portions of the Arabic text, and as all are found in the Dresden Ms., they establish the fact that the Vienna Ms. is not the ancestor of the Dresden Ms.; similarly the omissions of the Dresden Ms. are found in the Vienna Ms., proving that the Dresden could not be the parent Ms. of the Vienna Ms., entirely apart from the fact that the Vienna is quite definitely older.

In the Dresden Ms. some eleven omissions require examination. Of these eight are instances where the scribe has passed from one word or phrase to a similar word or phrase recurring in the adjacent text. Such omissions are, as we have noted, quite likely to be about the length of the line of the parent Ms., but may be materially shorter or longer. Five of these omissions also suggest a line of about forty letters in the parent Ms. Four of these omissions are rectified by marginal additions made by a second hand, but this does not change their value as indicating the length of line in the parent. Four of these omissions are indicated in the footnote to line 6, page 92. The first from *fuerint* to *fuerint;* the second from *sine re* to *sine re;* the third from *procreant* to *procreant,* and so also the fourth although here a 10 intervenes. The lengths in letters are 37, 43, 16, and 38, respectively. In line 7 on the same page is another short omission, from like to like, in the expression, et eius sextam in dragma et eius sextam; the Dresden Ms. omits the last five words. In the first line on the same page we have the evident omission of a full line, no recurring word appearing.

A much longer omission than any yet mentioned is found on page 94, lines 34–36:

. . . perueniet una substantia abiecta. Et si dixerit 10 sine re in re, dicas 10 in re 10 res procreant, et sine re in re substantiam generat diminutivam. Hoc igitur ad 10 res perueniet abiecta substantia.

The scribe passed from *perueniet* to *perueniet* two lines below. Similar omissions, in length about 16 and 25 letters, are found in the text, page 102, line 4, and page 40, line 19. The omission of *A quibus 21 demptis* after *producentur* 25 (page 110, line 8), is not easily explained, but the omission in the text of page 96, line 17, appears to be that of a complete line of 38 letters.

The single illustration of the repetition of a full line or more in either Ms. occurs in the text on page 114, line 29, where the scribe runs back from the *coequantes* at the end of one sentence (line 29), to the *coequantes* at the end of the preceding sentence; the length is 54 letters. A shorter repetition occurs in the text of page 116, line 6. In the text of page 84, lines 12–13:

Area igitur quam linee *t e*, *e a* circumdant similis est aree quam linee *m l*, *l d* circumdant. Area igitur *t a* similis est aree *m d*. . . . we have noted that the Vienna scribe passed from the one *Area igitur* to the second; the Dresden scribe writes *m* after the first *similis est aree*, passing to the second, but he corrects the error. The length here is 43 letters. Since one Ms. omits and the other starts to omit this line, it was possibly omitted in the body of the text of the parent, and supplied in the margin.

The Vienna manuscript contains on the first page two marginal additions, somewhat in the nature of titles (see Plate I). To these there correspond similar additions in the Dresden manuscript, but while the latter continues with numerous other marginal additions the Vienna manuscript does not follow that practise. Among the particularly noteworthy marginal additions by the first hand in the Dresden manuscript are the geometrical figures to be used in connection with the geometrical demonstrations of the solutions of quadratic equations. The Vienna text contains no such figures. But the Dresden figures are evidently not derived directly from the figures in the Arabic text; they give every evidence of being constructed by the writer of the Dresden manuscript upon the basis of the geometrical explanation given in the text. The lettering does not vary greatly from the Arabic, and two of the figures are left quite incomplete.

The second hand has made in the Dresden text several marginal additions based, in general, upon the Vienna manuscript, and once in agreement with Scheybl's reading as opposed to the Vienna reading. Thus on page 66 of our text the marginal notes to words in lines 17 and 18 *alii centenum* and *alii decem*, and on page 68 to a word in line 2, *alii coniunctis.* Four rather long marginal corrections are made, also apparently based upon the Vienna manuscript, in the text to page 92, line 1, in the text as given in note 6 on the same page, in the text given on page 94, lines 34–36, and on page 96, line 17.

We have found then in each of these two manuscripts seven definite indications of a parent manuscript with a line length of between 36 and 54 letters, not very different from the line length in these Mss. themselves. But aside from the line length of the parent there are other indications that the two Mss. have a common parent. In the text of page 86, line 15, both Mss. read *ex duobus et quarta in se ipsis* where the sense requires *ex unitate et medietate in se ipsis;* this error was evidently in the parent Ms., and possibly in the autograph. Similarly in the text of page 106, lines 11 and 14, both Mss. read plainly 36, and that twice, whereas 49 is the correct numerical result here. The scribe of the Dresden Ms. commonly (5 times) writes *hiis* and the other scribe *his*, but in the text of page 76, line 15, this procedure is reversed; the indication is that *hiis* was the form employed in the parent. The Dresden scribe writes *addicias* and the Vienna scribe *adicias*, except in the text of page 76, line 20 where the latter also writes *addicias;* probably this form was used in the parent Ms. In the text of page 102, line 26, the Vienna Ms. reads 10 *sine re*, and the Dresden Ms. *rem*, while the sense requires 10 *res sine substantia.* Either the translation was incorrect in the parent, or, more probably, the passage was illegible. In the text of page 112, line 22, the Vienna Ms. reads 2000, 500, 50 *et* 4^a, for 2550 *et* 4^a; the Dresden Ms. writes the expression in words. This corresponds to a direct translation from the Arabic, for most early mathematical Mss. in Arabic follow the practise of writing numbers in full numeral words, and not in Hindu-Arabic notation.

The discussion of Scheybl's text is somewhat more complicated than that of the two preceding, for Scheybl follows sometimes the one, sometimes the other, and frequently neither, of the two older texts. As we have indicated above, there is probability that the present Vienna text may have been in the library of the University at Vienna when Scheybl was a student there. Further, we know that the Dresden Codex was used by Johann Widmann of Eger towards the end of the fifteenth century when Widmann was lecturing on algebra in the University of Leipzig; and a little later this same manuscript was used by Adam Riese. Although Scheybl was connected as teacher and student with the University of Tübingen, his first work[1] was published at Leipzig. Scheybl may have been familiar, then, with both of our manuscripts, or with the parent. Scheybl's manuscript was prepared with care for the printer, and the few omissions do not throw any light upon his source.

Another difficulty in connection with the Scheybl text is the fact that he took many liberties with the text. I have noted in many places where Scheybl wrote the word which is given in the Vienna and Dresden Mss., and then deleted to substitute a word of his own. Thus, in the text of page 78, line 4, the words *aequalium* and *scilicet* both, though at first written, were crossed out by Scheybl, and on page 80 similar deleted words in the text of lines 3, 8, and 17 correspond also to words in the Vienna and Dresden Mss. In the text of page 76, line 15, of page 82, line 15, and page 100, line 19, the deleted word follows the Dresden Ms. and not the Vienna, while the reverse is true in the text of page 82, line 8, and page 102, line 12. The agreements of Scheybl's Ms. with the Vienna, as opposed to the Dresden readings, number about 125, whereas, the reverse agreements with the Dresden Ms. number about 70; these include phrases as well as single words. Notably, the two concluding paragraphs of the Vienna Ms. with the date and place of the translation, appear in Scheybl's Ms. and not in the Dresden Ms.

In the text of page 68, line 6, after *colligitur* Scheybl omits a passage of some fifty letters ending in *coniungitur;* while this may be an omission from like to like, yet it may also have been deliberately left out as the statement is a repetition, found in the Arabic, of a passage which precedes (lines 1–2). In the text of page 72, line 6, fuerint ut sunt due uel 3^s uel plures seu pauciores fuerint Scheybl passes from one *fuerint* to the second, an omission of some 36 letters. The omission of about ninety letters after *exten-*

---

[1] *De numeris et diversis rationibus seu regulis computationum opusculum* (Leipzig, 1545).

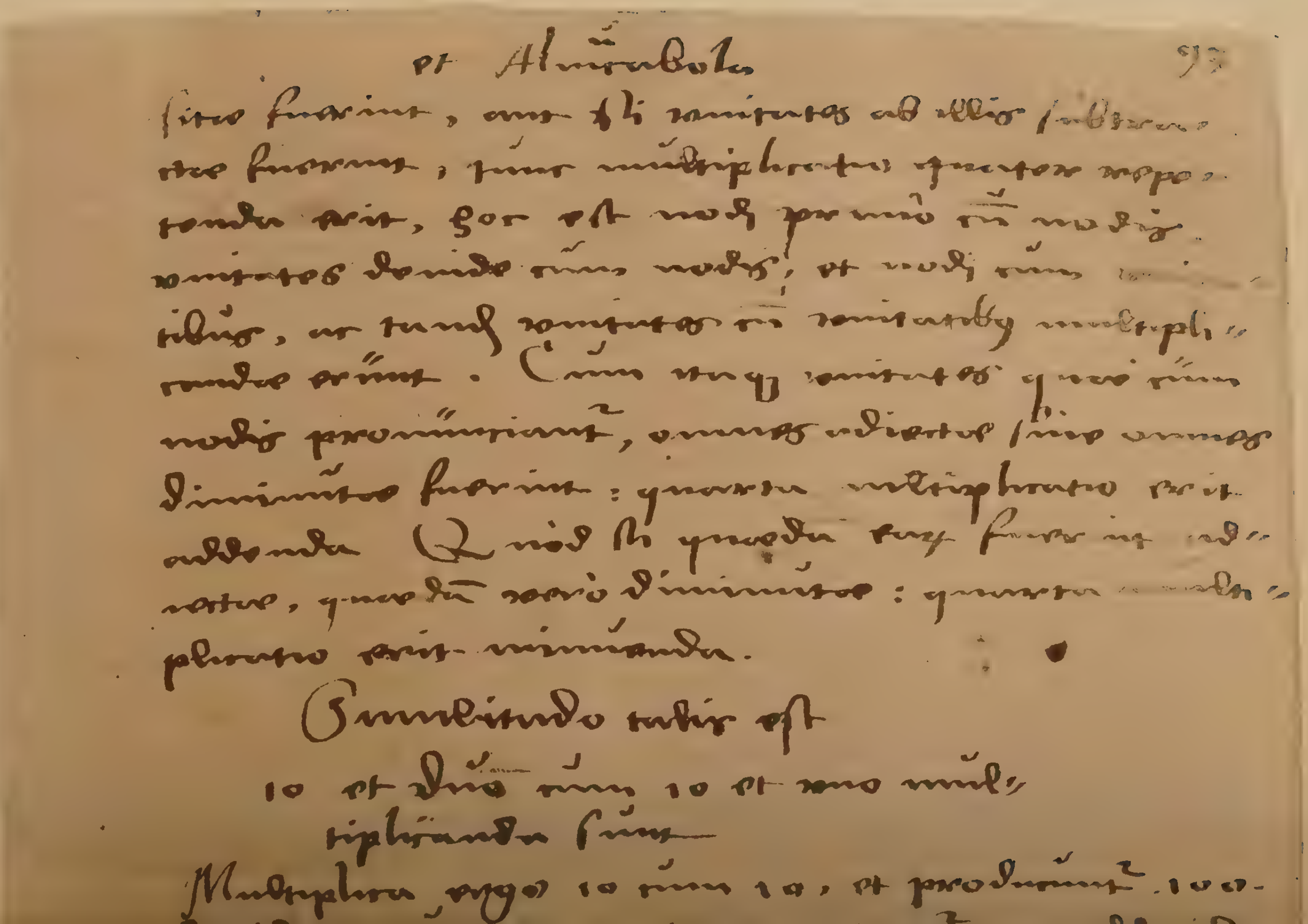
ditur in the text of page 96, line 23, appears to be deliberate, as the meaning of the passage is not clear. So, also, the omission in the text of page 116, line 13, may have been deliberate, as the multiplication is a simple repetition of work which precedes.

Another difficulty in any exact determination of the genesis of Scheybl's text is the fact that he had access to a copy of the algebra of Al-Khowarizmi in the translation which we have designated as the Libri text. The evidence of this familiarity is found in the *Addita*, written by Scheybl, which are printed on pages 128–156 of this work, for herein are contained portions of the algebra which were not translated by Robert of Chester, notably the problems involving the square root of two hundred (pages 142–144).

Only a fragment remains of the Romain version, constituting about 24 lines of our text. In this brief space there are some twelve agreements with the Dresden and Vienna Mss., and variations from the Scheybl text. However, one agreement with the Scheybl manuscript shows either familiarity with Scheybl's work, or a common source other than the Vienna and Dresden Mss. The title " Liber Algebrae et Almucabola, de quaestionibus arithmeticis et geometricis " appears only in the Romain fragment and in Scheybl's text; and somewhat similarly the word " creatori " after " Laus deo " in line 10, page 66, is common to the Romain version and the other Mss. except the Vienna Ms.

The fragment of our text in Codex Dresden C. 80^{m}, which we have reproduced above, follows exactly none of the other texts. Line omissions do not occur, but the spellings and transpositions agree sometimes with the Vienna, sometimes with the Dresden readings; in one instance *nil* for *nihil*, the agreement is with the Romain fragment as opposed to the Vienna and Dresden C. 80 readings. This fragment, then, appears also to be based on a parent of the extant Mss.

PLATE IV.



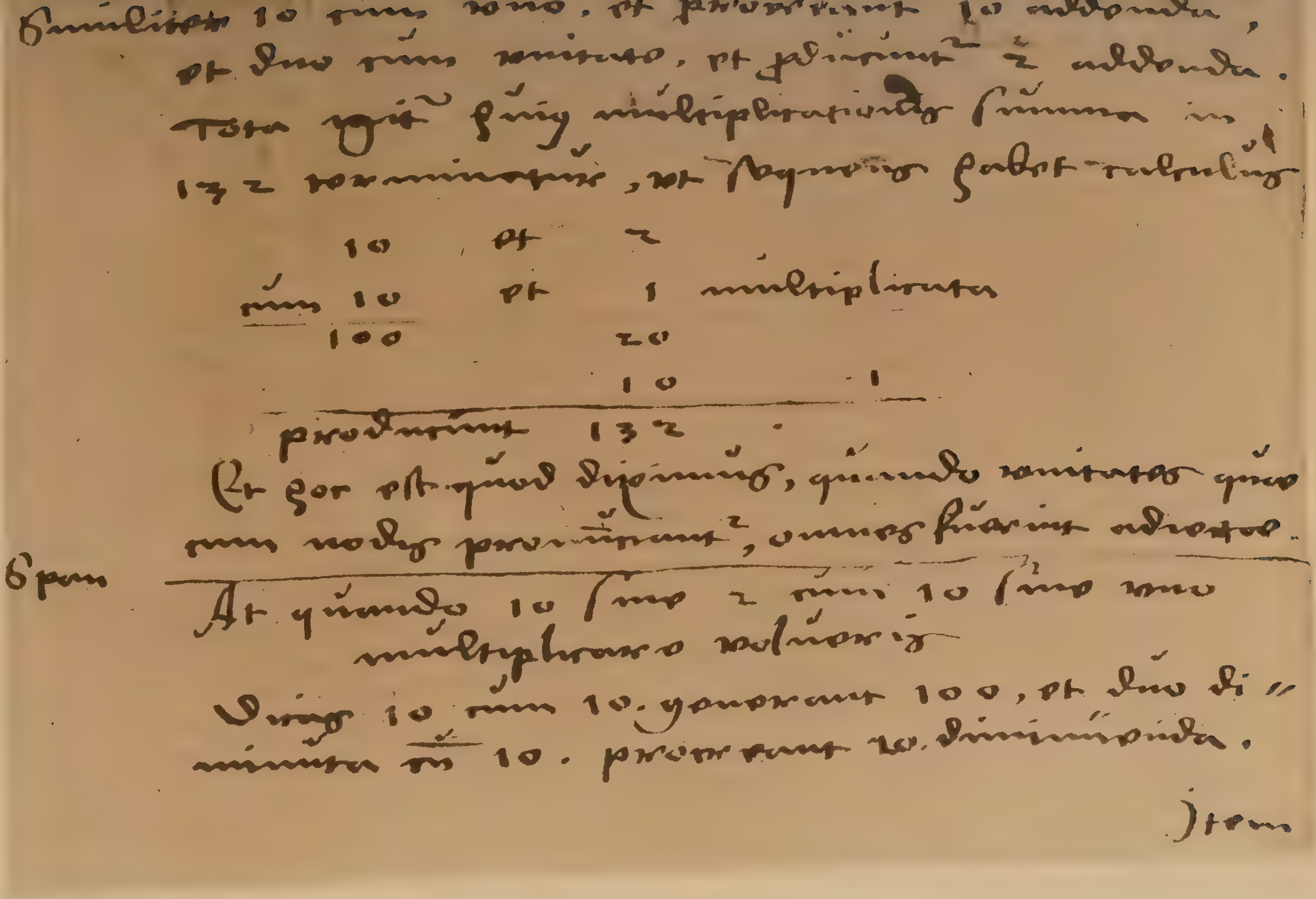
et Almucabola 97

sive fuerint, cum scilicet unitates ab illis subtractae fuerint, tunc multiplicatio quattuor ponenda erit, hoc est nodi primo cum nodis, unitates deinde cum nodis, et nodi cum unitatibus, ut tandem unitates cum unitatibus multiplicandae erunt. Cum itaque unitates quae cum nodis proveniunt, omnes adiectae sive omnes diminutae fuerint: quarta multiplicatio erit addenda. Quod si quaedam earum fuerint adiectae, quaedam vero diminutae: quarta multiplicatio erit minuenda.

Similitudo talis est

10 et duo cum 10 et uno multiplicanda sunt

Multiplica ergo 10 cum 10, et producuntur 100.



Similiter 10 cum uno, et proveniunt 10 addenda,
et duo cum uno, et proveniunt 2 addenda.
Tota igitur huius multiplicationis summa in
132 terminatur, et figura eius habet talem

10 et 2
cum 10 et 1 multiplicanda
100 20
10 1
producunt 132.

Et hoc est quod diximus, quando unitates que
cum nodis proveniunt, omnes fuerint addende.

Spem At quando 10 sine 2 cum 10 sine uno
multiplicare volueris
Duc 10 cum 10, generant 100, et duo di-
minuta cum 10, proveniunt 20 diminuenda.

Item

Columbia University Manuscript, X 512, Sch. 2. Q, Page 93.

ABBREVIATIONS

MANUSCRIPTS

C = Codex Universitatis Columbiae

D = Codex Dresdensis

R = Fragmentum Romain

V = Codex Vindobonensis

After the note to line 6 of the second page of the Latin text, the notes without any letter indicate the concurrence of V and D.

OTHER ABBREVIATIONS

add. *vel* + = additum

corr. = correctum

del. = deleta

man. = manus

marg. = margine

n. = nota

om. = omissum

quaest. = quaestio

ras. = erasura

relict. = relictum

spat. = spatium

superscr. = superscriptum

Tab. = Tabula

text. = textus

tit. = titulum

vac. = vacuum

φ = numerus

ꝶ = radix *vel* res

ʒ = substantia

⁊ = et

LIBER ALGEBRAE ET ALMUCABOLA

CONTINENS DEMONSTRATIONES AEQUATIONUM REGULARUM ALGEBRAE

Ab incerto authore olim arabice conscriptus atque deinde a Roberto Cestrensi, in ciuitate Secobiensi anno 1183, vt fertur, latino sermoni donatus.

LIBER ALGEBRAE ET ALMUCABOLA

Liber Algebrae et Almucabola, de quaestionibus arithmeticis et geometricis.

In nomine dei pii et misericordis incipit liber Restaurationis et Oppositionis numeri quem aedidit Mahomet, filius Mosi Algaurizin. Dixit Mahomet, Laus deo creatori, qui homini contulit scientiam inueniendi vim numerorum. Considerans enim omne id quo indigent homines, ex numeris componi, inueni illud totum esse numerum, et inueni nihil aliud esse numerum, nisi quod ex vnitatibus componitur. Vnitas ergo in omni numero reperitur. Inueni autem omnem numerum ita dispositum, vt omnis numerus vnitatem excedit vsque ad decem, denarius quoque numerus ad modum vnitatis disponitur, vnde et duplicatur et triplicatur, quemadmodum factum cum vnitate. Fiuntque ex eius duplicatione, 20: et triplicatione, 30. Et sic multiplicando denarium numerum, ad centenarium peruenitur. Ita centenarius numerus duplicatur et triplicatur, sicut denarius numerus. Et sic centenarium numerum duplicando et triplicando etc. millenarius excrescit numerus. Ad hunc modum numerum millenarium secundum ordinem numerorum multiplicando, vsque ad infinitam numeri inuestigationem peruenitur.

Postea inueni numerum restaurationis et oppositionis his tribus modis esse

---

1–6. *om.* VDR.

7. Liber . . . geometricis *om.* VD; + Praefatio R.

8. et^1 *om.* D. instauracionis D.

9. Mahumed filius moysi algaurizim V; Machumed filius moysi algaurizm D; Mahumed filius Moysis algaorizim R. Mahomet^2: Mahumed VR; machumed D.

10. creatori *om.* V.

11. ex numero VDR. componi *om.* VDR. id *pro* illud V.

12. exlun (?) *sed del. pro* et D. nil R.

13. omnem *om.* D. ita + essentialiter VD; + necessario R.

14. vnitatem *corr. ex* vnitas D *man.* 2. excedat VDR. decenus *pro* denarius *fere ubique* V; decimus *ubique* D; decenarius *ubique* R.

15. ad *in marg.* D *man.* 2 *pro* ex *del.* triplicatur + et D.

16. est ex *pro* cum VDR. eius *om.* V. et + ex VDR.

17. duplicando VDR. centum DR + alii centenum *in marg.* D *man.* 2; centenum V *et sic ubique.*

18. Ita: Proinde VD; Post modum R. centenus D. et *om.* V. sicut denarius numerus: ad modum 10^{ii} (= decimi) V; ad modum numeri decimi D *et in marg. man.* 2 alii decem; ad modum decenarii numeri R.

19. decenum *pro* centenarium VD *et corr. in* centenum D *man.* 2. triplicando multiplicando duplicando V. etc. *om.* VDR. millenus VD.

20. hunc + ergo VDR. millenus numerus ad modos VD; millenarius numerus ad modos R.

21. multiplicando *om.* VDR. infiniti V numeri *om.* V. conuertitur *pro* peruenitur VDR

22. hiis D.

THE BOOK OF ALGEBRA AND ALMUCABOLA[1]

CONTAINING DEMONSTRATIONS OF THE RULES OF THE EQUATIONS OF ALGEBRA

Written some time ago in Arabic by an unknown author and afterwards, according to tradition in 1183,[2] put into Latin by Robert of Chester in the city of Segovia.

THE BOOK OF ALGEBRA AND ALMUCABOLA

The Book of Algebra and Almucabola, concerning arithmetical and geometrical problems.

In the name of God, tender and compassionate, begins the book of Restoration and Opposition of number put forth by Mohammed Al-Khowarizmi, the son of Moses.[3] Mohammed said, Praise God the creator who has bestowed upon man the power to discover the significance of numbers. Indeed, reflecting that all things which men need require computation, I discovered that all things involve number and I discovered that number is nothing other than that which is composed of units. Unity therefore is implied in every number. Moreover I discovered all numbers to be so arranged that they proceed from unity up to ten. The number ten is treated in the same manner as the unit, and for this reason doubled and tripled just as in the case of unity. Out of its duplication arises 20, and from its triplication 30. And so multiplying the number ten you arrive at one-hundred. Again the number one-hundred is doubled and tripled like the number ten. So by doubling and tripling etc. the number one-hundred grows to one-thousand. In this way multiplying the number one-thousand according to the various denominations of numbers you come even to the investigation of number to infinity.

Furthermore I discovered that the numbers of restoration and opposition

---

[1] *Algebra* and *almucabola* are transliterations of Arabic words meaning 'the restoration,' or 'making whole,' and 'the opposition,' or 'balancing.' The first refers to the transference of negative terms and the second to the combination of like terms which occur in both members or to the combination of like terms in the same member. For a discussion of the terms *algebra* and *almucabola*, see Karpinski, *Algebra*, in *Modern Language Notes*, Vol. XXVII (1913), p. 93. Al-Karkhi included these two operations under *algebra* and the simple equating of the two members as *almucabola*, but Woepcke adds (*Extrait du Fakhri*, p. 64) that this is contrary to the common usage. The title *al-jebr w'almuqabala* is still used in Arabic. The Arabic verb stem *jbr*, from which *algebra* is derived, means 'to restore.' So in Spain and Portugal a surgeon was called an *algebrista*. See also note 3, p. 107.

[2] The date is given in the Spanish Era; 1145 A.D., according to our reckoning.

[3] Mohammed ibn Musa, Al-Khowarizmi. The word *algorism* is derived from his patronymic; the spelling and use in the Latin (see p. 76, line 18), indicate the process of evolution, although the term came into use through Al-Khowarizmi's arithmetic and not his algebra.

inuẹntum, scilicet radicibus, substantiis et numeris. Solus numerus tamen neque radicibus neque substantiis vlla proportione coniunctus est. Earum igitur radix est omnis res ex vnitatibus cum se ipsa multiplicata aut omnis numerus supra vnitatem cum se ipso multiplicatus: aut quod infra vnitatem diminutum cum se ipso multiplicatum reperitur. Substantia verò est omne illud quod ex multiplicatione radicis cum se ipsa colligitur. Ex his igitur tribus modis semper duo sunt sibi inuicem coaequantia, sicut diceres

Substantiae radices coaequant
Substantiae numeros coaequant, et
Radices numeros coaequant.

De substantiis radices coaequantibus. Ca [put] pri [mum].

Substantiae quae radices coaequant sunt, si dicas,

Substantia quinque coaequatur radicibus.

Radix igitur substantiae sunt 5, et 25 ipsam componunt substantiam, quae videlicet suis quinque aequatur radicibus. Et etiam si dicas,

Tertia pars substantiae quatuor aequatur radicibus:

Radix igitur substantiae sunt 12, et 144 ipsam demonstrant substantiam. Et etiam ad similitudinem,

Quinque substantiarum 10 radices coaequantium. Vna igitur substantia duabus radicibus aequiparatur, et radix substantiae sunt 2 — et substantiam quaternarius ostendit numerus.

Eodem namque modo, hoc quod ex substantiis excreuerit, aut minus ea fuerit, ad vnam conuertitur substantiam. Et similiter facies cum eo quod cum ipsis ex radicibus fuerit.

De substantiis numeros coaequantibus. Ca [put] II.

Substantiae verò numeros coaequantes hoc modo proponuntur.

Substantia nouenario coaequatur numero. Nouenarius igitur numerus mensurat substantiam cuius vnam radicem ternarius ostendit numerus. Eodem modo iuxta multitudinem et paucitatem substantiarum ipsae substantiae ad vnius

---

1–2. id est *pro* scilicet VR; et in D. Solis numeris VR. neque *bis supra versum* D *man.* 2, *pro* in *del. bis;* nec *bis* R. coniungitur DR *et in marg.* alii coniunctis D *man.* 2; coniunctis V. *In marg.* Quid ℞ V; Radix D.

3. in se *pro* cum se *fere ubique* VDR. ipsa *om.* V; ipsam R.

4. ipsa *pro* ipso *fere ubique* D; ipsum R *passim.*

5–6. vero *om.* D. ex radicis in se ipsa (ipsa *om.* V; ipsam R) multiplicacione VDR. *In marg.* Substantia VD. *Add. post* colligitur, Solus siquidem (quidem R) numerus neque (nec R) radicibus neque (nec R) substantiis ulla proporcione coniungitur VDR. ergo *pro* igitur *passim* VD. semper *om.* VD.

7. semet *pro* sibi. adinuicem V. sicuti.

8. coaequant + et.

9–10. et radices numeros coaequant *om.* D.

11. *Titulum om.*

12. Sed et substantie. sunt (+ est D) quasi diceres VD, *et* C *sed del.;* diceres *pro* dicas *fere ubique.* *In marg.* D

℞ ϕ coequantur
℞ assimilantur
ʒ

13. quinque: suis 5.

14. 35 V.

15. coequare *pro* aequare *passim.* quasi diceres.

17. ergo V.

22. ergo *pro* namque. hoc *om.* V; his D *et in marg. man.* 2 alii hoc. eis *pro* ea.

23. conuertas V; conuertatur D. facias de.

25. *Titulum om.*

26. In substantiis. coequantibus. proponitur D. *In marg.* ʒ assimilantur ϕ D.

27. equatur V. numerus *om.* V.

28. cuius scilicet. Eodem + ergo.

29. modo hoc (hĩc D) est. iuxta pluralitatem uel.

are composed of these three kinds: namely, roots, squares[1] and numbers. However number alone is connected neither with roots nor with squares by any ratio. Of these then the root is anything composed of units which can be multiplied by itself, or any number greater than unity multiplied by itself: or that which is found to be diminished below unity when multiplied by itself. The square is that which results from the multiplication of a root by itself.

Of these three forms, then, two may be equal to each other, as for example:

Squares equal to roots,
Squares equal to numbers, and
Roots equal to numbers.[2]

CHAPTER I

Concerning squares equal to roots[3]

The following is an example of squares equal to roots: a square is equal to 5 roots. The root of the square then is 5, and 25 forms its square which, of course, equals five of its roots.[4]

Another example: the third part of a square equals four roots. Then the root of the square is 12 and 144 designates its square.[5] And similarly, five squares equal 10 roots. Therefore one square equals two roots and the root of the square is 2. Four represents the square.[6]

In the same manner then that which involves more than one square, or is less than one, is reduced to one square. Likewise you perform the same operation upon the roots which accompany the squares.

CHAPTER II

Concerning squares equal to numbers[3]

Squares equal to numbers are illustrated in the following manner: a square is equal to nine. Then nine measures the square of which three represents one root.[7]

Whether there are many or few squares they will have to be reduced in the same manner to the form of one square. That is to say, if there

---

[1] Literally 'substances,' being a translation of the Arabic word *mal*, used for the second power of the unknown. Gerard of Cremona used *census*, which has a similar meaning.

[2] These are the three types designated as 'simple' by Omar al-Khayyami, Al-Karkhi, and Leonard of Pisa. They correspond in modern algebraic notation to the following:

$$ax^2 = bx,\ ax^2 = n, \text{ and } bx = n.$$

[3] These and the following chapter headings were doubtless supplied by Scheybl.

[4] $x^2 = 5x, \quad x = 5, \quad x^2 = 25.$

[5] $\frac{1}{3}x^2 = 4x, \quad x = 12, \quad x^2 = 144.$

[6] $5x = 10, \quad x = 2, \quad x^2 = 4.$

[7] $x^2 = 9, \quad x = 3.$

substantiae similitudinem erunt tractandae. Hoc est, si substantiae duae vel tres vel quatuor, siue etiam plures fuerint, earum cum suis radicibus coaequatio, sicut vnius cum sua radice, quaerenda est. Si verò minus vna fuerit, hoc est, si tertia vel quarta vel quinta pars substantiae vel radicis proposita fuerit, eodem modo ea tractetur, vt si dicas,

Quinque substantiae 80 coaequantur. Vna igitur substantia quintae parti numeri 80 coaequatur, quam videlicet 16 componunt. Et etiam si dicas,

Medietas substantiae 18 coaequatur. Haec igitur substantia 36 coaequatur. Hoc modo omnes substantiae, quotquot in vnum coniunctae, seu quae ab aliis diminutae fuerint, ad vnam conuertentur substantiam. Hoc idem cum numeris etiam, qui cum substantiis fuerint, agendum est.

De radicibus numeros coaequantibus. Ca [put] III.

Radices quae numeros coaequant sunt, si dicas,

Radix ternario aequatur numero. Huius igitur radicis substantiam nouenarius habet numerus. Et etiam si dicas,

Quatuor radices vigeno coaequantur numero. Vna igitur radix substantiae huius quinario coaequatur numero. Et etiam si dicas,

Media radix denario coaequatur numero. Tota igitur radix vigeno aequatur numero, cuius videlicet substantiam 400 demonstrant.

Radices igitur et substantiae et numeri solum, quemadmodum diximus, distinguuntur. Vnde et ex his tribus modis quos iam praemisimus, tria oriuntur genera tripliciter distincta; vt

Substantia et radices numeros coaequant
Substa[n]tia et numeri radices coaequant, et
Radices et numeri substantiam coaequant.

De substantiis et radicibus numeros coaequantibus. Ca [put] IIII.

Substantiae verò et radices quae numeros coaequant, sunt, si dicas,

Substantia et 10 radices 39 coaequantur drachmis. Huius igitur artis inuestigatio talis est: dic, quae est substantia, cui si similitudinem decem suarum radicum adiunxeris, ad 39 tota haec collectio protendatur. Modus hanc artem inueniendi est, vt radices iam pronunciatas per medium diuidas, sed radices in hac interrogatione sunt 10, accipe igitur 5, et iis cum se ipsis multiplicatis producuntur 25;

---

1. id est *pro* Hoc est V *passim*. si *om.* D.
2. seu quatuor seu. radicibus + queratur. equacio V.
4. vel¹ *om.* D. hoc modo tractetur. Sicut diceres.
7. numeri *om.*
9. substantias. coniunctae + fuerint.
10. conuertas. quod et de eo quod cum ipsis ex numeris fuerit *pro* Hoc idem . . . fuerint.
11. est *om.* D. 12. *Titulum om.*
13. Sed et radices. sicut diceres.
14. coequatur *pro* equatur.
16. equantur *pro* coaequantur. *In marg.*

D.

18. equatur *pro* coaequatur. coequatur *pro* aequatur.
20. soli. secundum quod iam diximus.
21. modis *om.* 22. Et *pro* vt.
23. Substantie.
24. Et substantie. radices et coequant D. et² *om.* D.
25. substantias.
26. *Titulum om.* V; Substantias et radicibus numeri coequantibus. D.
27. numerum coequantes. sicut.
28. dragmatibus *pro* drachmis; dragma *pro* drachma *ubique*.
29. Dic + ergo.
30. XXXVIII D.
32. eas V, ea D *pro* iis. ipsa *pro* ipsis V. multiplica et fient 25. facio *siue* sum *pro* produco *ubique*.

are two or three or four squares, or even more, the equation formed by them with their roots is to be reduced to the form of one square with its root. Further if there be less than one square, that is if a third or a fourth or a fifth part of a square or root is proposed, this is treated in the same manner.[1]

For example, five squares equal 80. Therefore one square equals the fifth part of the number 80 which, of course, is 16.[2] Or, to take another example, half of a square equals 18. This square therefore equals 36.[3] In like manner all squares, however many, are reduced to one square, or what is less than one is reduced to one square. The same operation must be performed upon the numbers which accompany the squares.

CHAPTER III

Concerning roots equal to numbers

The following is an example of roots equal to numbers: a root is equal to 3. Therefore nine is the square of this root.[4]

Another example: four roots equal 20. Therefore one root of this square is 5.[5] Still another example: half a root is equal to ten. The whole root therefore equals 20, of which, of course, 400 represents the square.[6]

Therefore roots and squares and pure numbers are, as we have shown, distinguished from one another. Whence also from these three kinds which we have just explained, three distinct types of equations[7] are formed involving three elements, as

A square and roots equal to numbers,
A square and numbers equal to roots, and
Roots and numbers equal to a square.[8]

CHAPTER IV

Concerning squares and roots equal to numbers

The following is an example of squares and roots equal to numbers: a square and 10 roots are equal to 39 units. The question therefore in this type of equation is about as follows: what is the square which combined with ten of its roots will give a sum total of 39? The manner of solving this type of equation is to take one-half of the roots just mentioned. Now the roots in the problem before us are 10. Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39, giving 64.

---

[1] Our modern expression "to complete the square," used in algebra, originally meant to make the coefficient of x^2 equal to unity, *i.e.* make one whole square.

[2] $5x^2 = 80;\ x^2 = 16.$

[3] $\frac{1}{2}x^2 = 18;\ x^2 = 36.$

[4] $x = 3;\ x^2 = 9.$

[5] $4x = 20;\ x = 5;\ x^2 = 25.$

[6] $\frac{1}{2}x = 10;\ x = 20;\ x^2 = 400.$

[7] 'Types of equations' = *genera*.

[8] Abu Kamil, Omar Al-Khayyami, Al-Karkhi, and Leonard designate these as 'composite' types. In modern notation: $ax^2 + bx = n$; $ax^2 + n = bx$; $ax^2 = bx + n$.

quae omnia 39 adiicias, et veniunt 64. Huius igitur radice quadrata accepta, quae est 8, ab ea medietatem radicum 5 subtrahas, et manebunt 3. Ternarius igitur numerus huius substantiae vnam ostendit radicem, quae videlicet substantia nouenario dinoscitur numero. Nouem igitur illam componunt substantiam.

Similiter quotquot substantiae propositae fuerint, omnes ad vnam conuertas substantiam. Similiter quicquid cum eis ex numeris siue radicibus fuerit, id omne eo modo quo cum substantiis existi, conuertas. Huius autem conuersionis talis est modus, vt si dicas,

Duae substantiae et 10 radices 48 drachmis coaequantur.

Huius artis talis est inuestigatio, vt dicas, Quae sunt duae substantiae inuicem collectae, quibus si similitudo 10 radicum earum adiuncta fuerit, ad 48 tota extendatur collectio. Nunc autem oportet, vt duas substantias ad vnam conuertas. Sed iam manifestum est, quoniam vna substantia medietatem duarum designat, igitur omnem rem in hac quaestione tibi propositam, ad medium conuertas, dicendo:

Substantia et 5 radices 24 drachmis coaequantur. Modus huius rei talis est, vt dicas, Quae est substantia, cui si quinque suas radices adiunxeris, ad 24 excrescat. Nunc etiam oportet, vt ad regulam supra datam animum conuertas, et diuidas radices per medium et veniunt 2 et vnius medietas, iis cum se ipsis multiplicatis, producuntur 6 et 1/4, illis 24 adiicias, veniunt 30 et vnius 1/4. Postea huius aggregati radicem quadratam accipias, quam scilicet 5 et vnius medietas componunt, ex qua medietatem radicum, 2 1/2, subtrahas et manebunt 3, quae vnam radicem substantiae exprimunt, quam substantiam nouenarius componit numerus. Et si diceretur,

Medietas substantiae et quinque radices 28 coaequantur drachmis.

Huius questionis talis est modus, vt dicas, Quae est substantia, cuius medietati si quinque suas radices adiunxeris, tota summa ad 28 excrescat, ita tamen vt substantia quae prius diminuta fuerat, perfecta compleatur. Igitur huius substantiae medietas cum radicibus secum pronunciatis, duplicanda est; veniunt autem,

1. quas (+ suis D) super 39 adicias (addicias *et sic ubique* D) et fient 64. Huius ergo collectionis. quadrata *om., et sic ubique.* assumpta *pro* accepta.
2. quae est: id est V; est D. id est quinque *pro* 5 *et sic passim.* diminuas *pro* subtrahas *passim.* remanebunt: remanere *pro* manere *ubique. In marg.* D, *figurae; vide Tab.* II.
3. scilicet D.
4. noscitur. iam *pro* illam.
6. fuerint + ut sunt due uel (uel *om.* D) 3^a uel plures seu (siue D) pauciores fuerint.
7. quotquot V; quidquid D. seu V. ad id cui (cuius D) substantiam conuertisti *pro* id omne . . . existi.
8. ergo *pro* autem.
9. si *om.*
11. Huius + autem D. est hec *pro* talis est. ut si dicas.
12. 10 rx. + unius V; *x* radicum + unius D. *In marg.* 2 ℨ + 10 ꝛ equantur 24. D.
13. ut ad unam substantiam duas conuertas V.
15. et hoc (hic D) est ut dicas *pro* dicendo.
16. substantiam *pro* substantia. 28 *pro* 24 V. coequant V.
17. sui *pro* suas D.
18. dictam *pro* datam. diuide.
19. erunt *pro* veniunt. eas + ergo *pro* iis. in seipsa D. multiplica.
20. et fient 6 et 1/4 (numeri *pro* 1/4 *bis* D) quas videlicet super 24 adicias fientque (fiantque D). summe *pro* aggregati.
21. accipies V. componit D.
22. qua + videlicet. radicis substantiam *pro* radicem substantiae.
24. diceret.
26. est^1 *om.* D. medietati + cui *supra versum* D *man.* 2.
27. rx *pro* radices V. sumas *et* suas *supra versum* D *man.* 2. 25 *pro* 28 V. quod *pro* vt.
29. fietque *pro* veniunt autem.

Having taken then the square root of this which is 8, subtract from it the half of the roots, 5, leaving 3. The number three therefore represents one root of this square, which itself, of course, is 9. Nine therefore gives that square.[1]

Similarly however many squares are proposed all are to be reduced to one square. Similarly also you may reduce whatever numbers or roots accompany them in the same way in which you have reduced the squares.

The following is an example of this reduction: two squares and ten roots equal 48 units.[2] The question therefore in this type of equation is something like this: what are the two squares which when combined are such that if ten roots of them are added, the sum total equals 48? First of all it is necessary that the two squares be reduced to one. But since one square is the half of two, it is at once evident that you should divide by two all the given terms in this problem. This gives a square and 5 roots equal to 24 units. The meaning of this is about as follows: what is the square which amounts to 24 when you add to it 5 of its roots? At the outset it is necessary, recalling the rule above given, that you take one-half of the roots. This gives two and one-half, which multiplied by itself gives $6\frac{1}{4}$. Add this to 24, giving $30\frac{1}{4}$. Take then of this total the square root, which is, of course, $5\frac{1}{2}$. From this subtract half of the roots, $2\frac{1}{2}$, leaving 3, which expresses one root of the square, which itself is 9.

Another possible example: half a square and five roots are equal to 28 units.[3] The import of this problem is something like this: what is the square which is such that when to its half you add five of its roots the sum total amounts to 28? Now however it is necessary that the square, which here is given as less than a whole square, should be completed.[4] Therefore the half of this square together with the roots which accompany it must be doubled. We have then, a square and 10 roots equal to 56 units. There-

[1] $x^2 + 10x = 39$; $\frac{1}{2}$ of 10 is 5, 5^2 is 25, $25 + 39 = 64$.
$\sqrt{64} = 8$; $8 - 5 = 3$. $x = 3$; $x^2 = 9$.

For the general type $x^2 + bx = n$, the solution is $x = \sqrt{\left(\frac{b}{2}\right)^2 + n} - \frac{b}{2}$; the negative value of the square root is neglected, as that would give a negative root of the equation.

[2] $2x^2 + 10x = 48$, reducing to $x^2 + 5x = 24$; $\frac{1}{2}$ of 5 is $2\frac{1}{2}$, $(2\frac{1}{2})^2 = 6\frac{1}{4}$, $24 + 6\frac{1}{4} = 30\frac{1}{4}$, $\sqrt{30\frac{1}{4}} - 2\frac{1}{2} = 3$.

The general type $ax^2 + bx = n$ is reduced to the preceding by division, giving $x^2 + \frac{b}{a}x = \frac{n}{a}$, and the solution is, as before, $x = \sqrt{\left(\frac{b}{2a}\right)^2 + \frac{n}{a}} - \frac{b}{2a}$.

[3] $\frac{1}{2}x^2 + 5x = 28$, reducing to $x^2 + 10x = 56$. $x = \sqrt{5^2 + 56} - 5$, or $x = 4$. Note that the value of x^2 is not given here as it usually is.

[4] Attention is called to the force of the expression, "completing the square," as here used with the meaning to make the coefficient of the second power of the unknown quantity equal to unity or making one whole square. See also page 81, footnote 1.

Substantia et 10 radices 56 drachmis aequales.

Diuide igitur radices per medium, et veniunt 5, quibus cum seipsis multiplicatis, producuntur 25; illa adde 56, et colliguntur 81; huius collecti radicem quadratam accipias, quam nouenarius componit numerus, atque ex ea medietatem radicum 5 subtrahas et manebunt 4, substantiae radix.

Hoc modo cum omnibus substantiis quotquot ipsae fuerint, cum radicibus item et drachmis agendum est.

De substantiis et numeris radices coaequantibus. Ca [put] V.

Propositio huius rei talis est, vt dicas,

Substantia et 21 drachmae 10 radicibus coaequantur.

Ad hoc inuestigandum talis datur regula, vt dicas, Quae est substantia, cui si 21 drachmas adiunxeris, tota summa simul decem ipsius substantiae radices exhibeat. Huius quaestionis solutio hoc modo concipitur, vt radices primum per medium diuidas, et veniunt in hoc casu 5, haec cum seipsis multiplicata, producuntur 25. Ex illis 21 drachmas, quas paulo ante cum substantiis commemorauimus, subtrahas, et manebunt 4, horum radicem quadratam accipias, vt sunt 2, quae ex medietate radicum 5 diminuas, et manebunt 3, vnam radicem huius substantiae constituentia, quam scilicet substantiam nouenarius complet numerus. Quod si libuerit, poteris ipsa 2, quae a medietate radicum iam diminuisti, medietati radicum 5 scilicet addere, et veniunt 7; quae vnam substantiae radicem demonstrant quam substantiam 49 adimplent. Cum igitur aliquod huius capitis exemplum tibi propositum fuerit, ipsius modum cum adiectione, quemadmodum dictum est, inuestiga, quem si cum adiectione non inueneris, procul dubio cum diminutione reperies. Hoc enim caput solum adiectione simul et diminutione indiget quod in aliis capitibus praemissis minime reperies.

Sciendum est etiam, quando radices iuxta hoc caput mediaueris, et medietatem deinde cum seipsa multiplicaueris, si quod ex multiplicatione tollitur vel procreatur, minus fuerit drachmis cum substantia pronunciatis: quaestio tibi proposita nulla erit. At si drachmis aequale fuerit vel procreatur vna radix

1. drachmis *om.* equiparantes *pro* aequales.

2. erunt 5 et postea eas in seipsis multiplica fientque 25. adde igitur eas super 56 eruntque (que *om.* D) 81 *pro* veniunt 5 . . . colliguntur 81.

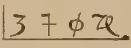
3. ergo summe *pro* collecti.

4. et *pro* atque. 5. substantiae radix *om.*

6. Hoc + ergo. fuerint + et.

7. et numeris secum pronunciatis *pro* item et drachmis.

8. *Titulum om.* V; radicem *pro* radices D.

9. autem huius artis *pro* huius rei. talis *om.* V. *In marg.*  D.

11. hoc + ergo.

12. similis *pro* simul. radicibus procreetur.

13. Huius + autem. hac regula *pro* hoc modo. vt *om.* Primum ergo radices.

14. diuides D. et fient. in hoc casu *om.*

14–16. eas ergo in se multiplica et erunt 25. Ex his (hiis D) ergo 21 minue (diminuas D) que cum substantia iam pretaxauimus *pro* haec cum . . . subtrahas.

16. id est *pro* vt sunt.

18. nouenus *pro* nouenarius.

19. Et si volueris. poteris *om.* ipsi medietati et (id est D) 5 *pro* medietati radicum 5.

20. addicias et fient 7; addicias *corr. in* adicias V. quae vnam: Hec igitur (ergo D) 7 unam.

21. quam + scilicet. adimplet D. aliquid. capitis: capitulum *pro* caput *ubique.* exemplum *om.*

22. addicione D *et sic passim.* secundum quod.

23. quam D. 24. Hic D. enim *om.*

25. tribus *pro* aliis. preter hunc (hoc *superscr.* D *man.* 2) solum in quibus radices mediantur *pro* praemissis.

26. est *om.* quoniam quando. eas *pro* medietatem.

27. deinde *om.* seipsas V; seipsis D. colligitur V. vel procreatur *om. et sic infra.*

28. pronunciatis + fuerit. quaestio + que.

29. facta fuerat adnullata est, ac si ipsa simul dragmatibus fuerit *pro* proposita . . . vel procreatur. facta C *sed del.*

fore take one-half of the roots, giving 5, which multiplied by itself produces 25. Add this to 56, making 81. Extract the square root of this total, which gives 9, and from this subtract half of the roots, 5, leaving 4 as the root of the square.

In this manner you should perform the same operation upon all squares, however many of them there are, and also upon the roots and the units.

CHAPTER V

Concerning squares and numbers equal to roots

The following is an illustration of this type: a square and 21 units equal 10 roots.[1] The rule for the investigation of this type of equation is as follows: what is the square which is such that when you add 21 units the sum total equals 10 roots of that square? The solution of this type of problem is obtained in the following manner. You take first one-half of the roots, giving in this instance 5, which multiplied by itself gives 25. From 25 subtract the 21 units to which we have just referred in connection with the squares. This gives 4, of which you extract the square root, which is 2. From the half of the roots, or 5, you take 2 away, and 3 remains, constituting one root of this square which itself is, of course, 9.[2]

If you wish you may add to the half of the roots, namely 5, the same 2 which you have just subtracted from the half of the roots. This give 7, which stands for one root of the square, and 49 completes the square.[3] Therefore when any problem of this type is proposed to you, try the solution of it by addition as we have said. If you do not solve it by addition, without doubt you will find it by subtraction. And indeed this type alone requires both addition and subtraction, and this you do not find at all in the preceding types.[4]

You ought to understand also that when you take the half of the roots in this form of equation and then multiply the half by itself, if that which proceeds or results from the multiplication is less than the units above-mentioned as accompanying the square, you have no equation.[5] If equal

---

[1] $x^2 + 21 = 10x$. For this type of equation both solutions are presented since both roots are positive. A negative number would not be accepted as a solution by the Arabs of this time, nor indeed were they fully accepted until the time of Descartes.

[2] For the general type, $x^2 + n = bx$ the solution is $x = \frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - n}$, and both positive and negative values of the radical give positive solutions of the equation proposed. In this problem we have $x^2 + 21 = 10x$; $\frac{1}{2}$ of 10 is 5, 5^2 is 25, $25 - 21 = 4$.

$\sqrt{4} = 2$; $5 - 2 = 3$, one root; $5 + 2 = 7$, the other root.

[3] Another use of the expression, "completing the square."

[4] The Vienna MS. defines, 'in which the roots are halved.'

[5] This corresponds to the condition, $b^2 - 4ac < 0$, in the equation $ax^2 + bx + c = 0$; in this event the roots are imaginary.

substantiae simul etiam medietas radicum, quae cum substantia sunt, pronunciatur, adiectione simul et diminutione abiectis. Quicquid igitur duarum substantiarum, aut plus, aut minus substantia propositum fuerit, ad vnam conuertas substantiam, sicut in primo capite praediximus.

De radicibus et numeris substantiam coaequantibus. Ca [put] VI.

In hoc capite sic proponitur,

Tres radices et quatuor ex numeris coaequantur substantiae.

Ad hoc inuestigandum talis datur regula, quod scilicet radices per medium diuidas et venit vnum et alterius medietas, hoc deinde cum seipsis multiplices, et producuntur 2 1/4, his 4 ex numeris adiicias, et veniunt 6 1/4, huius postea radicem quadratam accipias, hoc est 2 1/2. Atque eam tandem medietati radicum, vni scilicet et dimidio, adiicias, et veniunt 4, quae vnam substantiae radicem componunt, quam deinde substantiam numerus 16 adimplet. Quicquid igitur plus siue minus substantia tibi propositum fuerit, ad vnam conuertas substantiam. Ex his igitur modis, de quibus in principio libri mentionem fecimus, sunt tres priores, in quibus radices non mediantur, in tribus vero posterioribus vel residuis mediantur radices prout superius liquet.

DIXIT ALGAURIZIN

Sex sunt modi de quibus, quantum ad numeros pertinet, sufficienter diximus. Nunc verò oportet, vt quod per numeros proposuimus, ex geometrica idem verum esse demonstremus. Nostra igitur prima propositio talis est,

Substantia et 10 radices 39 coaequantur drachmis.

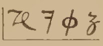
Huius probatio est, vt quadratum cuius latera ignorantur, proponamus. Hoc autem quadratum quod loco substantiae ponimus, et eius radicem scire volumus atque designare. Sit igitur quadratum *a b* cuius vnumquodque latus vnam ostendit radicem. Iam manifestum est, quoniam, quando aliquod eius latus cum

1. similis erit medietati. sunt *om.* pronunciantur V.

2. Quidquid.

3. substantiarum, aut minus substantia seu (siue D) maius; seu *pro* siue *ubique* V.

4. iam diximus.

5. *Titulum om.* V. *In marg.*  D.

6. hoc + autem. primum sic proponuntur.

7. Tres + autem. ex numero uni.

8. Huius igitur *pro* Ad hoc inuestigandum. quantus *pro* quod scilicet.

9. et fiet una radix et. igitur *pro* deinde. multiplicata, fientque duo et 4^a. Hec igitur (ergo D) super 4 adicias et erunt 6 et 4^a.

10. 6 1/2 C. ergo summe *pro* postea.

11. id est *pro* hoc est. duo et medium *pro* 2 1/2 *et sic saepius.*

11–12. quam super medietatem radicum adicias id est super unum et alterius medietatem fientque (que *om.* D) 4 *pro* Atque . . . 4.

12. quae + scilicet D.

13. 16 ergo substantiam adimplent *pro* quam . . . adimplet. Quotquot V; Quidquid D. maius *pro* plus.

14. fuerat D.

15. Hiis (His D) ergo 6 modis de; His *sed del. et* Ex his *in marg.* C.

16. primi *pro* priores. mediant *bis* D. posterioribus vel *om.*

18. + Nunc vero oportet quod numero proposuimus geometrice idem verum esse probemus *ante* Dixit D *man.* 2. alguarizim V; alghuarizim D.

19. Sex + autem. numerum. *In marg.* De $\tilde{4}$ prima D.

20. numero *pro* per numeros. ex *om.* geometrice.

21. probemus.

22. 4 *pro* 10, 29 *pro* 39 V; xxx et nouem D.

23. rumbus *pro* quadratum *ubique.* Rumbum *in text* D *et* rumbum *in marg. man.* 2.

24. Hic igitur rumbus substantiam quam *pro* Hoc . . . ponimus. cuius radices V.

25. atque *om.* designet (designetur D). et ipse est *pro* Sit igitur. unumquotque V. vnam + eius.

26. ostendet. Et iam. aliquid.

to the units, it follows that a root of the square will be the same as the half of the roots which accompany the square, without either addition or diminution.[1] Whenever a problem is proposed that involves two squares, or more or less than a single square, reduce to one square just as we have indicated in the first chapter.

CHAPTER VI

Concerning roots and numbers equal to a square

An example of this type is proposed as follows: three roots and the number four are equal to a square.[2] The rule for the investigation of this kind of problem is, you see, that you take half of the roots, giving one and one-half; this you multiply by itself, producing $2\frac{1}{4}$. To $2\frac{1}{4}$ add 4, giving $6\frac{1}{4}$, of which you then take the square root, that is, $2\frac{1}{2}$. To $2\frac{1}{2}$ you now add the half of the roots, or $1\frac{1}{2}$, giving 4, which indicates one root of the square. Then 16 completes the square.[3] Now also whatever is proposed to you either more or less than a square, reduce to one square.

Now of the types of equations which we mentioned in the beginning of this book, the first three are such that the roots are not halved, while in the following or remaining three, the roots are halved, as appears above.

GEOMETRICAL DEMONSTRATIONS

We have said enough, says Al-Khowarizmi, so far as numbers are concerned, about the six types of equations. Now, however, it is necessary that we should demonstrate geometrically the truth of the same problems which we have explained in numbers. Therefore our first proposition is this, that a square and 10 roots equal 39 units.

The proof is that we construct a square of unknown sides, and let this square figure represent the square (second power of the unknown) which together with its root you wish to find. Let the square, then, be *a b*, of which any side represents one root.

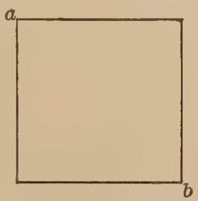


Fig. 1. — This figure appears only in the Columbia manuscript.

---

[1] Condition for equal roots, $b^2 - 4ac = 0$.

[2] $3x + 4 = x^2$; $\frac{1}{2}$ of 3 is $1\frac{1}{2}$, $(1\frac{1}{2})^2 = 2\frac{1}{4}$, $2\frac{1}{4} + 4 = 6\frac{1}{4}$, $\sqrt{6\frac{1}{4}} = 2\frac{1}{2}$, $2\frac{1}{2} + 1\frac{1}{2} = 4$, the root.

[3] The solution of the general type $bx + n = ax^2$, reduced by division to $\frac{b}{a}x + \frac{n}{a} = x^2$, is $x = \sqrt{\left(\frac{b}{2a}\right)^2 + \frac{n}{a}} + \frac{b}{2a}$, and only the positive value of the radical is taken, since the negative value would give a negative root of the proposed equation.

numero numerorum multiplicauerimus, tunc hoc quod ex multiplicatione colligitur erit numerus radicum radici ipsius numeri aequalis. Quoniam ergo decem radices cum substantia pronunciantur, quartam igitur partem numeri decem accipimus, atque vnicuique lateri quadrati aream aequedistantium laterum applicamus quarum longitudinem quidem longitudo quadrati primò descripti, latitudinem verò duo et dimidium, quae sunt numeri 10 quarta pars, demonstrant. Quatuor igitur areae laterum aequedistantium primo quadrato *a b* applicantur. Quarum singularum longitudo longitudini vnius radicis quadrati *a b* aequalis erit, latitudo etiam singularum 2 et medium, vt iam dictum est, demonstrat. Sunt autem hae areae *c d e f*. Ex hoc igitur quod diximus, sit area laterum inaequalium, quae similiter ponuntur ignota, in cuius videlicet quatuor angulis quorum vniuscuiusque quantitas areae, quam 2 et dimidium cum duobus et dimidio multiplicata perficiunt, imperfectionem maioris seu totius areae ostendunt. Vnde fit vt circunductionem areae maioris, cum adiectione duorum et dimidii cum duobus et dimidio quatuor vicibus multiplicatorum compleamus, generat autem haec tota multiplicatio 25. Et iam manifestum est, quoniam primum quadratum, quod substantiam significat, et quatuor areae ipsum quadratum circundantes, 39 perficiunt, quibus quando 25, hoc est quatuor minora quadrata, quae scilicet super quatuor angulos quadrati *a b* ponuntur, adiecerimus, quadratum maius, *G H* vocatum, circunductione complebitur. Vnde etiam haec tota numeri summa vsque ad 64 excrescet, cuius summae radicem octonarius obtinet numerus, quo etiam vnum eius latus compleri probatur. Igitur vbi ex numero octonario quartam partem numeri denarii, sicut in extremitatibus quadrati maioris *G H* ponuntur, subtraxerimus bis, 3 ex ipsius latere manebunt. Quinque ergo ex octo subtractis, 3 manere necesse est; quae simul vni lateri quadrati primi, quod est *a b*, aequiparantur.

Haec igitur 3 quadrati vnam radicem, hoc est vnam radicem substantiae

---

1. tollitur D.

2. consimilis *pro* aequalis. Quando igitur V.

3. substantia primum 10 (xiii D) radices proposuimus 4^{am} partem denarii id est (numeri *pro* id est D) 2 et medium accipiemus (accipiens D) *pro* substantia . . . accipimus.

4. At (ac D) eciam unicuique 4^{or} laterum rumbi primi aream equalium laterum applicabimus; aequalium *sed del.* C. quarum + scilicet; scilicet *sed del.* C.

5. quidem *om.* unius lateri rumbi primi demonstrent. Earum (+ earum *in marg.* D *man.* 2) vero latitudinem *pro* quadrati . . . verò.

6. medium *pro* dimidium *ubique.* numerum obtineant que videlicet 4^{am} partem denarii numeri *pro* quae . . . demonstrant.

7. equalium *pro* aequedistantium. *a b:* id est *a b.* omni *pro* singularum.

8. une radicum V. manet consimilis *pro* aequalis erit. quarumque latitudinem *pro* latitudo etiam singularum.

9. 2 *om.* D. demonstrant. Et hee (*om.* D) sunt.

10. *g d h c et passim.* sint D, sit D *man.* 2. aree *pro* area D. equalium *pro* inaequalium.

11. id est in cuius unoquoque angulo *pro* quorum vniuscuiusque. quantum D.

12. cum duobus et dimidio *om.* V; in duo et medium quantitas D. multiplicata *om.* D. circumdant *pro* perficiunt.

13. seu totius *om.* areae + idem V. in *pro* vt D

14. in *pro* cum^2.

15. ut *pro* generat autem; perficit *sed del.* C.

16. 25 + pariat. primus rumbus, qui *et sic passim.*

17. significat: signare *pro* significare *ubique.* quadratum *om.* perficiunt (faciunt D) + numerum.

18. hoc est quatuor: et 4 V; id est iiii D. minora *om.* quatuor^2 *om.*

19. adiciemus. id est rumbus (*om.* D) *e c pro G H* vocatum.

20. Vnde etiam + et V; + ad D.

21. unius octonarius. qui.

22. Quando igitur *pro* Igitur vbi. similitudinem 4^e partis denarii numeri simul in *pro* quartam . . . sicut in.

23. qui est *e c pro G H et sic postea.*

25. contra similiter *pro* quae simul.

26. vnam + huius. id est.

When we multiply any side of this by a number (of numbers) [1] it is evident that that which results from the multiplication will be a number of roots equal to the root of the same number (of the square). Since then ten roots were proposed with the square, we take a fourth part of the number ten and apply to each side of the square an area of equidistant sides, of which the length should be the same as the length of the square first described and the breadth $2\frac{1}{2}$, which is a fourth part of 10. Therefore four areas of equidistant sides are applied to the first square, *a b*. Of each of these the length is the length of one root of the square *a b* and also the breadth of each is $2\frac{1}{2}$, as we have just said. These now are the areas, *c, d, e, f*. Therefore it follows from what we have said that there will be four areas having sides of unequal length, which also are regarded as unknown. The size of the areas in each of the four corners, which is found by multiplying $2\frac{1}{2}$ by $2\frac{1}{2}$, completes that which is lacking in the larger or whole area. Whence it is that we complete the drawing of the larger area by the addition of the four products, each $2\frac{1}{2}$ by $2\frac{1}{2}$; the whole of this multiplication gives 25.

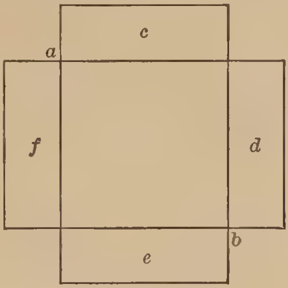


FIG. 2. — From the Columbia manuscript.

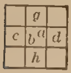


FIG. 3. — From the Dresden manuscript.

And now it is evident that the first square figure, which represents the square of the unknown (x^2), and the four surrounding areas ($10\,x$) make 39. When we add 25 to this, that is, the four smaller squares which indeed are placed at the four angles of the square *a b*, the drawing of the larger square, called *G H*, is completed. Whence also the sum total of this is 64, of which 8 is the root, and by this is designated one side of the completed figure. Therefore when we subtract from eight twice the fourth part of 10, which is placed at the extremities of the larger square *G H*, there will remain but 3. Five being subtracted from 8, 3 necessarily remains, which is equal to one side of the first square *a b*.[2]

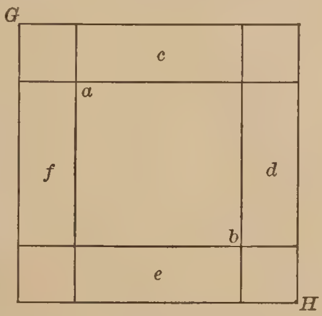


FIG. 4. — Final completed figure. From the Columbia manuscript.

This three then expresses one root of the square figure, that is, one root of the proposed square of the unknown, and 9 the square itself.

---

[1] Evidently meaning a pure number.

[2] The proportions of the figures are not correct to scale.

propositae: nouenarius deinde numerus ipsam substantiam exprimit. Ergo numerum denarium mediamus, et alteram eius medietatem cum seipsa multiplicamus, deinde totum multiplicationis productum numero 39 adiicimus, vt maioris quadrati *G H* circunductio compleatur. Nam eius quatuor angulorum diminutio totam huius quadrati circunductionem imperfectam reddebat. Manifestum enim est, quod quarta pars omnis numeri cum suo aequali, ac deinde cum quatuor multiplicata, eandem perficiat numerum, quem medietas numeri cum seipsa multiplicata, perficit. Igitur si radicum medietas cum seipsa multiplicetur, huius multiplicationis summa, multiplicationem quartae partis cum seipsa ac deinde cum quatuor multiplicatae, sufficienter euacuet, adaequabit vel delebit.

Ad hoc etiam idem demonstrandum, altera datur formula, quae talis est. Quadrato *a b* substantiam significante aequalitatem decem radicum addimus; has radices deinde per medium diuidamus, venient 5, ex quibus duas areas ad duo latera quadrati *a b* constituamus, hae autem vocentur *a g* et *b d*, et erit vtriusque vtraque latitudo vni lateri quadrati *a b* aequalis; vtramque denique longitudinem numerus quinarius adimplebit. Superest iam, vt ex multiplicatione 5 cum 5, quae medietatem radicum quas ad duo latera quadrati prioris substantiam significantis, addidimus, quadratum faciamus. Vnde iam manifestum est, quòd duae areae, quae supra duo latera ponuntur, et quae 10 radices substantiae significant, simul cum quadrato priori, quod est substantia, 39 ex numero adimpleant. Manifestum etiam, quòd area maioris seu totius quadrati per multiplicationem 5 cum 5 perficiatur. Hoc ergo quadratum perficiatur, atque ad perfectionem eius numerus 25 ad priora 39 adiiciatur: tota igitur haec summa vsque ad 64 excrescet. Nunc summae huius radicem quadratam, quae vnum latus quadrati maioris designat, accipiamus, atque inde aequalitatem eius quod ei addidimus, hoc est 5,

---

1. proposito obtinent, quam videlicet substantiam nouenarius adimplet numerus (+ si D *man.* 2). Igitur.

2. unam *pro* alteram eius. in seipsam. multiplicauimus V; multiplicauerimus D.

3. et *pro* deinde. totam multiplicationis summam super (sunt D) 39 addidimus; summam 39 C *sed del.*

4. compleretur. Nam + et.

6. quam *pro* quod *saepius.* consimili ducta, deinde in 4 V; consimili *a e* ductum in 4^{or} D.

7. eundem perficiet. quem + si V; + et si D. medietas + unum D. numeri *om.*

8. semel duceretur *pro* multiplicata; semel C *sed del.* perficeret. si *om.* D. medietas + semel. ducatur *pro* multiplicetur; ducere *pro* multiplicare *saepius.*

10. iii D, *sed* 4 *superscr. man.* 2. adaequabit vel delebit *om.*

11. demonstrandum: comprobandum V; probandum D. Sit (que est D) *a b* ipsam signans substantiam (+ ad D) cuius similitudinem 10 *pro* quae talis est . . . decem.

12. radices. 10 ergo (igitur D) *pro* has.

13. deinde *om.* et fient *pro* venient. quibus + similiter. super *pro* ad.

14–16. constituas, proponimus et ipse sunt aree *b d* et *b g* (et *b g add. in marg.* D *man.* 2), quare simul (similes *sed del.* D) latitudines (latitudinem D) uni lateri rumbi *a b* habentur consimiles. Earum vero (earum Nam D) longitudines quinarius complet numerus, qui medietatem 10 radicum super duo latera rumbi *a b* adiectarum adimplet.

16. Restat igitur *pro* Superest iam. vt + rumbum.

17. radicum: 10 radicum designant. super *pro* ad; sup C *sed del.* primi D; extremi V. substantiam significantis *om.*

18. addidimus qui et substantiam signet faciamus. Et iam. est *om.* V.

20. primo. adimplent.

21. Manifestum + est. quam ex perfectione aree *pro* quòd area. seu totius *om.* ex multiplicatione.

22. 5^2 + que (et D) sunt 25 nondum. perficitur. et *pro* atque. perfectam *pro* perfectionem.

23. rumbi maioris qui est rumbus *a h* super 39 adiciatur *pro* eius . . . adiiciatur. Toto ergo D. vsque *om.* V.

24. Huius ergo summe.

25. accipias et ex ea similitudinem eius quod ei addidisti id est 5.

Hence we take half of ten and multiply this by itself. We then add the whole product of the multiplication to 39, that the drawing of the larger square *G H* may be completed;[1] for the lack of the four corners rendered incomplete the drawing of the whole of this square. Now it is evident that the fourth part of any number multiplied by itself and then multiplied by four gives the same number as half of the number multiplied by itself.[2] Therefore if half of the roots is multiplied by itself, the sum total of this multiplication will wipe out, equal or cancel[3] the multiplication of the fourth part by itself and then by four.

Another method[4] also of demonstrating the same is given in this manner: to the square *a b* representing the square of the unknown we add ten roots and then take half of these roots, giving 5. From this we construct two areas added to two sides of the square figure *a b*. These again are called *a g* and *b d*. The breadth of each is equal to the breadth of one side of the square *a b* and each length is equal to 5. We have now to complete the square by the product of 5 and 5, which, representing the half of the roots, we add to the two sides of the first square figure, which represents the second power of the unknown. Whence it now appears that the two areas which we joined to the two sides, representing ten roots, together with the first square, representing x^2,[5] equals 39. Furthermore it is evident that the area of the larger or whole square is formed by the addition of the product of 5 by 5. This square is completed and for its completion 25 is added to 39. The sum total is 64. Now we take the square root of this, representing one side of the larger square and then we subtract from it the equal of that which we added, namely 5. Three remains,

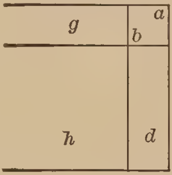


FIG. 5. — Incomplete figure. From the Dresden manuscript.

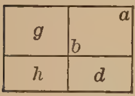


FIG. 6. — From the Dresden manuscript.

---

[1] This corresponds to our algebraic process of completing the square. The correspondence of the geometrical procedure to the terminology and the methods employed in algebra makes it highly desirable to present the geometrical and algebraical discussions together to students of elementary mathematics.

[2] $4\left(\frac{a}{4}\right)^2 = \left(\frac{a}{2}\right)^2$.

[3] The words *adaequabit vel delebit* are doubtless added by Scheybl to explain the force of *euacuet*.

[4] A method slightly different from either of these is given by Abu Kamil and also in the Boncompagni version of Al-Khowarizmi's algebra, ascribed to Gerard of Cremona. This consists in applying to one side of the square a rectangle with its length equal to 10, while the other dimension is the same as that of the square. The two together represent $x^2 + 10x$, or 39. Bisect the side whose length is 10. Now by Euclid II, 6, the square on half the side 10 plus the side of the original square, $(x + 5)^2$, equals the whole rectangle (39) plus the square 25 of half the side 10. The rest of the demonstration is similar to that here given.

[5] I use x^2 for *substantia* here and in the following demonstrations.

subtrahamus, et manebunt 3; quae latus quadrati *a b* hoc est vnam radicem substantiae propositae complere probantur. Tria igitur huius substantiae sunt radix; et substantia nouem.

De substantia et drachmis res coaequantibus.

Substantia et 21 drachmae 10 rebus aequiparantur.

Propositio haec seu quaestio in capite quinto proposita fuit, cuius hic demonstratio docetur. Quadratum igitur *a b*, quod latera habet ignota, substantiam pono, atque ei parallelogrammum rectangulum, cuius vtraque latitudo vni lateri quadrati *a b* aequalis sit, cuiusque longitudo vtraque rerum seu radicum medietatem referat, applico. Deinde vero huius rectanguli summam 21 ex numeris constituo, qui numerus cum ipsa substantia propositus est. Haec autem area vel rectangulum *b g* inscribitur, cuius simul latitudo *g d* dinoscitur, longitudo ergo duarum arearum inuicem coniunctarum, in *h d* terminatur. Et iam manifestum est quod haec longitudo denarium obtinet numerum, quoniam omnis area quadrilatera et rectorum angulorum, ex multiplicatione sui lateris cum vnitate semel, vnam obtinet radicem: et si cum binario numero, duae eiusdem areae nascuntur radices.

Quoniam igitur primò sic proposuit, vna substantia et 21 drachmae, 10 radicibus aequiparantur, manifestum est, quòd longitudo lateris *h d* in denario terminetur numero, quia latus *h b* vnam substantiae radicem obtinet, latus igitur *h d* super punctum *e* per medium diuide. Vnde et linea *e h* lineae *e d* fiet aequalis, atque ducta ex puncto *e* linea perpendiculari *e t*: haec eadem perpendicularis ipsi *h a* lineae aequalis erit. Lineae ergo *e t* portionem, quae ipsaque *d e* linea breuior est, in rectum adiicio *e c*, et fiet linea *t c* aequalis *t g* lineae, vnde quadratum *t l*, quod ex multiplicatione medietatis radicum cum se ipsa multiplicatae, id est ex multiplicatione quinarii cum quinario (in hoc casu) colligitur, nobis eueniet. Et iam manifestum etiam est, quoniam area *b g* 21, quae substantiae addidimus, in se obtinet, ex a ea igitur *b g* per lineam *t c*, quae est vnum latus areae *t l* extrahamus atque ex eadem area *b g* aream *b t* minuamus, deinde verò super lineam *e c* quae

---

1. subtrahas. quae + simul. qui *pro* hoc. vnam radicem *om.* substantia complere probatur. + Hoc igitur (ergo D) unam substantie (substantiam D) radicem adimplet (adimplent D).
3. et *om.* V. et nouem D.
4. *Titulum om.* V; 21 dragmatibus 4 res D.
5. Substantia + vero.
6. Propositio . . . docetur *om.*
7. habeat.
8. cui V; tibi D; cui C *sed del.* *pro* atque ei. aream laterum equalium equidistancium V; aream lateris equalium D *pro* parallelogrammum rectangulum. vtraque *om.*
9. similis; similis *pro* aequalis *saepius.* eiusque V. vtraque *om.* ad (*om.* D) quamlibet hanc (*om.* D) quantitatem *pro* rerum . . . referat.
10. Summa igitur huius aree 21 ex numero.
11. qui + simul est. eciam *pro* autem. vel rectangulum *om.*
12. *a g.* inscribi D. *d in g d om.* C. cognoscitur V.
13. iam *om.* D. quam sua *pro* quod haec.
14. obtineat. 4ta V; quadrata D.
15. angulorum equalium *pro* rectorum angulorum. cum *om.* D. unum *pro* vnitate V. semel *om.* V; semel + deducta D.
16. binario + ducatur.
17. primum. drachmae *om.*
18. est *om.* D.
19. quoniam *pro* quia. *a b* V; *h* D *pro h b.*
20. diuido. et linee *e d* D. fiet similis V; similes sunt D.
21–23. atque ducta ex puncto *e* linea . . . in rectum adiicio *e c om.* *Add.* Sed et iam manifestum est quam linea *e c* similis sit *h b.* In linea ergo *e t* simile residui *d e* super *e t* adiciam ut et aree circumduccie (circumduccione D) adimpleatur.
24. qui V; est D *pro* quod. in seipsam V; *om.* D.
25. in hoc casu *om.* colligitur (colligi D) + et fiunt 25. nobis eueniet *om.* D.
26. *a g.* quae + in simul.
27. obtineat. igitur *om.* D. *a g pro b g.* linea *pro* per lineam.
28. at V, ac D *pro* atque. eadem *om.* at V, ac D *pro* deinde verò.

which proves to be one side of the square $a\,b$, that is, one root of the proposed x^2. Therefore three is the root of this x^2, and x^2 is 9.

Concerning a Square and Units Equal to Unknown Quantities

A square and 21 units are equal to ten unknowns. This proposition or problem was proposed in the fifth chapter and here a geometrical demonstration is presented.

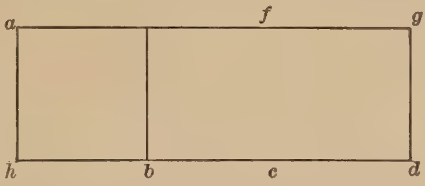


FIG. 7.—Incomplete figure. From the Columbia manuscript, where it twice appears.

Suppose that the square $a\,b$, having unknown sides, represents x^2 and apply to it a rectangular parallelogram of which the breadth is equal to one side of the square $a\,b$ and the length is any quantity you please.[1] Then the numerical value of this rectangle is 21, which number accompanies the same x^2. Moreover this area or rectangle is called $b\,g$, of which one side is $g\,d$ and the length of the two areas together is finally $h\,d$. And it is now evident that this length represents 10, since every quadrilateral having right angles (every square) gives for the product of one of its sides by unity one root, and if multiplied by two gives two roots of its area.

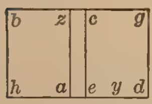


FIG. 8.[2]—From the Dresden manuscript.

Therefore since the problem was given, x^2 and 21 units equal 10 roots, it is evident that the length of the side $h\,d$ is 10, for the side $h\,b$ designates one root of x^2. Therefore bisect the side $h\,d$ at the point e[3] so that the line $e\,h$ is equal to the line $e\,d$. From the point e draw the perpendicular $e\,t$. This perpendicular equals $h\,a$. Add to the prolongation of the line $e\,t$ a part $e\,c$ equal to the amount by which it is less than $d\,e$ and then $t\,c$ will equal $t\,g$. Whence we arrive at the square $t\,l$ which is the product of half of the roots multiplied by itself, that is the product, in this instance, of 5 and 5. Moreover we know that the area $b\,g$ which we add to x^2 amounts to 21. Therefore we cut across the area $b\,g$ with the line $t\,c$, which is one side of the area $t\,l$, and thus decrease the area $b\,g$ by the amount of the area $b\,t$. Then we form

---

[1] Scheybl's text is incorrect here.

[2] If the lettering of this figure is made to conform to that of our text the demonstration will be seen to be not materially different; it is based more directly on Euclid II, 5. This proposition, following Heath, *The Thirteen Books of Euclid's Elements*, reads as follows: "If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half." This is one of the propositions of Euclid which connect very directly with the geometrical solution of the quadratic equation.

[3] The completed figure (Fig. 9) appears on p. 85. The lettering of Fig. 7 does not correspond to that of the completed figure.

est in quo linea *t e c* lineam *a h* in quantitate deuincit, quadratum *e n m c* ponamus. Vnde et iam manifestum, cum linea *t c* aequalis sit linea *l c*, nam ipsae in quadrato *t l* aequales protenduntur, similiter et linea *e c* lineae *m c* aequalis, quia ipsae quadratum *e m* aequali dimensione circundant: aequalium igitur ab aequalibus lineis subtractione facta, linea *t c* lineae *l m* aequalis relinquetur quod est notandum.

Rursus manifestum est, quoniam linea *g d* aequalis est lineae *a h*, cum ipsae in latitudine areae *h g* aequali dimensione tenduntur, sed linea *a h* aequalis est lineae *h b* cum ipsae in vno quadrato appareant. Item quoniam linea *g l* aequalis est lineae *d e*, cum ipsae in vno quadrato reperiantur aequales, sed linea *d e* aequalis est *h e*, cum ipsae decem radices per medium diuidant; linea igitur *d l* residuum lineae *g l*, lineae *e b* ex linea *e h* residuae aequalis erit: atque tandem, cum linea *t e* lineae *l m* ex superiori demonstratione, non sit inaequalis, area quam linea *t e* et *e b* circundant, comprehensae sub *l m* et *d l* lineis areae aequalis erit. Area igitur *t b* aequalis est *m d* areae. Et iam manifestum est, quoniam quadratum *t l* 25 in se continet, cum ergo ex eodem quadrato *t l* areas *d t* et *m d*, quae videlicet duabus areis *g e* et *t b*, 20 et vnum in se continentibus, sunt aequales, subtraxerimus, quadratum *n c* nobis manebit, qui simul numerum, qui est inter 25 et 21, obtinet. Et hic numerus est quaternarius, cuius videlicet radicem duo designant, quae latus *e c* adimplent. Hoc autem latus aequale est lineae *e b*, quoniam *e c* aequale est *d l* lateri, cum ipsa in latitudine areae *d c* aequalia protendantur. Iam manifestum est, quoniam *d l* aequalis est lineae *e b*, quando igitur *e b* quae sunt duo, ex *e h*, quae sunt radicum medietas, quam quinarius ostendit numerus, abstulerimus, linea *b h* ternarium ostendens numerum restabit. Ternarius igitur numerus radicem primae demonstrat substantiae.

Quòd si contrà lineam *e c* lineae *e h*, quae medietatem radicum continet, addiderimus, colligentur 7, quae lineam *n h* ostendunt. Et tunc radix substantiae maior

---

1. uincit. *e* et *n m c* faciamus *pro* *e n m c* ponamus.

2. Sed *pro* Vnde. manifestum est, quam et linea *c t* similis. nam + et.

3. similis est. quoniam et *pro* quia.

4. diuisio *pro* dimensio *ubique*. aequalium . . . facta *om.*

5. linea + igitur V; + ergo D. restat consimilis. quod est notandum *om.*

6. Et iam eciam *pro* Rursus. quoniam + et. similis sit *et sic saepius*. *b h pro a h et sic infra*. quoniam *pro* cum. ipsae + est V.

8. *d e* V; *g d* D *pro* *h b*. cum ipsae . . . lineae *d e om.* V; Nam quoniam linea *g l* et linea *h e* in quantitate habentur consimiles quoniam linea *g l* similis est linee *d e* D.

9. nam *pro* cum. reperiuntur. aequales *om.* sed *om.* lineaque.

10. Nam eciam et *pro* cum. diuidunt. *d l* residuum lineae *g l*, lineae *om.*

11. *e a pro e b*. residua. erit + linee *d l* ex linea *g l* residue. Sed et *pro* atque tandem, cum.

12. ex superiori demonstratione *om.* iam fuerat consimilis *pro* non sit inaequalis. area quam *t e* et *e b* circundant *om.* V; area igitur quam linee *t e*, *e a* circumdant D.

13. comprehensae sub . . . aequalis erit *om.* V; similis est aree quam linee *m l*, *l d* circumdant. D.

14. *t a pro t b*. area *pro* quadratum.

15. contineat. area *pro* eodem quadrato. areas + areas D. et *om.*

16. *e t*, *e g* 21 V; *a t*, *e g* 20 et unum D. consimiles V; similes D.

17. similitudinem *pro* simul V. numeri.

19. et hoc (hic D). autem *om.* simile est *e a* V; similis est *e a* D *pro* aequale est lineae *e b*. similis *pro* aequale.[2]

20. lateri *om.* Nam et ipse *pro* cum ipsa aequalia *om.* continentur. Et iam.

21. *d* vel (?) *pro* *d l* D lineae *om.* *e a pro e b*. quando igitur *e b om.* V; Cum *e a* D.

22. radicis. quaternarius D *et* quinarius *in marg. man.* 2.

23. designans *pro* ostendens.

25. Vel *pro* Quòd D. contrà *om.* linea *pro* lineam D. *e n* (*e h* D) super lineam *c h* (*e h* D) addiderimus. quae + solum V; quae + semel D.

26. et (*om.* V) fiunt 7. a^{or} *pro* maior D.

the square *e n m c* upon the line *e c*, which is of the length by which the line *t e c* exceeds the line *a h*. Whence, since *t c* equals *l c*, being found in the square *t l*, and similarly since *e c* equals *m c*, these being equal dimensions of the square *e m*, and further equal lines being subtracted from equal lines, it is evident that *t e* is left equal to *l m*. This is to be noted.

Again it is evident that the line *g d* is equal to *a h*, since they represent in breadth equal dimensions of the area *h g*, and the line *a h* equals *h b* as they appear in one square. Also since the line *g l* is equal to *d e*, being found in the same square, and *d e* is equal to *h e*, each being the half of ten roots, therefore the line *d l*, the residuum of the line *g l*, is equal to *e b*, the residuum of the line *e h*. And so, as the line *t e*, by the above demonstration, is not unequal to the line *l m*, the area which is included by the lines *t e* and *e b* is equal to the area comprehended by the lines *l m* and *d l*. Therefore the area *t b* equals the area *m d*. The square *t l* equals 25. Therefore when we subtract from this same square *t l* the areas *d t* and *m d*, which are of course equal to the two areas *g e* and *t b*, containing 21, it is evident that we have left the square *n c*, which amounts to the difference between 25 and 21. This number is four, of which the root is two, and this gives the line *e c*. Moreover *e c* equals *d l*, since each represents the breadth of the area *d c*. Since *d l* equals *e b*, it is evident that when *e b*, which is two, is taken from *e h*, which is half of the roots, or five, three remains for the line *b h*. Therefore three is the root of the first x^2.

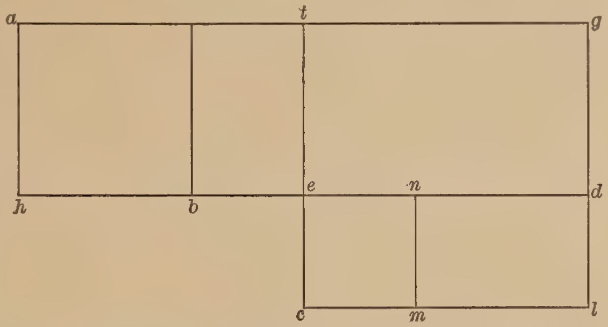


FIG. 9. — Completed figure. From the Columbia manuscript, where it appears twice.

On the contrary if we add the line *e c* to the line *e h*, representing half of the roots, we get 7 which is *n h*. And so the root of the square is greater than (the root of) the

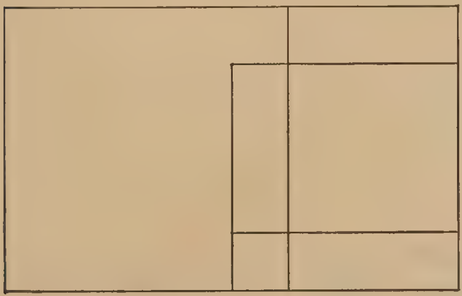


FIG. 10. — Unlettered figure. From the Columbia manuscript.

priore substantia erit, quia videlicet super ipsam 21 addidimus. Et similiter ipsa suis 10 radicibus aequalis fiet, quod demonstrare voluimus.

De radicibus et numeris substantiam coaequantibus.

Tres radices et 4 ex numeris coaequant substantiam.

Quadratum igitur cuius latera ignota ponuntur propono, quod sit *a b c d*, atque hoc quadratum tribus radicibus et quatuor ex numeris, vt diximus, aequale constituo. Quoniam autem manifestum est, quòd si vnum latus omnis quadrati semel in vnitatem ducamus vna radix eiusdem quadrati necessario nascatur ex quadrato igitur *a b c d* per lineam *e f* aream *a f* resecemus, atque vnum huius areae latus numerum ternarium, id est numerum radicum, significare constituamus. Sit autem hoc latus linea *a e*. Nobis igitur manifestum quoniam area *e c* numero quaternario, quem supra radices adiecimus, adimpletur, igitur super punctum *g* latus *a e* tres radices signans, per lineam in duo media diuidamus, ex quorum vno quadratum, quod est *g k l e* faciamus, quadratum inquam, quod ex multiplicatione medietatis radicum in seipsa ductae perficitur, hoc est ex vnitate et medietate cum seipsis multiplicatis producitur.

Deinde lineam *k m* quae sit lineae *e d* aequalis, lineae *g k* adiiciamus, fietque linea *g m* aequalis lineae *g d*, vt quadratum, quod est *g o*, inde nascatur. Et iam manifestum est, quoniam linea *a d* aequalis est lineae *e f*, sed *g d* aequalis est *e n* cum ipsae in quadrato *g o* tenduntur aequales, linea igitur *g a* lineae *n f* manebit aequalis. Quia verò *g a* aequalis est lineae *g e* cum ipsae radices medient, id est per medium diuidant, linea insuper *g e* lineae *k l* aequalis, nam ipsae in latitudine areae *e m* continentur aequales: linea igitur *k l* lineae *n f* aequalis erit. Rursus manifestum est, quoniam linea *d g* aequalis est lineae *e n* cum ipsae in quadrato *g o* tendantur aequales. Sed linea *g e* aequalis est lineae *e l* cum ipsae quadratum *g l* aequali

---

1. ista *pro* priore. erit *om.* quando *pro* quia. addidimus, fiet ipsa substantia 10 suis consimilis radicibus et hoc est (tamen D) quod explanare voluimus.

2. *Add.* Sequitur huius alterius partis pro additione figura geometrica C.

3. *Titulum om.* V; De tribus radicibus et 4^{or} ex numero D.

4. Tres + autem. numero D.

5. ponuntur + substantiam. sit + rumbus *a d.* Sed et totum (+ habet D) *pro* atque hoc.

6. numero. que prediximus. equalem. constituto D.

7. Et iam *pro* Quoniam autem.

8. simul D. duxerimus V; duximus D. eius V. nascetur. Aream igitur (ergo D) *h d* ex area *a d* resecemus (repetemus D *sed* resecemus *superscr. man.* 2) et unum aliquod (aliud D) *pro* ex quadrato . . . huius areae.

10. signans constituamus.

11. Erit quoque (que D) hoc latus (+ simile D) *z d.* area *h b.* 4^{or} ex numero que.

12. adimpleat. *e pro g.*

13. *e h g* V; *h g* D. per lineam *om.* quibus *pro* quorum vno.

14. area *e c pro g k l e.* Eritque hec area que *pro* quadratum inquam, quod.

15. ductae: deducte V; deductam D. perficitur *om.* 2^{bus} et 4^{a}V; duobus et 4^{ta} D *pro* vnitate et medietate.

16. deductis perficitur.

17. *t l pro k m.* quae sit *om.* *a h pro e d.* consimilem. *e t pro g k.* et ffietque D.

18. *a e pro g m.* *e l pro g d.* area *e m pro g o.*

19. *a g pro a d.* *h z*, sed *h e* (*a e* D). *h n pro e n.*

20. quoniam *pro* cum. rumbo *e m.* *e g pro g a.* *n c* V; *z n* D. remansit consimilis *pro* manebit aequalis.

21. Sed et linea *pro* Quia verò. *e g pro g a.* *h e pro g e.* Nam et *pro* cum. mediant.

22. diuidunt. quoque *h e* (ergo *h a sed a corr. ex e man.* 1 D) similis est linee *t c pro* insuper . . . aequalis.

23. *h l pro e m.* *n z pro k l.* *c t pro n f.* Et iam *pro* Rursus.

24. *a h pro d g.* lineae *om.* *m n pro e n.* quoniam in latitudine aree *a z* proponuntur *pro* cum ipsae . . . tendantur.

25. Sed + et. *a e pro g e.* quoniam *pro* cum. rumbum *e m* V; rumbos eos D *pro* quadratum *g o.*

first square. Of course when you add 21 to it the sum is equal likewise to ten of its roots which we desired to demonstrate.[1]

Concerning Roots and Numbers Equal to a Square

Three roots and four are equal to x^2.

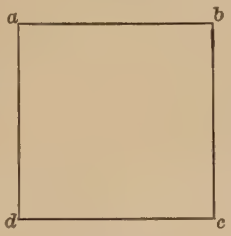


Fig. 11.

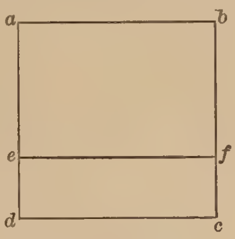


Fig. 12.

Incomplete figures. From the Columbia manuscript.

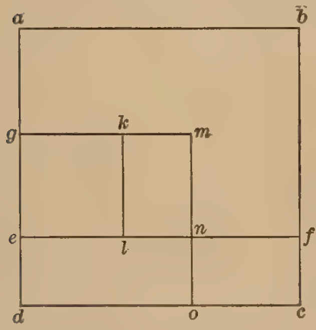


Fig. 13. — Completed figure. From the Columbia manuscript.

I suppose a square, which is *a b c d*, of which the sides are unknown; this square, as we have said, equals three roots and four in number. If one side of any square is multiplied by unity you necessarily obtain one root of the same square. Therefore we cut off from the square *a b c d* the area *a f* by the line *e f* and one side of this area we take to be three, constituting the number of the roots; let this side be the line *a e*. Now it is clear to us that the area *e c* amounts to the four which is added to the roots. Hence we bisect the side *a e*, representing three roots, at the point *g*. Upon this half construct a square, which is *g k l e*. I say that this square is made by the multiplication of half of the roots by itself, that is, produced by one and one-half multiplied by itself. Then to the line *g k* we add the line *k m*, which is equal to the line *e d*. The line *g m* equals the line *g d*, thus forming a square which is *g o*. Since *a d* is equal to *e f* and *g d* equal to *e n*, as they occur in the square *g o*, it is evident that *g a* is left equal to *n f*. The line *g a* is equal to the line *g e*, being half of the roots, that is, bisecting the roots, and it is further true that *g e* equals *k l*, for each measures the breadth of the area *e m*. Therefore *k l* equals *n f*. Again since *d g* equals *e n*, being in the square *g o*, and *g e* equals *e l*, as they measure the same dimension of the square *g l*, it is evident that the line *e d* is

---

[1] This paragraph is not found in the Libri version but appears in the Arabic as published by Rosen. The translation follows the Vienna version. The figure to be used in the geometrical demonstration to obtain by addition the second root of the given quadratic equation appears at the bottom of the preceding page. (Fig. 10.) The Boncompagni version (*loc. cit.* p. 35) varies by letting the middle point fall first within the side of the first square, and secondly without: Cum itaque dividitur per medium linea *b e* ad punctum *z*, cadet ergo inter puncta *g e* aut *b g*: sit hoc prius inter puncta *b g*.

dimensione circundant; linea igitur *e d* lineae *l n* aequalis manebit. Quia verò *e d* lineae aequalis est lineae *n o*, cum ipsae in latitudine areae proponantur aequales; et linea igitur *n o* lineae *l n* aequalis erit, atque tandem cum linea *k l* lineae *n f* ex superiore demonstratione non sit inaequalis, area quam lineae *k l* et *l n* circundant, comprehensae sub *n f* et *n o* lineis areae aequalis erit. Area igitur *k n* aequalis est *n c* areae. Et iam manifestum, cum area *e c* quatuor ex numeris, quae supra tres radices addidimus, in se contineat, duae areae *e o* et *k n* vni areae *e c*, quae quatuor ex numeris in se continet, in quantitate aequales fiunt. Manifestum est igitur nobis, quadratum *g o* ex multiplicatione medietatis radicum, id est $1\frac{1}{2}$ ex numeris, cum suo consimili, et adiectione numeri quatuor, cui duae areae *e o* et *k n* aequales sunt, compleri. Est autem totum hoc quadratum numero senario et vni quartae aequale, atque eius radicem duo et medium designant, quae in latere *d g* continentur. Restat igitur nobis ex latere quadrati primi, quod est area *a b c d* quae totam substantiam significat, radicum medietas, quae vnum et medium in se continet, latus etiam *g a*. Cum ergo linea *d g*, quae est latus quadrati *g o*, continentis in se id quod ex multiplicatione medietatis radicum cum seipsa colligitur, cuiusque adiectio sunt quatuor, quae diximus. Et hoc totum sex et vnam quartam in se continet, quarum radicem $2\frac{1}{2}$ super lineam *g a* quae est medietas trium radicum, vnum et medium in se continens, addimus. Tota igitur haec summa, ad quaternarium excrescet numerum, quae est linea *a d*; item radix substantiae atque insuper etiam quadratum *a c*. Tota autem substantia in 16 terminatur. Et hoc est quod exponere voluimus.

Haec igitur geometrica compendiose diximus vt ea quae alioquin oculis mentis difficultate quadam concipiuntur, his geometricè perspectis, ad eadem intelligenda facilior huius disciplinae aditus paretur.

MAHOMET ALGAURIZIN DE ADDITIS ET DIMINUTIS

Et inueni (inquit Mahomet Algoarizin) omnem numerum restaurationis et

---

1–12. diuisione (+ diuisione D *et* diuisione *superscr.* D *man.* 2) circumdant. Et linea *h e* similis est linee *e t*. Nam et ipse rumbum *e c* equali longitudine circumdant. Remanet eciam linea *a h* similis linee *t l*, linea ergo *t l* similis sit (est D) linee *m n*. Sed et linea *c t* linee *n z* iam fuerat consimilis. Area igitur quam *m n*, *n z* circumdant similis est aree (+ quam *c t*, *t l* similis quam *t e* circumdant. Area ergo *n z* similis est aree D; *n z pro n t del.*) *c l*. Et iam manifestum est quoniam area *a z* 4 ex numero que sunt (super D) 3 radices addidimus in se obtineat due igitur aree *a n* et *c l* vni aree *a z* que 4 ex numero in se continet in quantitate fiunt consimiles. Manifestum ergo est nobis quoniam rumbus *e m* qui (numeri D) ex multiplicatione radicum in suo consimili deducte que scilicet duobus (duo habet D) et medium colligitur totum adimpleat (adimplet D) id est duo et ¼ que simul rumbum *e c* perficiunt cuius videlicet adiectio sunt 4 ex numero que due aree *a n*, *c l* adimplent. Fietque (Sitque D) hoc totum senario numero et vnius 4^e coequale (coequales D), cuius simul radicem (radicis D) duo et.

13. *a e et sic infra.*

14. *a d.* quae + et.

15. duo *pro* vnum. que simul lineam *e g* perficiunt (perfit D). Cum ergo lineam *a e* que est radix rumbi *e m* que hoc *pro* latus . . . id.

17. radicem D. in suo consimili + deducte. colligitur + continent.

18. obtinet. quare radix duo sunt et medium (+ que si V).

19. *e g.* in se *om.*

20. addiderimus. igitur *om.*

21. *a g* que est radix substantie que est area *a d.*

22. igitur V; ergo D *pro* autem. 16 + ut subiecta docet descripcio.

23. geometrice. que oculis mentis quasi quadam difficultate concipiuntur, perspectis his (hiis D) geometrice figuris, ad.

25. faciliorem disciplinem D. prebeant aditum; aditur paratur C. *Pagina fere tota vac.* V.

26. Mahumed Algorismus V; Mauhumed algaorisim D. *Titulum om.* C.

27. Mahumed Alguarizmi (algaorismi D).

left equal to $l\,n$. Further since $e\,d$ equals $n\,o$, as they measure the breadth of the same area, therefore $n\,o$ equals $l\,n$. Now as from the above demonstration $k\,l$ is not unequal to $n\,f$, it follows that the area comprehended by $k\,l$ and $l\,n$ equals the area included by the lines $n\,f$ and $n\,o$. Therefore the area $k\,n$ equals the area $n\,c$. The area $e\,c$ amounts to the number four which we added to the three roots, and it is evident that the two areas $e\,o$ and $k\,n$ are equal in quantity to the one area $e\,c$, containing four in number. It is clear to us that the square $g\,o$ consists of the product of one-half of the roots, *i.e.* $1\frac{1}{2}$, by itself, with the addition of the two areas $e\,o$ and $k\,n$. Thus the sum total of this square is $6\frac{1}{4}$ and the root of it is given by $2\frac{1}{2}$, which is contained by the side $d\,g$. We have left of the side of the first square, which is the area $a\,b$, representing x^2, the half of the roots amounting to $1\frac{1}{2}$ which is the side $g\,a$. The line $d\,g$ is a side of the square $g\,o$, containing the product of one-half of the roots by itself with the addition of four, as we have said. This total amounts to $6\frac{1}{4}$. We add the root of this, $2\frac{1}{2}$, to the line $g\,a$ which is one-half of the three roots, amounting to $1\frac{1}{2}$. Hence this sum total reaches four, which is the line $a\,d$. This is the root of x^2 and also further of the square $a\,c$. The whole of x^2 is finally 16. This is what we desired to explain.

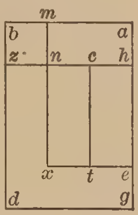


FIG. 14. — From the Vienna manuscript.[1]

We have now explained these things concisely by geometry in order that what is necessary for an understanding of this branch of study might be made easier. The things which with some difficulty are conceived of by the eye of the mind are made clear by geometrical figures.[2]

POSITIVES AND NEGATIVES

And I found, says Mohammed Al-Khowarizmi, that all problems of restoration and opposition are included in the six chapters which we have set forth in the beginning of this book.[3]

---

[1] The writer of the Vienna manuscript took no pains to make the proportions of his figures correct. Thus in this figure $a\,b\,d\,g$ is intended to represent a square, while e is supposed to be the middle point of the line $h\,g$. Further $e\,c$ is to be a square and likewise $e\,m$, and also the rectangle $n\,t$ is intended to be equal to the rectangle $z\,m$. Moreover the proof in the Vienna manuscript is not consistent with the lettering of the figure, showing that the copyist did not succeed in following closely the argument of the text. Similarly the figures in the Boncompagni version do not have the correct proportions.

For the proof based directly on Euclid II. 6, see page 133. The Boncompagni version and Abu Kamil make explicit reference to the propositions of Euclid.

[2] This paragraph is not found in either the Libri or the Arabic versions; nor in the Boncompagni text.

[3] The evident meaning of this passage is that all problems leading to equations of the first or second degree can be solved by the methods set forth in the preceding text.

oppositionis in sex capitibus, quae in principio huius libri praemisimus, contineri. Nunc porro, quomodo res vel radices, quando vel solae vel cum illis numeri fuerint, aut quando ex eis numeri extracti, seu cum ipsae ex numeris extractae fuerint, ad inuicem multiplicentur, vel quomodo ad inuicem iungantur, vel ex aliis diminuantur, deinceps dicendum est.

In primis ergo sciendum est, quòd numerus cum numero multiplicari non possit, nisi cum numerus multiplicandus toties sumatur, quoties in numero cum quo ipse multiplicatur, vnitas reperitur. Cum ergo nodi numerorum et cum illis aliquot vnitates propositae fuerint, aut si vnitates ab illis subtractae fuerint, tunc multiplicatio quater repetenda erit; hoc est nodi primò cum nodis, vnitates deinde cum nodis, et nodi cum vnitatibus, ac tandem vnitates cum vnitatibus multiplicandae erunt. Cum itaque vnitates quae cum nodis pronunciantur, omnes adiectae siue omnes diminutae fuerint, quarta multiplicatio erit addenda. Quòd si quaedam earum fuerint adiectae, quaedam verò diminutae; quarta multiplicatio erit minuenda.

Similitudo talis est, 10 et duo cum 10 et vno multiplicanda sunt. Multiplica ergo 10 cum 10, et producuntur 100; deinde 2 cum 10, et procreantur 20 addenda. Similiter 10 cum vno, et procreantur 10 addenda, et duo cum vnitate, et producuntur 2 addenda. Tota igitur huius multiplicationis summa in 132 terminatur. Et hoc est quod diximus, quando vnitates quae cum nodis pronunciantur, omnes fuerint adiectae.

At quando 10 sine 2 cum 10 sine vno multiplicare volueris, dicas 10 cum 10 generant 100, et duo diminuta cum 10 procreant 20 diminuenda. Item 10 cum vno procreant 10 diminuta. Hoc autem totum 70 complectitur. Sed duo diminuta cum vno diminuto, duo procreant addenda. Tota ergo haec summa in 72 terminatur. Et hoc est quod diximus, quando omnes diminutae fuerint.

Si autem 10 et 2 cum 10 sine vno multiplicare volueris: dicas 10 cum 10 100,

---

1. capitulo D.

2. Nunc ergo dicendum (addendum D) est. id est radices quando sole fuerint uel quando cum.

3. uel *pro* aut V. numeros extraxerint. quando *pro* cum.

4. et qualiter adinuicem iungantur, et qualiter quidam (qui D) ex aliis diminuantur *pro* vel^1 . . . est.

6. est *om.* V. quam. in numerum.

7. cum^1 *om.* duplicandus D. tociens duplicetur quociens. in (*om.* D) quem.

8. multiplicatio D. modi D *saepius;* C *add.:* seu incerta et obscura cognitio.

10–12. id est nodos in (+ prima D) nodis et unitates in nodis (*om.* D) et iterum unitates in nodis et unitates cum unitatibus. Cum ergo.

13. adiectiuus *pro* adiectus *siue* addendus *et saepius.* su^a *pro* siue D. diminutiuus *pro* diminutus *siue* diminuendus *et saepius.*

14. alie V; alia D *pro* quaedam^2. multiplico V; multiplicato D.

16–19. Dicam ergo 10 in 10, 100 procreant, et unum in 10, 10 generat adiectiua et 2 in 10, 20 procreant adiectiua, et duo in 1 duo generant adiectiua.

19. 132 creantur D. C *add.:* vt sequens habet calculus.

10 et 2
cum 10 et 1 multiplicata
100 20
10 1
producunt 132

20. qui.

22. 10 sine uno in 10 sine uno (+ in 10 sine uno V).

23–25. et unum diminutum in 10 deductum, 10 procreant diminutiua. Item unum diminutum (diminutiuum D) in 10 deductum, 10 generat diminutiua. Hoc ergo totum 80 (81, *et* 80 C *sed del.*) amplectitur. Sed et unum diminutum in uno diminutiuo unum procreant adiectiuum.

26. C *add.:* vt sequitur,

10 sine 2
cum 10 sine 1 multipli.
100 mi. 20
mi. 10
Summa prod. 70
plus 1
71

27. ducas *pro* Dicas D. fiunt 100 V.

Now further the method is to be explained by which you multiply unknown quantities or roots, either when alone, or when numbers are joined to them or subtracted from them, or when they are subtracted from numbers; also in what manner they are added to, or, in turn, subtracted from, each other.

In the first place you should understand that the only way to multiply a number by a number is to take the number to be multiplied as many times as there are units in the number by which it is to be multiplied.[1] When therefore the nodes [2] of numbers are proposed either with some units or if units are subtracted from them, then the multiplication is fourfold, *i.e.*, first the nodes are multiplied by the nodes, then the units by the nodes and the nodes by the units, and finally the units by the units. When therefore the units which accompany the nodes are both added or both subtracted the fourth product is to be added. But if one is added and the other subtracted then the fourth product is to be subtracted.

A problem of this kind is given by the following: 10 and 2 are to be multiplied by 10 and 1. Hence multiply 10 by 10, giving 100; then 2 by 10, giving 20 to be added; likewise 10 by 1, giving 10 to be added. Two by 1 gives 2 to be added. The sum total of this multiplication is finally 132.[3] And this illustrates what we have said in respect to the type in which the units which accompany the nodes are both to be added.[4]

But when you wish to multiply 10 less 2 by 10 less 1, you say 10 by 10 gives 100; 2 to be subtracted by 10 gives 20 to be subtracted; also 10 by 1 gives 10 to be subtracted. This total, then, amounts to 70. But negative 2 multiplied by negative one gives positive 2. Therefore the sum total is finally 72.[5] This illustrates what we have said when both (binomials) involve negatives.

Moreover if you wish to multiply 10 and 2 by 10 less 1, you say 10 by

---

[1] To this definition Al-Khowarizmi refers in his arithmetic (*Trattati*, I, p. 10).

[2] Scheybl adds that this is a word of uncertain and obscure meaning. The Arabic word *ʿuqud* is connected with the verb meaning "to knot," referring to tying knots on a string to indicate numbers. The Libri and Boncompagni texts use *articuli*, while Rosen translates 'greater numbers.' See also F. Woepcke, in *Journal Asiatique*, Vol. I (6), 1863, p. 276.

[3] The Columbia manuscript continues with the following addition by Scheybl:

The calculation is as follows:

$$\begin{array}{rrl} 10 + & 2 & \\ 10 + & 1 & \\ \hline 100 & 20 & \\ & 10 & 1 \text{ [written by mistake for 2]} \\ \hline & 132 & \end{array}$$

[4] This is one of the early attempts at a discussion of the multiplication of binominals, including $(x+a)$ times $(x+b)$, $(x-a)$ times $(x-b)$, and $(x-a)$ times $(x+b)$.

[5] Scheybl's text contains by error 81 for 72. The problem as given in the Arabic and Libri versions is $(10-1)$ by $(10-1)$ with the product 81. This undoubtedly was given by Robert of Chester, and is so recorded in the Vienna and Dresden MSS. Scheybl evidently varied from the text before him, but neglected to make necessary changes in the numerical computation.

et 2 adiecta cum 10 multiplicata 20 generant addenda. Item 10 cum vno diminuto multiplicata, 10 procreant diminuenda. Haec autem summa vsque ad 100 et 10 protenditur. Sed 2 adiecta cum vno diminuto multiplicata, 2 procreant diminuenda. Vnde tota multiplicationis summa ad 108 extenditur. Et hoc est quod etiam diximus, quando quaedam earum fuerint adiectae quaedam vero diminutae.

Similiter in fractionibus, si dicas: drachma et eius sexta cum drachma et eius sexta. Dicas, drachma cum drachma drachma[m], et drachmae sexta cum drachma drachmae sextam procreat. Item sexta cum drachma sextam procreat, et sexta cum sexta sextam sextae, id est tricesimam sextam drachmae procreat. Erit autem hoc totum, drachma $\frac{1}{3}$ et $\frac{1}{36}$ drachmae.

Si eodem modo drachma[m] sine sexta cum drachma sine sexta multiplicares, tantum fiet quantum si $\frac{5}{6}$ cum suo aequali multiplicares. Vnde et haec multiplicatio ad 25 partes ex tricesimis sextis partibus vnius drachmae extendetur, id ad $\frac{2}{3}$ et $\frac{1}{6}$ sextae.

Modus autem multiplicationis est, vt drachmam cum drachma multiplices, et producetur drachma; deinde sine sexta cum drachma, sextam procreat diminuendam; item drachmam cum sine sexta, et producetur vna sexta diminuenda. Duae igitur tertiae vnius drachmae supersunt. Et sine sexta cum sine sexta, sextam sextae generat addendam. Tota igitur haec summa ad $\frac{2}{3}$ et sextam sextae extenditur.

---

1. et (*om.* V) unum diminutum in 10 multiplicatum, 10 generat diminutiua, 2 quoque (ergo D) adiectiua in 10 deducta, 20 procreant adiectiua; duo . . . adiectiua^2 *in marg.* D *man.* 2.

3. 10 *pro* 2^2 C. Hec igitur.

4. Vnde + et. 180. C *add.* Haec autem sequenti calculo patent:

$$\begin{array}{rcr} 10 & \text{et} & 2 \\ 10 & \text{sine} & 1 \\ \hline 100 & \text{plus} & 20 \\ & \text{mi.} & 10 \\ \hline & \text{Summa pro.} & 110 \\ & \text{Minus} & 2 \\ \hline & & 108 \end{array}$$

Et *om.*; Est C.

6. diminutiue *et add.* Hoc igitur modo res inuicem multiplicantur. Quando cum ipsis numeri fuerint (+ et quando ipse sine numeris fuerint. Et quando numeri sine ipsis propositi fuerint V). Et si tibi propositum fuerit 10 sine re (+ in 10 deducta quantum constituunt. Dicas 10 in 10, 100 et sine re V; + et expositio rei est re in 10 deducta. Dicas 10 in 10, 100 et sine re *in marg.* D *man.* 2) in 10, 10 generat (generant D) radices diminutiuas. Dicas ergo quod tota hec summa usque ad 100 extenditur 10 rebus abiectis. Et si dixerit 10 et res in 10 deducta quantum procreant. Dicas 10 in 10 centum, et res adiectiua decies deducta 10 res generat adiectiuas. Tota igitur (ergo D) hec summa ad 100 et 10 res extenditur. Si autem dixerit 10 et res in suo consimili quantum multiplicata faciunt. Dicas 10 in 10, 100 procreant (+ et 10 in re 10 res procreant V · + res D). Item 10 in re, 10 res procreant et res in re substantiam generat adiectiuam. Ergo tota hec summa ad 100 ex numero et 20 res et substantiam extenditur adiectiuam. Si autem sic (si D) proponat, 10 sine re in 10 sine re quantum faciunt. Dicas 10 in 10, 100 et sine re in 10, res procreant 10 (+ diminutiuas et sine re in 10, 10 similiter diminutiuas res procreant V) et sine re in (+ x D) sine re substantiam generat (generant D) adiectiuam. Erit ergo hoc totum 100 et substantia 20 rebus abiectis; *Vide infra pag.* 94.

7. Eodem modo si dragma (dragmam D) et eius sextam (+ in dragma et eius 6^{tam} V) duxeris, quantum fiet.

8. et *om.* dragma in 6^{tam}, sextam dragmatis.

9. forma *pro* sexta D.

10. et *pro* cum D. $\frac{1}{36}$ (xxxvi D) dragmatis generat adiectiuam. Erit ergo hoc totum dragma et $\frac{2}{6}$ vel $\frac{1}{3}$ (+ dragma D) et sextam (due D) 6^{te}.

12. duxeris. 16. huius *pro* autem. ut dragma.

17. fietque. et *pro* deinde. in dragma + multiplicatum.

18. et sine 6^{ta} (*om.* D) in dragma, sextam procreat similiter diminutiuam.

19. vnius *om.* in sine sexta + deductum.

21. C *add.*: Sequitur calculus.

$$\begin{array}{ll} \text{drachma} & \text{sine sexta} \\ \text{drach.} & \text{sine sex.} \\ \hline \text{drach.} & \text{sine } \frac{1}{6} \\ & \text{sine } \frac{1}{6} \\ \hline \end{array}$$

manent $\frac{2}{3}$. Accedit $\frac{1}{6}$ de $\frac{1}{6}$. Vnde multiplicationis productum tandem ad $\frac{2}{3}$ plus $\frac{1}{36}$ sese extendat.

10, 100, and positive 2 multiplied by 10 gives positive 20. Also 10 multiplied by negative 1 gives negative 10. This sum, moreover, amounts to 110. But positive 2 multiplied by negative 1 gives negative 2. Whence the sum total of this multiplication equals 108.[1] And this illustrates the type of process when units are to be added and others to be subtracted.[2]

Likewise in the case of fractions, if the problem is a unit and one-sixth (to be multiplied) by a unit and one-sixth. You say, unit by unit gives unit; and one-sixth of a unit by a unit gives one-sixth of a unit. Also one-sixth by a unit gives one-sixth and one-sixth by a sixth gives one-sixth of a sixth, *i.e.* one thirty-sixth. The total will be a unit and $\frac{1}{3}$ and $\frac{1}{36}$ of a unit.

In the same manner, if you multiply a unit less one-sixth by a unit less one-sixth, the product will be the same as $\frac{5}{6}$ multiplied by its equal. Whence this product equals 25 thirty-sixths of one unit, *i.e.* $\frac{2}{3}$ and $\frac{1}{6}$ of one-sixth.

Now the method of this multiplication is that you multiply unit by unit, giving unit; then negative one-sixth by unit, giving negative one-sixth; then you multiply a unit by negative one-sixth, giving one-sixth negative. Therefore two-thirds of one unit remain. And negative one-sixth multiplied by negative one-sixth produces one-sixth of one-sixth positive. The sum total therefore amounts to $\frac{2}{3}$ and one-sixth of one-sixth.[3]

---

[1] Scheybl adds: Moreover this is evident by the following calculation:

$$\begin{array}{r} 10 + 2 \\ \underline{10 - 1} \\ 100 + 20 \\ \underline{-10} \\ 110 \\ \underline{-2} \\ 108 \end{array}$$

Similarly to the example which immediately precedes this Scheybl adds: as follows:

$$\begin{array}{r} 10 - 2 \\ \underline{10 - 1} \\ 100 - 20 \\ \underline{-10} \\ 70 \\ +1\,[2] \\ 71\,[72] \end{array}$$

[2] Evidently considerable interchange of text was made at this point by Scheybl. The passage inserted in the footnote to line 6 should be compared with the 28 lines of Scheybl's text on page 94, which are not found at that point in the Dresden and Vienna manuscripts.

[3] Scheybl adds: The calculation follows:

$$\begin{array}{l} \text{unit} - \frac{1}{6} \\ \underline{\text{unit} - \frac{1}{6}} \\ \text{unit} - \frac{1}{6} \\ \underline{\qquad\quad - \frac{1}{6}} \\ \text{giving } \frac{2}{3} \\ \text{Add } \ \frac{1}{6} \text{ of } \frac{1}{6}. \end{array}$$

Whence the product of this multiplication finally amounts to $\frac{2}{3}$ plus $\frac{1}{36}$.

Sequuntur nunc similes nodorum multiplicationes, per res seu radices et numeros expositae. Eodem modo res inter se multiplicantur, quando cum ipsis numeri, vel ipsae sine numeris fuerint, vel quando sine ipsis numeri propositi fuerint. Dicendo multiplicetur 1 res et 10 cum 1 re et 10, dicas igitur res cum re substantiam, et res cum 10 multiplicata 10 res generat. Item 10 cum re 10 res, et 10 cum 10 multiplicata 100 generant. Erit ergo totum 1 substantia 20 res et 100 ex numeris.

Et si dicas, multiplicetur 1 res cum 1 re: dicas, res cum re multiplicata producit substantiam. Atque tantum quidem est multiplicationis productum.

Similiter res sine 10 cum re sine 10: dic, res cum re substantiam producit; sine 10 vero cum re 10 res producit diminuendas. Item res cum re 10 res diminuendas; sine 10 verò cum sine 10 100 ex numeris addendas procreant. Vnde totum multiplicationis productum ad 1 substantiam sine 20 radicibus, additis vero ex numeris 100, sese extendit.

Vel etiam si dicas, 10 cum 10; item 10 sine re cum 10 sine re: dic 10 cum 10 multiplicata procreant 100. Atque tantum est productum multiplicationis prioris. Dic deinde 10 cum 10 centum addenda; sine re verò cum 10, 10 res generat diminuendas. Item dic 10 cum sine re 10 res diminuendas; sine re verò cum sine re, substantiam procreat addendam. Vnde totum multiplicationis posterioris productum ad 100 absque 20 rebus, vna substantia verò adiecta, sese extendit.

Si autem quaesieris 10 et res cum suo aequali multiplicata, quantum producunt? Dic 10 cum 10, 100; et res cum 10, 10 res procreat. Item 10 cum re, 10 res; et res cum re, substantiam generat. Tota autem haec multiplicatio ad 100 ex numero, 20 res et vnam substantiam sese extendet.

Quòd si sic quaesieris decem sine re cum 10, vel decem et res cum 10: producet multiplicatio prior 100 ex numeris absque 10 rebus, posterior verò 100 ex numeris et 10 res.

Si autem dixerit aliquis, decem sine re cum 10 et re multiplicata quantum faciunt? Dicas 10 cum 10, 100 drachmas; et sine re cum 10, 10 res procreat diminuendas. Item 10 cum re, res 10 generant addendas; et sine re cum re, substantiam procreat diminuendam. Hoc ergo totum ad 100 drachmas proueniet, vna substantia abiecta.

Et si dixerit, decem sine re cum re: dic 10 cum re, 10 res procreant, et sine re cum re, substantiam generat diminuendam. Hoc ergo ad 10 res perueniet abiecta substantia.

1–28. *om.* *Vide pag.* 92, *n.* 6.
7. C *add.*: vt sequitur,
1 res et 10
cum 1 re. et 10
1 sub. et 10 res
10 res et 100 N
Summa pro. 1 sub. 20 res 100 ex nu.
14. C *add.*: Sequitur calculus.
1 res sine 10
cum 1 re. sine 10
1 sub. sine 10 re.
sine 10 re. plus 100
1 sub. sine 10(20) res plus 100

29. aliquis *om.*
30. Ducas D. drachmas *om.*
31. et res in 10, 10 res. generat.
32. procreant D. Hoc *om.* D. perueniet.
33. vna *om.* D. C *add.*: Sequitur calculus.
10 sine re.
cum 10 et re.
100 sine 10 reb.
10 res sine substantia
100 sine substantia
34–36. Et . . . substantia *in marg.* D *man.* 2.
34. dicas *pro* dic *saepe.*
35. perueniat.

Similar multiplications of nodes,[1] illustrated by things or roots and numbers, follow. In the same manner the unknowns[2] are multiplied by themselves, either when numbers added to them, or when numbers are subtracted from them, or when they are to be subtracted from numbers. For example, to multiply $x + 10$ by $x + 10$, you proceed thus: x by x, x^2, and x multiplied by 10 gives $10x$; also 10 by x, $10x$, and 10 multiplied by 10 gives 100. The sum total is then x^2, $20x$, and 100.[3]

Another example: multiply x by x. You say that x by x gives x^2, and this is the product of the multiplication.

Similarly, $x - 10$ by $x - 10$: x by x gives x^2 and negative 10 by x gives negative $10x$. Also negative[4] 10 by x gives $10x$ negative, and negative 10 by negative 10 gives positive 100. Whence the total product of this multiplication amounts to x^2 less $20x$, with 100 to be added.[3]

Or also if you multiply 10 by 10, and again $10 - x$ by $10 - x$: 10 multiplied by 10 gives 100; so much is the product of the first multiplication. Then, 10 multiplied by 10 gives positive 100; negative x by 10, $10x$ negative; also 10 by negative x gives negative $10x$; negative x by negative x gives positive x^2. Whence the total product of the second multiplication extends to 100 less $20x$, with x^2 to be added.

Again, if you try to find the product of $10 + x$ multiplied by its equal you proceed thus: 10 by 10, 100; and x by 10 gives $10x$; also 10 by x, $10x$, and x by x gives x^2. The total product amounts to 100, $20x$ and x^2.

Now if you try to find the product of either $10 - x$ by 10 or $10 + x$ by 10, the first product is $100 - 10x$ and the other $100 + 10x$.

Further some one may ask, how much is the product of $10 - x$ by $10 + x$? You proceed thus: 10 by 10, 100 units, and negative x by 10 gives negative $10x$; also 10 by x gives $10x$ positive, and negative x by x gives negative x^2. This total then equals 100 units less x^2.[3]

Another problem: $10 - x$ by x. You proceed thus: 10 by x gives $10x$, and negative x by x gives x^2 to be subtracted. This then equals $10x$ less x^2.

---

[1] See p. 91, footnote 2.

[2] *Res* (literally 'thing') is used in such a technical sense that it seems better to translate by 'unknown,' as in this instance, or by x as in much of the following work.

[3] Scheybl adds to this, and to two problems below, the following calculation forms which approach the modern symbolism for the product of binomials. Scheybl prefaces with the words, 'as follows,' or 'the calculation follows':

$$\begin{array}{l} x + 10 \\ x + 10 \\ \hline x^2 + 10x \\ \qquad 10x + 100N \\ \hline x^2, \quad 20x, \quad 100 \end{array} \qquad\qquad \begin{array}{l} x - 10 \\ x - 10 \\ \hline x^2 - 10x \\ \qquad - 10x + 100 \\ \hline x^2 - 20x + 100 \end{array} \qquad\qquad \begin{array}{l} 10 - x \\ 10 + x \\ \hline 100 - 10x \\ \qquad + 10x - x^2 \\ \hline 100 - x^2 \end{array}$$

Scheybl uses $100\,N$ here for the number 100, following the notation employed by him in his printed works on algebra as well as in the algebra text which is found in the same manuscript with this version of Robert of Chester.

[4] In the Latin text (l. 11) Scheybl has *res cum re* instead of *sine 10 cum re*.

Si autem dixerit, decem et res cum re sine 10, quantum procreant? Dic 10 cum re multiplicata, 10 res generant; et res cum re, substantiam generat addendam. Item 10 cum sine 10, 100 drachmas procreant subtrahendas; et res cum sine 10 multiplicata res 10 generat diminuendas. Dicas ergo quòd haec tota summa vsque ad vnam substantiam, 100 drachmis abiectis, sese extendat.

Si autem quis dixerit, decem drachmae et rei medietas cum medietate drachmae, quinque rebus abiectis, multiplicatae, quantum procreant? Dicas, decem cum medietate drachmae multiplicata, 5 drachmas procreant, et medietas rei cum medietate drachmae, quartam rei procreat addendam. Item 10 cum sine 5 rebus multiplicata, 50 procreant res diminuendas. Vnde tota haec multiplicationis summa ad 5 drachmas, 49 rebus et tribus quartis vnius rei abiectis excrescet. Postea medietate rei cum sine 5 rebus multiplicata, duae substantiae et media producentur diminuendae. Tota igitur multiplicationis summa ad 5 drachmas, duabus substantiis et media nec non etiam 49 rebus et tribus rei quartis abiectis, excrescet.

Et si dixerit, decem et res cum re absque 10 multiplicata, quantum faciunt? Est quasi diceres, res et 10 cum re sine 10. Vnde sic respondeas: res cum re multiplicata, substantiam generat; et 10 cum re, 10 radices generant addendas. Item res cum sine 10 multiplicata, 10 res procreat diminuendas. Vnde 10 res adiectae et 10 res diminutae seu ablatae, cum prius tantum tribuat quantum posterius aufert, negligunt; et relinquitur substantia sola. Porro 10 cum sine 10, 100 drachmas generant ex omni substantia diminuendas. Tota igitur haec multiplicatio ad substantiam, 100 drachmis abiectis, extenditur.

CAPUT ADIECTIONIS ET DIMINUTIONIS

Sciendum est, quòd omnis radix substantiae propositae est ignota; duplicatur etiam et triplicatur et caet., atque ex ipsius duplicatione et triplicatione cum sua substantia talis nascitur numerus cuius videlicet vna radix duabus siue tribus radicibus suae substantiae aequiparatur. Quod totum euenire videtur iuxta multiplicationem numeri supra vnitatem naturaliter dispositi. Nam si radices

---

1. Dicas res in 10.

2. generat. res in re + ducta (deducta D).

3. et sine 10 in 10 + deducta. diminutiua. sine (cum D) 10 in re.

4. substantia absque 100, illi (id D) cum quo opposuisti equatur (coequatur D). Quod id circo contingere videtur quia proiecisti 10 res diminutiuas cum 10 rebus (+ et D) adiectiuis. Unde eciam substantia absque 100 dragmatibus permansit *pro* quòd haec . . . extendat.

7–10. multiplicata, quantum procreant. Dicas medietas dragmatis in 10 ducta dragmatibus, 5 dragmata progenerat (progenerant D) et medietas dragmatis in rei medietate (medietatem D) deducta, quartam rei procreat (procreant D) adiectiue, et sine 10 rebus in 10 multiplicatum dragmatibus, 50 res procreat diminutiuas. Unde et.

11. 36 *pro* 49 *bis*. excrescit.

12. Postea multiplica medietatem dragmatis absque 5 rebus in medietatem rei adiectiue fientque due substantie et medium diminutiue.

13. diminutiua *corr. in* diminuendae C.

17. et *pro* Est D. diceret. sine 10 . . . multiplicata *in marg.* D *man.* 2.

18. re + multiplicata. 10 res. adiectiuas.

19. et sine re (x D) in re. diminutiuas.

19–22. unde (+ re in 100 *superscr.* D *man.* 2) adiectiua cum diminutiuis adnullantur (adnullatur D) et remanet substantia, ac 10 in sine 10 100 generantur ex omni substantia diminutiua.

23. extenditur + Et quotquot (quidquid D) fuerit in multiplicatione adiectum seu diminutum semper in lance consimili apponitur id est si unum fuerit adiectiuum alterum erit diminutiuum.

24. *Titulum* Radicum algorismus V; *om.* D.

25. quam V; quando D *pro* quòd. seu V; siue D *pro* est^{3}. ignote.

26. aut *pro* etiam et. ut *pro* atque.

27. substantia + multiplicatione. nascatur.

28. seu V; siue D *pro* suae. equiparantur D.

29. super sola unitate V; super solam unitatem D. Nam + et. radicem.

Yet another problem: how much is $10 + x$ by $x - 10$? 10 multiplied by x gives $10x$, and x by x gives positive x^2; also 10 by negative 10 gives 100 units negative, and x multiplied by negative 10 gives negative $10x$. You can say, therefore, that this sum total amounts to x^2 less 100 units.

If moreover some one asks what is the product of 10 units and one-half x multiplied by one-half a unit less $5x$, you proceed thus: 10 multiplied by one-half a unit gives 5 units and one-half of x by one-half a unit gives one-fourth x; also 10 by negative $5x$ gives negative $50x$. Whence the sum total of this multiplication amounts to 5 units, from which are to be subtracted[1] $49x$ and $\frac{3}{4}x$. Then $\frac{1}{2}x$ multiplied by negative $5x$ gives two and one-half x^2 negative. The sum total of the multiplication amounts to 5 units, with two and one-half x^2, and $49x$ and $\frac{3}{4}x$ to be subtracted.[2]

Another problem: how much is $10 + x$ multiplied by $x - 10$? This is the same as $x + 10$ by $x - 10$. Whence you proceed in this manner: x multiplied by x gives x^2, and 10 by x gives 10 positive roots; also x multiplied by negative 10 gives negative $10x$. Whence the $10x$ to be added (positive) and the $10x$ to be subtracted (negative), or taken away, cancel each other, since the first adds as much as the second takes away and x^2 alone remains. Then 10 by negative 10 gives 100 units to be subtracted from x^2. This total product therefore amounts to x^2 less[3] 100 units.

On Increasing and Diminishing[4]

The fact must be recognized that every root of any given square is unknown; it is also doubled or tripled, etc. in such a way that by doubling and tripling it, by the multiplication of its square, a number is formed of which one root is equal to two or three roots of the given unknown square. All of this turns out to be like the multiplication of any number beyond unity, all in natural order. For if you wish to double the roots, you multiply

---

[1] Error made by Scheybl, who writes "added" instead of "subtracted."

[2] $\left(10 + \frac{x}{2}\right)\left(\frac{1}{2} - 5x\right) = 5 - 49\frac{3}{4}x - 2\frac{1}{2}x^2.$

[3] Scheybl writes *adiectis* for *abiectis*. The problem is, of course, that $x + 10$ multiplied by $x - 10$ gives $x^2 - 100$.

[4] An algebraical work with this title is supposed to have been written by Al-Khowarizmi.

The Libri and the Arabic versions follow with four problems which do not occur in Robert of Chester's translation. However, Scheybl takes up three of these problems in his additions, on pages 142–144 of this work.

These problems, following Rosen, *op. cit.*, p. 27, are as follows: "Know that the root of two hundred minus ten, added to twenty minus the root of two hundred, is just ten. The root of two hundred, minus ten, subtracted from twenty minus the root of two hundred, is thirty minus twice the root of two hundred; twice the root of two hundred is equal to the root of eight hundred. A hundred and a square minus twenty roots, added to fifty and ten roots minus two squares, is a hundred and fifty, minus a square and minus ten roots. A hundred and a square, minus twenty roots, diminished by fifty and ten roots minus two squares, is fifty dirhems and three squares minus thirty roots. I shall hereafter explain to you the reason of this by a figure, which will be annexed to this chapter."

duplicare volueris, binarium cum binario multiplices et quod ex multiplicatione excreuerit, cum ipsius radicis substantia multiplices; et is excrescet numerus, cuius vna radix duabus ipsius substantiae radicibus fiet aequalis. Quòd si radicem triplicare volueris, ternarium cum ternario multiplices, et quod ex multiplicatione excreuerit, cum ipsius radicis substantia multiplices; et is tibi nascetur numerus, cuius vna radix tribus radicibus primae substantiae aequiparatur. Si autem medietatem radicis habere volueris, oportet vt medietatem cum medietate, ac cum producto postea ipsam substantiam multiplices; et erit radix quae tollitur medietati radicis substantiae aequalis. Natura enim numeri hoc exigit, vt quemadmodum in numeris integris multiplicatur, ita etiam et in numeris diminutis, hoc est in fractionibus. Eodem modo cum tertiis, cum quartis, atque omni eo quod ipso integro minus est, agendum erit. Similitudo autem multiplicationis huius talis est.

Similitudo multiplicationis prima. Accipiamus exempli gratia radicem numeri nouenarii, ac deinde multiplicemus. Quòd si duplationem radicis numeri nouem habere volueris, dicas, bis duo procreant 4, que cum nouenario multiplicata, ad 36 excrescet multitudo. Huius itaque multitudinis accipias radicem, id est numerum senarium, qui duabus radicibus numeri nouenarii, hoc est numeri ternarii duplo aequalis reperitur; idem est enim numerum senarium semel accipere.

Similitudo multiplicationis secunda. Si autem radicem numeri nouem triplicare volueris, ternarium cum ternario multiplices, et fient 9; quae si cum seipsis, hoc est cum nouenario multiplicaueris, vsque ad 81 excrescet numerus, cuius vnam radicem nouenarius complet numerus, qui tribus radicibus nouenarii, hoc est numero nouem, videtur aequalis.

Similitudo multiplicationis tertia. Sed si medietatem radicis saepe dicti numeri, habere volueris, medium cum medio multiplica, et fiet quarta, quam si cum 9 multiplicaueris, duo et quartam vnius perficies. Horum igitur radicem accipias, id est vnum et medium, quae medietatem radicis numeri nouenarii, hoc est medietatem numeri ternarii, adimplent. Nam vnum cum sui

---

1. radicare *pro* duplicare D. binario in binario D. dicas V; ducas D *pro* multiplices. numerum ut (et D) *pro* et.

2. substantiam. talis *pro* is.

4. ducas (+ productum in V) substantiam ut talis tibi nascatur *pro* multiplices . . . nascetur.

6. equiparantur D.

7. assumere volueris. et *pro* ac.

8. cum producto *om.* in *pro* ipsam. tollitur + hoc est elicitur C.

10. multiplico D. diminutis + multiplicetur.

11. hoc est in fractionibus *om.* Hoc ergo (quo- D) modo seu (siue D) in 3^{is} seu (siue D) in 4^{is} seu (siue D) in eo quod minus est.

12–14. igitur huius multiplicacione prima talis est ut *pro* autem . . . gratia.

15. Et *pro* Quòd D. duplicationem. numeri nouem habere *om.*

16. bis bini. ducta (ducitur D) + numero.

17. multiplicatio *pro* multitudo; multitudo *corr. ex* multipli C. summe *pro* multitudinis.

18. senarium + accipe. unum *pro* numeri^1 D. id est numero 3^{ris}.

19. bis sumpto similis V; simul D *pro* duplo aequalis. ternarium que est radix nouenarii, bis accipere et numerum semel accipere senarium V; senarium semel accipere quod si numerum ternarium que radix est numeri nouenarii, bis accipere D *pro* senarium semel accipere.

21. Similitudo multiplicationis secunda *om et sic infra.* numeri nouem *om.*

22. ducas numerum, et fiunt 9 que si (similiter D).

23. hoc est cum nouenario *om.* cuius + videlicet.

24. qui scilicet tribus radicibus nouenarii numeri videtur equalis (*om.* D).

26. Et *pro* Sed D. radicis *om.* V. saepe dicti numeri *om.*

27. multiplicare *pro* habere; multiplicare C *sed del.* fietque. quia *pro* quam.

29. quam *pro* quae.

30. medietatem *om.* numeri *om.* V.

2 by 2, and the product by the unknown square of the same root. The result will be a number of which one root will be equal to two roots of the given unknown square.[1] And if you wish to triple the root, you multiply 3 and 3, and the product by the square of the root; so you obtain a number of which one root is equal to three roots of the first square.[2] Moreover, if you wish to take one-half of a root, it is necessary to multiply one-half by one-half, and then the product by the square itself. The root which is taken will be one-half of the root of the given square.[3] Indeed the nature of numbers requires that just as integral numbers are multiplied so also are lesser numbers, *i.e.* fractions. You proceed then in the same manner with thirds, with fourths, and so with every number less than an integer; illustrations follow.[4]

First illustration[5]: Take the root of nine to be multiplied. If you wish to double the root of nine you proceed as follows: 2 by 2 gives 4, which you multiply by 9, giving 36. Take the root of this, *i.e.* 6, which is found to be two roots of nine, *i.e.* the double of three. For three, the root of nine, added to itself gives 6.[6]

Second illustration: If you wish to triple the root of 9, you multiply 3 by 3, giving 9, which multiplied by itself, *i.e.* by 9, gives 81. Of this number 9 is the root, and this is seen to be equal to 3 roots of 9, *i.e.* 3 × 3.[7]

Third illustration: If, however, you wish to take one-half of the root, multiply one-half by one-half, giving $\frac{1}{4}$, which when multiplied by 9 will give $2\frac{1}{4}$. Take the root of this, *i.e.* $1\frac{1}{2}$, which is one-half of the root of 9,

---

[1] $2\sqrt{x} = \sqrt{2^2 \cdot x}$.

[2] $3\sqrt{x} = \sqrt{3^2 \cdot x}$.

[3] $\frac{1}{2}\sqrt{x} = \sqrt{\frac{1}{2} \cdot \frac{1}{2} \cdot x} = \sqrt{\frac{x}{4}}$.

[4] This section begins in the Arabic with the four problems which we have given in footnote 4 on the preceding page of the translation. The part which corresponds to this paragraph, following Rosen, pp. 27–28, is as follows: 'If you require to double the root of any known or unknown square (the meaning of its duplication being that you multiply it by two), then it will suffice to multiply two by two, and then by the square; the root of the product is equal to twice the root of the original square.

If you require to take it thrice, you multiply three by three, and then by the square; the root of the product is thrice the root of the original square.

Compute in this manner every multiplication of the roots whether the multiplication be more or less than two.

If you require to find the moiety of the root of the square, you need only multiply a half by a half, which is a quarter; and then this by the square: the root of the product will be half the root of the first square.

Follow the same rule when you seek for a third, or a quarter of a root, or any larger or smaller quota of it, whatever may be the denominator or the numerator. Examples of this . . .'

[5] In translating I omit some words added by Scheybl.

[6] $2\sqrt{9} = \sqrt{4 \cdot 9} = 6$. The problems may appear trivial, but the reader should note that this is the first approach to an algebraic treatment in systematic form of surd quantities. Al-Khowarizmi proceeds admirably from known to unknown.

[7] $3\sqrt{9} = \sqrt{3 \cdot 3 \cdot 9} = 9$.

ipsius medio bis acceptum ternarium complet numerum. Secundum ergo hunc modum in huius modi multiplicationibus cum omnibus radicibus, quotquot integrae vel fractae fuerint, agendum erit.

Modus diuidendi

Si autem radicem numeri nouenarii in radicem quaternarii diuidere volueris. Diuide 9 in 4 et exeunt $2\frac{1}{4}$, quorum radix, quae in vno et dimidio terminatur, vnam complet particulam.

Si autem e contrario diuidere volueris, id est radicem numeri quaternarii in radicem numeri nouenarii, diuide 4 in 9, et exeunt quatuor nonae vnius; harum radicem, id est duas tercias, accipe. Et hoc est quod vni particulae scilicet contingit.

Alius diuisionis modus

Quòd si radicem numeri nouenarii in radicem numeri quaternarii diuidere volueris, ita tamen vt substantia in substantiam non diuidatur, radicem numeri nouenarii, quoties volueris, duplica vel collige, et scias cuius numeri numerus ex collectione proueniens radix habeatur. Hunc ergo modum numeri in 4, aut in alium numerum, in quem radicem primam diuidere voluisti diuide. Nam eius radix vni eueniet. Iuxta ergo hunc modum si tres radices vel quatuor, seu pauciores, seu medietatem radicis, aut minus, aut quotquot fuerint, numeri nouenarii diuidere volueris, cum omnibus iis agendum erit. Operare ergo secundum modum quem proposuimus, et rem ita se habere inuenies, si deus voluerit.

Sequuntur nunc similes multiplicationes, per res seu radices et numeros expositae

Sed si radicem numeri nouenarii cum radice numeri quaternarii multiplicare volueris, multiplicemus 9 cum 4 et producentur 36. Sume ergo horum radicem, id est 6. Et hoc est quod producitur ex radice numeri nouenarii cum radice numeri quaternarii multiplicata. Eodem modo si multiplicare volueris radicem numeri quinarii cum radice numeri denarii. Multiplica ergo 5 cum 10, et producentur 50, quorum radix substantiam, hoc est ipsum quod voluisti, significat.

---

1. mediabis *pro* medio bis D.

2. modi *om.* D. multiplicatione quotquot radices seu (*om.* D) integre seu diminute fuerint.

4. *Titulum om. et sic infra.*

5. radices *pro* radicem ^{1} D. numeri *om.* V. numeri quaternarii D. diuide per *vel* diuide super *pro* diuide in *saepe*.

6. Deinde *pro* Diuide D. fientque duo et 4^{a}. quarum D. quae + scilicet radix.

8. econuerso. super *pro* in.

9. numeri *om.* D. fientque. nouene D. tunc earum radicem assume id est duas unius accipe 3^{as} *pro* vnius . . . accipe.

10. particulae scilicet *om.*

13. super.

14. tamen quod substantiam super.

15. quotiens. duplica V; multiplica duplica D. vel collige *om.* scito.

16. ex duplicatione concretus radix habeatur. Per hunc.

17. primo V; primum D.

18. Nam + et. ergo *om.* D. aut plures *pro* vel quatuor.

19. aut quotquot fuerint, numeri nouenarii *om.* V; aut quod volueris, numeri nouenarii D; quotquot + volueris C *sed del.*

20. cum omnibus iis *om.* est. Operacio D. secundum quod diximus et inuenies si deus voluerit.

23. Sed + et. radicem *om.* D. numeri ^{2} *om.*

24. Multiplica. fientque 36.

25. quod excreuit.

26. multiplica D. numeri *om.* D.

27. numeri *om.* senarii D. ergo *om.*

28. quorum videlicet radix substantiam quam voluisti signat.

i.e. one-half of 3. For $1\frac{1}{2}$ taken twice gives 3.[1] You proceed then in the same manner with such multiplications with all roots, whether they are integral or fractional.

Method of dividing

If, moreover, you wish to divide the root of 9 by the root of 4, you divide 9 by 4, giving $2\frac{1}{4}$, and the root of this, which is finally $1\frac{1}{2}$, completes the division.[2]

If you wish to perform the reciprocal division, *i.e.* divide the root of 4 by the root of 9, divide 4 by 9, giving $\frac{4}{9}$ of a unit, and take the root of this, *i.e.* $\frac{2}{3}$. This is, of course, the result of the division.[3]

Another method of division

You may desire to divide the root of 9 by the root of 4, without dividing the square by the square.[4] Double or gather up the root of 9 as many times as desired and of the resulting number you find the root. This number divide by 4 or by any other number by which you wished to divide the first root; for in this way the root of it will be found. You proceed then in like manner if you wish to divide the root of 9 by 3 or 4 or less, or by $\frac{1}{2}$ or less, or by anything else; with all of these the rule is the same. Follow then the rule which we have explained and so you will find the result, if God will.

Similar multiplications explained by things, or roots, and numbers

Now if you wish to multiply the root of 9 by the root of 4, multiply 9 by 4, giving 36. Take the root of this, *i.e.* 6. This is the product of the root of 9 multiplied by the root of 4. Likewise if you wish to multiply the root of 5 by the root of 10, you multiply 5 by 10 giving 50.[5] The root of this is the desired product.

---

[1] $\frac{1}{2}\sqrt{9} = \sqrt{\frac{9}{4}} = \frac{3}{2}$.

[2] $\frac{\sqrt{9}}{\sqrt{4}} = \sqrt{\frac{9}{4}}$.

[3] $\frac{\sqrt{4}}{\sqrt{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$.

[4] There seems to be something incorrect about Robert's translation. Probably this should be as in the Arabic, according to Rosen, *op. cit.*, p. 30: "If you wish to divide twice the root of 9 by the root of 4, or of any other square, you double the root of nine in the manner above shown to you in the chapter on Multiplication, and you divide the product by four, or by any number whatever. You perform this in the way pointed out.

In like manner, if you wish to divide three roots of nine, or more, or one-half or any multiple or sub-multiple of the root of nine, the rule is always the same: follow it, the result will be right."

Rosen here follows the custom of modern translators of Arabic in leaving out the reference to the Deity which is actually given in the Arabic text.

[5] $\sqrt{5} \cdot \sqrt{10} = \sqrt{50}$, both quantities being surds.

LIBRARY

Quòd si radicem tertiae cum radice medietatis multiplicare volueris, tertiam cum medietate multiplica, et producitur vna sexta. Radix igitur sextae ipsum est quod ex radice tertiae cum radice medietatis multiplicata, nobis excreuit.

Si autem duas radices numeri nouenarii, cum tribus radicibus numeri quaternarii multiplicare volueris, accipe duas radices numeri nouenarii secundum quod iam diximus, vt scias cuius substantiae radicem compleant. Similiter de tribus radicibus numeri quaternarii facias, quatenus cuius substantiae sint radix reperias. Has igitur substantias inter se multiplica, vnam videlicet earum cum altera, atque huius producti radicem accipe, quoniam hoc est quod ex duabus radicibus numeri nouenarii cum tribus radicibus quaternarii multiplicatis excreuit.

Quotcunque igitur radices simul colligere, vel quas a quibusdam minuere volueris, cum errore abiecto iuxta hoc exemplar multiplicare poteris.

INCIPIUNT CAPITUM QUAESTIONES

Dixit Mahomet Algoarizim, hactenus praemisimus numerorum capita, quae sub sex quaestionibus pro numero capitum in libri principio a nobis proposita sunt atque ibi etiam diximus, numerum restaurationis et oppositionis in his sex capitibus omnino versari. Sed quoniam ea quasi sub inuolucris te edicta sunt, igitur haec ipsa, quò omnium studium exerceatur, et scientia facilius elucescat, adducemus ac fusius explicamus.

Caput primum, quaestio prima

Modus huius quaestionis est, vt dicas: denarium numerum in duo diuide vt eius vna pars cum altera multiplicata, numerum ex multiplicatione concreet seu producat, qui quater acceptus aequalis fuit numero, ex multiplicatione vnius partis semel cum se ipsa generato.

Similitudo talis est, vt vnam partem numeri denarii rem constituas, et alteram 10 sine re. Multiplica igitur rem cum 10 sine re, fient 10 res absque substantia. Item multiplica 10 res absque substantia cum quatuor, quoniam quater dixisti,

---

2. fietque $\frac{1}{6}$ (6^{a} D). igitur *om.* D. ipsa.

4. cum tribus radicibus numeri quaternarii *om.* D.

5. extrahe.

7. fuit D.

8. inuicem *pro* inter se. et unam (id est una D) earum in alteram ducas, et huius summe radicem accipias.

9. erit.

10. radicibus *om.* numeri 4^{rii}.

11. Quotquot. in simul colligere volueris. inuicem *sed del. et* minuere *superscr.* D *man.* 2.

12. omni *pro* cum. poteris + laus deo etc. V; poteris *om.* D.

13. *Titulum om.*

14–16. Mahumed Algoarizim, primum quidem capitula numeri proposuimus (premisimus D) que sub sex questionibus ad similitudinem 6 capitulorum in libri principio propositorum constituimus. Vbi eciam diximus quam numerus.

17. procul dubio versatur.

17–19. ea que in questionibus quasi sub quodam tegumento sunt dicta, hic illud in quo sciencia et animi studium faciliori exerceantur aditu elucescet introducimus. C *add.:* Quid si quaestio capitis primi? te *pro* tibi C.

20. *Titulum om. et sic ubique.*

21. *In marg.* C Textus, *et sic ubique in has quaest.* Modus huius capituli. decenum *pro* denarium *fere ubique.* ita in duo *et sic saepe.* diuido.

22. diuisio *pro* pars *vel* pars diuisionis *saepius.*

22–24. deducta, numerus ex multiplicatione increet et quater acceptus similis sit numero (non D) ex multiplicatione unius diuisionis semel in seipsam deducte generato.

25. *In marg.* C Minor, *et sic ubique.* decem *pro* denarii. alteram + eiusdem (eius D) diuisionem.

26. re^{1} + proponas V; + propones D. re^{2} *om.* V. fientque 10 radices.

27. 10 sine re V; rem D *pro* 10 res absque substantia.

If you wish to multiply the root of $\frac{1}{3}$ by the root of $\frac{1}{2}$, you multiply $\frac{1}{3}$ by $\frac{1}{2}$, giving $\frac{1}{6}$. The root of this one-sixth is that which we obtain by the multiplication of the root of $\frac{1}{3}$ by the root of $\frac{1}{2}$.

Again if you wish to multiply two roots of 9 by three roots of 4,[1] take two roots of 9 according to the method which we have explained so that you may know the square of which this is the root. Treat similarly the three roots of 4 in order to find the square of which this is the root. Then multiply these squares by each other, *i.e.* one of them by the other, and take the root of this product since this is the result of the multiplication of two roots of 9 by three roots of 4.

Therefore using this process you are able, casting aside error, to multiply as many roots as you wish to join together or as many as you wish to subtract from other quantities.[2]

Problems Illustrating the Chapters[3]

Says Mohammed Al-Khowarizmi: Up to this point we have set forth the chapters on numbers which we proposed in the beginning of this work, under six problems, one for each chapter, and we also mentioned in that place that every problem of restoration and opposition necessarily falls within the scope of one of these six chapters. But since the explanations were somewhat involved we present and more fully explain these further problems, by which each type is illustrated and the science is more easily elucidated.[4]

First problem, illustrating the first chapter

Divide ten into two parts in such a way that one part multiplied by the other and the product, or result, taken four times, will be equal to the product of one part by itself.[5]

The method is to let x represent one part of ten, and the other $10 - x$. Therefore multiply x by $10 - x$, giving $10x - x^2$. Also multiply $10x - x^2$ by 4, as it was to be taken four times, giving $40x - 4x^2$ as four times the

---

[1] $2\sqrt{9}$ by $3\sqrt{4} = \sqrt{36}$ by $\sqrt{36} = \sqrt{1296} = 36$.

[2] Rosen, *op. cit.*, p. 31, translates this paragraph: "You proceed in this manner with all positive or negative roots"; he follows with the geometrical explanation of the two problems given in footnote 4, p. 33, and a further elucidation of the third problem of that set.

[3] Scheybl adds to the statement of each problem the marginal word, *Textus*, and to the explanation, *Minor;* also the numbering of the problems appears to be his addition.

[4] Rosen, *op. cit.*, p. 35: "Of the six problems.

Before the chapters on computation and the several species thereof, I shall now introduce six problems, as instances of the six cases treated of in the beginning of this work. I have shown that three among these cases, in order to be solved, do not require that the roots be halved, and I have also mentioned that the calculating by completion and reduction must always necessarily lead you to one of these cases. I now subjoin these problems, which will serve to bring the subject nearer to the understanding, to render its comprehension easier, and to make the arguments more perspicuous."

[5] $4x(10 - x) = x^2$; $5x^2 = 40x$; $x = 8$.

fientque quatuor aequales multiplicationes partis vnius cum altera, 40 res absque 4 substantiis. Postea rem cum re, id est alteram partem cum seipsa, multiplica, et producetur substantia, rebus 40 absque 4 substantiis aequalis. Restaura igitur numerum, hoc est substantiae substantias quatuor adiicias, venientque quinque substantiae 40 res coaequantes. Vnius ergo substantiae radicem octonarius numerus assignat. Ipsa deinde substantia in 64 terminatur, cuius scilicet radix vnam partem numeri denarii cum seipsa multiplicatam, demonstrat, et residuum numeri 10 in binario terminatur numero. Duo itaque alteram partem numeri 10 obtinent. Iam igitur haec quaestio ad vnum sex illorum capitum, illud nimirum in quo diximus, substantiae radices coaequant, te perduxit.

Caput secundum, quaestio secunda

Denarium numerum sic in duo diuido, vt si denarius numerus semel cum seipso multiplicetur, numerus qui ex multiplicatione producetur aequalis sit duplo numeri eius, qui ex multiplicatione vnius partis cum seipsa producitur, septem nouenis partibus eiusdem producti numeri superadditis.

Huius rei expositio est, vt numerum 10 cum seipso multiplices, fientque 100, duas substantias et septem vnius substantiae nonas coaequantia. Haec ad vnam conuertas substantiam, id est ad nouem vigesimas quintas partes quae quintam et $\frac{4}{5}$ quintae continent, atque centenum numerum ad eius quintam et quatuor quintae quintas, id est ad 36, vnam substantiam coaequantes conuerte; et erit radix substantiae 6, vnam diuisionis partem numeri decem exprimens, vnde altera deinde in quaternario numero procul dubio terminatur. Igitur haec quaestio ad secundum sex capitum te perduxit, in quo diximus, substantiae numeros coaequant.

Caput tertium, quaestio tertia

Denarium numerum ita in duo diuido, vt, vna eius parte in alteram diuisa, inde exiens particula in quarternario numero finietur.

Expositio talis est, vt vnam partem, rem constituas; atque alteram, 10 sine re proponas, deinde 10 sine re in rem diuidas, et exeunt 4. Iam autem manifestum est, si id quod ex aliquo diuiso exierit, cum eo in quod ipsum diuiditur, multipli-

---

1. similitudines multiplicationis. in alteram + et erunt.

2. re + multiplica. deducas.

3. fiet. 4 + 4^{or} D. coequales V.

4. igitur + vel comple C. et super substantiam, substantias (res D) adicias, fientque 40 res 5 substantias.

6. et ipsa. deinde *om.* 8 *pro* scilicet V.

8. ternario D. Nam et (*om.* D) duo.

9. obtinebit C. ad unum capitulorum te perduxit in quo diximus, Substantia radices coequat.

13–15. ducatur, numerus qui ex multiplicatione excreuerit (decreuerit D) similis sit numero qui ex multiplicatione unius diuisionis in semetipsam bis deducte (+ tollitur D) vii nouenis superadditis.

16. semetipso V. -que *om.*

17. nouenas. coequantur D. Haec + igitur.

18. 9 partes ex (et D) 25 que (+ numerum D).

19. 4^{or} 5^{te} quintas. continent, et centenum numerum ad eius quintam V; *om.* D.

20–22. Radix igitur huius (*om.* D) substantie id est 6 unum numeri 10 diuisionem ostendunt. Altera uero eius diuisio in 4^{rio}.

22–24. Igitur hec questio quo ad unum vi capitulorum usque perduxit in quo diximus, substantia numeri coequat D; *om.* V.

26. et eiuus (?) una D. 4 eueniant V; Vnaquoque particula in ternario numero finiatur D *pro* inde . . . finietur.

29. ergo *pro* deinde. super rem diuide ut fiant 4. Et iam. autem *om.*

30. quam *pro* si. exierit + si V. multiplicatur V.

product of one part by the other. Then multiply x by x, *i.e.* one part by itself, giving x^2, which equals $40x - 4x^2$. Therefore restore[1] or complete the number, *i.e.* add four squares to one square, and you obtain five squares equal to $40x$. Hence 8 is the root of the square which itself is 64. The root of this is that part of 10 which is to be multiplied by itself, and the difference between this number and ten is 2. So that 2 is the other part of 10. Now this problem has led you to one of the six chapters[2] and, indeed, to that one in which we treat the type, squares equal to roots.

Second problem, illustrating the second chapter

I divide 10 into two parts in such a way that the product of 10 by itself is equal to twice the product of one part by itself, adding seven-ninths of the same product.[3]

Explanation. You multiply 10 by itself, giving 100, which is equal to $2x^2$ and $\frac{7}{9}x^2$. You reduce this to one square, *i.e.* to $\frac{9}{25}$, equal to $\frac{1}{5}$ and $\frac{4}{5}$ of $\frac{1}{5}$ of itself. And so reduce $\frac{1}{5}$ and $\frac{4}{5}$ of $\frac{1}{5}$ of 100, *i.e.* 36, to one square. The root of the square is 6, representing one part of the division of 10, whence then the other part is necessarily 4. Therefore this problem has led you to the second of the six chapters in which we treated the type, squares equal to numbers.

Third problem, illustrating the third chapter

I divide 10 into two parts in such a way that when one part is divided by the other, the resulting fraction equals 4.[4]

Explanation. You let x represent one part, and consequently the other you propose as $10 - x$. Then you divide $10 - x$ by x, giving 4. Now it is evident that if you multiply the quotient by the divisor you

---

[1] This is strictly the operation corresponding to the term *algebra*, and the verb used in the Arabic text is in fact from the same stem *jbr* as the word 'algebra'; the quantity $40x$ above is regarded as incomplete by the amount $4x^2$.

[2] The reference is to the six types of quadratic equations which are discussed extensively in the first part of the work, namely,

$$\begin{array}{ll} ax^2 = bx, & ax^2 + bx = n, \\ ax^2 = n, & ax^2 + n = bx, \\ ax = n, & ax^2 = bx + n. \end{array}$$

The first six of the problems in this set are chosen to illustrate each of these six types, in order.

[3] $2x^2 + \frac{7}{9}x^2 = 100$; $\frac{25}{9}x^2 = 100$; $x^2 = 36$ and $x = 6$.

The Arabic text of this problem is somewhat lengthier, and includes the statement of a second problem which does not appear either in our text or in the Libri version. Following Rosen's translation, *op. cit.*, p. 36: "I have divided ten into two portions: I have multiplied each of the parts by itself, and afterwards ten by itself: the product of ten by itself is equal to one of the two parts multiplied by itself, and afterwards by two and seven-ninths; or equal to the other multiplied by itself and afterwards by six and one-fourth." But a solution is given only for the first part of the problem.

[4] $\frac{10 - x}{x} = 4$; $10 - x = 4x$; $10 = 5x$, and $x = 2$.

caris,' quòd tum substantia quae diuisa est, adimpleatur. Et in hac quaestione 4 diuisum obtinet et id per quod diuisum diuiditur, rem proposuimus; numerus igitur 4 cum re multiplicandus erit, et producentur 4 res, substantiam quam diuisimus, hoc est 10 sine re exequantes. Restaura igitur 10 sine re, et ipsam rem rebus 4 adde, et venient 10 quinque res coaequantia. Haec autem res in binario finitur numero. Vnde haec quaestio ad tertium sex capitum te perduxit, vbi etiam diximus, radices numeros coaequant.

Caput quartum, quaestio quarta

Tertiam rei et vnam drachmam cum quarta rei et vna drachma sic multiplico, vt huius multiplicationis productum in 20 terminetur.

Expositio talis est, vt tertiam rei cum quarta rei multiplices, et producetur medietas sextae vnius substantiae, et drachma cum quarta rei multiplicata, quartam rei generat addendam. Similiter tertia rei cum drachma, tertiam rei procreat atque tandem drachma cum drachma, drachmam producit. Haec porro multiplicatio ad medietatem sextae vnius substantiae, et ad tertiam ac quartam rei atque ad vnam drachmam vinginti drachmas coaequantia extenditur. Vnam igitur drachmam ex 20 subtrahas, et manebunt 19 drachmae medietatem sextae vnius substantiae simul tertiam ac quartam rei coaequantes. Iam ergo substantiam compleas, hoc est, quicquid habueris cum duodecim multiplices, et producetur substantia et 7 res, 228 ex numeris coaequantes. Radices igitur media, id est per medium diuide, et vnam medietatem cum seipsa multiplica, et producentur 12 et $\frac{1}{4}$. Haec 228 ex numeris adiicias, et veniunt 240 et $\frac{1}{4}$; hinc radicem accipe, 15 et $\frac{1}{2}$; atque ex ea 3 et dimidium subtrahe, et manebunt 12, radicem substantiae adimplentia. Iam igitur haec quaestio ad quartum sex capitum te perduxit, in quo diximus, substantia et radices numeros coaequant.

Caput quintum, quaestio quinta

Denarium numerum ita in duo diuido, vt vnaquaque diuisionis parte cum seipsa multiplicata, duorum productorum summa ad 58 perueniat.

---

1. substantiam cuius diuisiones (diuisionis D) fuerint adimplebit *pro* quod . . . adimpleatur. hac diuisione D.

2. cum quatuor, ipsum exeuntem, id deinde in quod diuiditur, rem proponamus C. Multiplica ergo rem per 4 et fient 4 res V; Multiplicatam ergo iiii^{or} et fient xi res D.

4. coequantes. cum re D *pro* sine re. et super 4 res ipsam adde, fientque.

5. ergo.

6. Ergo *pro* Vnde. usque ad unum sex. ut et V; nos (*sed del.*) ubi et D *pro* vbi etiam.

9. Terciam substantie et unum dragma in 4^{am} substantie et uno dragmate.

10. summa *pro* productum *ubique*.

11. fietque.

12–15. in dragmate ductum, dragma generat (generatur D) adiectiuum et 3^{a} rei in dragmate ducta 3^{am} rei procreat. Sed et 4^{a} rei in dragmate ducta, 4^{am} (4^{te} D) rei progenerat. Hec igitur summa.

15. et rei 4^{am} et etiam.

16. drachmas *om.*

18. sextam D. et 3^{am} et. coequantia V; coe + adequantia D. Sic igitur substantiam compleas et quidquid habueris.

20. 7 radices. numero V; *om.* D *sed spat. relict.*

21. in ipsam. fientque.

22. Hec igitur super 228 adicias et fient. Huius (Hanc D) ergo.

23. et ex eis tria et medium diminuas.

24. Hec ergo V; Iam in hoc D. ad unum 6.

25. substantie. numerum.

27. sic *pro* ita. *In marg.* $\phi + 3$ ℞ D. unaquaque diuisione in semetipsam deducta, tota multiplicationis summa ad 58 tendatur.

obtain thus the quantity[1] which was divided. And as in this problem the quotient is 4 and the divisor is given as x, you multiply 4 by x, giving $4x$ for the quantity which was divided, *i.e.* equal to $10 - x$. Therefore complete $10 - x$ by adding x to $4x$, giving 10 equal to $5x$. Whence it follows that x is 2. Thus this problem has led you to the third of the six chapters in which we treated the type, roots equal to numbers.

Fourth problem, illustrating the fourth chapter

Multiply $\frac{1}{3}x$ and one unit by $\frac{1}{4}x$ and one unit so as to give as the product 20.[2]

Explanation. You multiply $\frac{1}{3}x$ by $\frac{1}{4}x$, giving $\frac{1}{2}$ of $\frac{1}{6}x^2$, and a unit multiplied by $\frac{1}{4}x$ gives $\frac{1}{4}x$ to be added. Similarly $\frac{1}{3}x$ multiplied by a unit gives $\frac{1}{3}x$ and then a unit by a unit gives a unit. Then this multiplication amounts to $\frac{1}{2}$ of $\frac{1}{6}x^2$, and $\frac{1}{3}x$ and $\frac{1}{4}x$ and one a unit, equal to 20 units. You subtract one unit from 20 units, giving 19 units equal to $\frac{1}{2}$ of $\frac{1}{6}x^2$ together with $\frac{1}{3}x$ and $\frac{1}{4}x$. Now then you complete the square,[3] *i.e.* you multiply throughout by 12. This gives x^2 and $7x$ equal to 228. Then halve the roots, *i.e.* divide them equally, and multiply one-half by itself, giving $12\frac{1}{4}$. You add this to 228, and you will have $240\frac{1}{4}$. From the root of this, $15\frac{1}{2}$, subtract $3\frac{1}{2}$, leaving 12 as the root of the square. Now then this problem has led you to the fourth of the six chapters in which we treated the type, a square and roots equal to numbers.

Fifth problem, illustrating the fifth chapter

I divide ten into two parts in such a way that the sum of the products obtained by multiplying each part by itself is equal to 58.[4]

---

[1] Robert of Chester employs *substantia* here in the non-technical sense of "substance" or "quantity." So also *census* in the Libri version and *mal* in the Arabic version are used with the same significance. This was a common usage of Arabic writers. Abu Kamil followed this practice, on occasion, and Leonard of Pisa, who drew extensively from Abu Kamil, copied this peculiarity from the Arabs: see *Scritti di Leonardo Pisano*, Vol. I, p. 422, and my article, *The Algebra of Abu Kamil Shoja' ben Aslam*, *Bibliotheca Mathematica*, third series, Vol. XII (1912–1913), p. 53. In particular, also, *census* appears in this sense in the *Liber augmenti et diminutionis vocatus numeratio divinationis, ex eo quod sapientes Indi posuerunt, quem Abraham compilavit et secundum librum qui Indorum dictus est composuit*, published by Libri, *Histoire des sciences mathématiques en Italie*, Vol. I, Paris, 1838, pp. 304–371. This work is probably by Abraham ibn Esra.

[2] $(\frac{1}{3}x + 1)(\frac{1}{4}x + 1) = 20$; $\frac{1}{12}x^2 + \frac{1}{3}x + \frac{1}{4}x + 1 = 20$; $x^2 + 7x + 12 = 240$; $x^2 + 7x = 228$; $\frac{1}{2}$ of 7 is $3\frac{1}{2}$, $(3\frac{1}{2})^2 = 12\frac{1}{4}$, $228 + 12\frac{1}{4} = 240\frac{1}{4}$, $\sqrt{240\frac{1}{4}} = 15\frac{1}{2}$, $15\frac{1}{2} - 3\frac{1}{2} = 12$, which is the value of x.

[3] The present usage of the expression "to complete the square" is quite different from that of our text. Here it means, of course, to make the coefficient of x^2 unity and this also corresponds to the operation termed by the Arabs, *algebra*, as opposed to the operation of *almuqabala;* see the article *al-Djabr wā-'l-Mukābala* by Professor H. Suter in *The Encyclopedia of Islam*, Vol. I (Leyden, 1913), pp. 989–990.

[4] $x^2 + (10 - x)^2 = 58$; $2x^2 - 20x + 100 = 58$; $2x^2 + 42 = 20x$; $x^2 + 21 = 10x$, which is a problem that appears earlier in the text.

Expositio talis est, vt 10 sine re cum seipso multiplices, fientque 100 et substantia absque 20 rebus. Postea multiplica rem cum suo aequali et producetur substantia. Deinde haec duo multiplicationis producta in vnum collige, et veniunt 100 et 2 substantiae absque 20 rebus, 58 coaequantes. Comple igitur 100 et 2 substantias absque 20 rebus, et ipsas 20 res 58 ex numeris adiicias, et venient 100 drachmae et 2 substantiae, 58 drachmas et 20 res coaequantia. Hoc iam ad vnam conuertas substantiam, atque oppositione deinde 29 ex 50 proiicias, et manebunt 21 et substantia, 10 res coaequantia. Res igitur mediabis, et veniunt 5; haec sum seipsis multiplicata, producunt 25. Ex his 21 abiicias, et relinquentur 4. Accipe horum radicem 2, atque hanc a 5, id est à medietate radicis, subtrahe, et manebunt 3 quae videlicet vnam partem numeri decem adimplent. Igitur haec quaestio ad quintum sex capitum te perduxit, in quo diximus, substantia et numeri radices coaequant.

Caput sextum, quaestio sexta

Rei tertiam et eius quartam sic multiplico, vt multiplicationis productum ipsam rem, viginti quatuor drachmis superadditis, coaequent.

Expositio est, vt primum scias, quòd quando tertiam rei cum quarta rei multiplicaueris, medietas sextae vnius substantiae, rem et 24 drachmas coaequans, oriatur. Multiplicatio igitur medietatis sextae substantiae cum duodecim, substantiam reddet completam. Similiter multiplicatio rei et 24 drachmarum cum 12, radices 12 ducenta et 88, substantiam coaequantes, adimplebit. Diuide igitur radices per medium, et mediam partem cum seipsa multiplica, atque multiplicationis productum numero 288 iunge, et venient 324. Horum nunc radicem accipe, id est 18, quibus medietatem radicum etiam adde, et radix in 24 finietur. Haec igitur quaestio ad sextum sex capitum iam te perduxit, in quo diximus, radices et numeri substantiam coaequant.

Ad huc restat, vt de sedecim aliis tractemus quaestionibus, quae ex sex praemissis oriri videntur, vt quicquid ex numero huic arti addicto opifici propositum fuerit, omni errore abiecto facilius elici queat.

1. in semetipsis.

2. radicibus.

3–10. has multiplicationis summas in unum collige et habebis 100 et duas substantias absque 20 radicibus 58 coequantes. Comple igitur 100 et duas substantias absque 20 radicibus cum re quam diximus, et adde eam super 58, et fient 100 et due substantie, 58 et 20 (10 D) res coequancia. Res ergo mediabis et erunt 5. Hec igitur in seipsis (semetipsis D) multiplica et erunt 25. Ex his ergo 21 (*om.* D) abicias et remanebunt 4. Sume ergo horum radicem (harum radices D) id (*om.* D) est (*om.* D) duo qui ab 5 prius positis, id est a medietate radicum, subtrahas.

11. videlicet *om.*

12. unum 6. te + iam. numeri *om.* D. R¹ *pro* radices.

15. Substantie *pro* Rei. in *pro* et. et *pro* vt *et sic infra* (3). *In marg.* φ + ℞ 3 D.

16. substantiam. coequant.

18. et *om.* D.

19. orietur.

20. Similiter + et. 34 V. in (+ in D) 12.

21. ducentos et 88 et radices xii D. coequans V; *om.* D.

22. ac *pro* atque.

23. summam super 288 adicias (addicias D *et sic saepius*) et erit hoc totum 324; erit hoc totum C *sed del.* igitur *pro* nunc.

24. accipias. quibus videlicet medietatem radicum adicias, id est 6, et sic substantia (substantiam D) in 24 (cxiiii D) terminetur.

25. unum sex.

27. Hee ergo sunt (+ vi D) questiones de quibus superius me tractaturum promisi. Sed adhuc *pro* Ad huc. primis *pro* praemissis.

28. quidquid V; quid D. huius artis intento.

29. abiecte D. eliciatur *pro* elici queat.

Explanation. You multiply $10 - x$ by itself, giving 100 and x^2 less $20x$. Then multiply x by itself, giving x^2. Collecting the products of these two multiplications you obtain 100 and $2x^2$ less $20x$ equal to 58. Complete the 100 plus $2x^2$ less $20x$ by adding the $20x$ to 58. This gives 100 units $+ 2x^2$ equal to 58 units and $20x$. Now you reduce this to one square, and then by opposition[1] you take 29 from 50, leaving $21 + x^2$ equal to $10x$. Therefore you halve the roots, giving 5. Multiply this by itself, giving 25. From this you subtract 21 and 4 is left. Take the root of this, 2, and subtract it from 5, *i.e.* from the half of the roots. This gives 3 which represents, of course, one part of ten. So this problem has led you to the fifth of the six chapters in which we treated the type, a square and numbers equal to roots.

Sixth problem, illustrating the sixth chapter

I multiply $\frac{1}{3}x$ and $\frac{1}{4}x$ in such a way as to give x itself plus 24 units.[2]

Explanation: first you observe that when you multiply $\frac{1}{3}x$ by $\frac{1}{4}x$ you obtain $\frac{1}{2}$ of $\frac{1}{6}x^2$ equal to $x + 24$ units. The multiplication of $\frac{1}{2}$ of $\frac{1}{6}x^2$ by 12 gives the complete square. Similarly, the multiplication of $x + 24$ units gives $12x + 288$[3] which equal x^2. Therefore take one-half of the roots and multiply the half by itself. Add the product of this multiplication to 288, giving 324. Take now the root of this, *i.e.* 18, and add it to the half of the roots. The root finally is 24. So this problem has led you to the sixth of the six chapters in which we treated the type, roots and numbers equal to a square.[4]

Sixteen Additional Problems[5]

It now remains for us to treat sixteen other problems which seem to arise out of the six which we have set forth, in order that the craftsman versed in this art may more easily and without any error solve any problem proposed.[6]

---

[1] This operation corresponds to the Arabic term *almuqabala*. In this instance the 29 on the right balances or cancels an equal amount of the 50 on the left.

[2] $\frac{1}{3}x \cdot \frac{1}{4}x = x + 24$; $x^2 = 12x + 288$; $\frac{1}{2}$ of 12 is 6, $6^2 = 36$, $288 + 36 = 324$, $\sqrt{324} = 18$, $18 + 6 = 24$, which is the value of the unknown.

[3] Evidently Robert of Chester wrote the word for two-hundred out in full, or in Roman numerals, and the 88 in Hindu-Arabic numerals, for both the Dresden and Scheybl versions use this form, separating the two-hundred from the 88.

[4] The above six problems appear in this order in all of the versions.

The terminology of the fourth and sixth problems is the same in the Arabic text, although Rosen, *op. cit.*, pp. 38, 40 translates differently the same expression: "I have multiplied one-third of thing and one dirhem by one-fourth of thing . . ."; "I have multiplied one-third of a root by one-fourth of a root . . ." The Libri text has *census* in both problems, and the Boncompagni text has *multitudo;* the Arabic word is *mal* with the meaning (footnote 1, p. 107) "unknown quantity."

[5] I have added this title to correspond to the Arabic (Rosen, *op. cit.*, p. 41).

[6] This paragraph is an addition by Robert of Chester.

Quaestio prima

Denarium numerum sic in duo diuido, vt vna parte cum altera multiplicata, productum multiplicationis in 21 terminetur.

Iam ergo vnam partem, rem proponimus quam cum 10 sine re, quae alteram partem habent, multiplicamus, et producuntur 10 res absque substantia, drachmas 21 coaequantes. Comple igitur 10 res cum substantia, et substantiam numero 21 adde, et venient 10 res, substantiam et 21 drachmas coaequantes. Accipe medietatem rerum, hoc est 5, et eam cum seipsa multiplica, et producentur 25. Ex his 21 subtrahe, et manebunt 4. Horum radicem, 2 scilicet, accipe, atque eam tandem ex medietate radicum subtrahe, et manebunt 3, quae vnam partem diuisionis demonstrant.

Quaestio secunda

Numerum denarium sic in duo diuido, vt vtraque parte cum seipsa multiplicata, si productum partis minoris ex producto partis maioris auferatur, quadraginta maneant.

Exempli expositio talis est, vt 10 sine re cum suo aequali multiplices, et producentur 100 ex numero, vna substantia absque 20 rebus. Multiplices etiam rem cum se, et producetur substantia, quam ex 100 et substantia absque rebus diminuas, et manebunt 100 absque 20 rebus, quadraginta drachmas coaequantia. Comple igitur 100 drachmas cum 20 rebus, et eas drachmis 40 adiicias et habebis 100, quadraginta drachmas et 20 res coaequantes. Igitur 40 ex 100 auferas, et manebunt 60 drachmae 20 res aequantes. Res igitur ternario aequantur numero, qui vnam partem diuisionis demonstrat.

Quaestio tertia

Denarium numerum sic in duo diuido, vt vtraque parte cum seipsa multiplicata, et multiplicationum productis simul collectis, ac quantitate deinde, quae est inter duas partes, illis addita, tota summa ad 54 drachmas excrescat.

Huius exempli expositio talis est, vt 10 sine re cum suo aequali multiplices, et

---

2. vt *om.* D. deducta, summa. *In marg.* ʒ + φ D.

5. fiuntque. drachmas *om.*

6. sine re *pro* res. super *pro* numero.

7. Dic ergo 10 res. coequant. Sume medietatem radicum.

8. in semetipsam (semetipsa D).

9. A quibus 21 demptis V; *om.* D. et *om.* V. Accipe eorum (earum D) radicem, id est 2, et eam ex.

10. et *om.* D.

13. ut (et D) unaquaque diuisione in semetipsa (semetipsam D) deducta, si multiplicatio minoris diuisionis ex multiplicatione maioris tollatur 40 (*om.* D) remaneant.

16. Expositio huius talis. consimili.

17. 100 et substantia absque (+ absque vel minus *in marg.* D *man.* 1) 20 radicibus. Multiplica igitur rem in re, et fiet.

18. que V. absque *om.* V; et D. 20 rebus.

20. 100 cum 20, et eas super 40 adicias.

21. drachmas *om.* Hoc ergo centeno opponas (apponas D) numero et 40 ex. inferas D.

22. 60 (4, + cum D) 20 res coequancia.

23. unam mensurat diuisionem.

25. unaquaque. semetipsa. multiplica D. *In marg.* 3 + φ ℞ D.

26. et utriusque multiplicatione in unam collecta (+ a D) quantitate que.

27. addita + nam et unamquamque earum in semetipsa multiplicasti D.

28. Inde igitur talis datur expositio, ut. consimili.

First Problem[1]

I divide ten into two parts in such a way that the product of one part multiplied by the other gives 21.[2]

Now then we let x represent one part, which we multiply by $10 - x$, representing the other part. The product $10x - x^2$ is equal to 21 units. Complete $10x$ by x^2 and add this x^2 to 21. This gives $10x$ equal to $x^2 + 21$ units. Take one-half of the unknowns, *i.e.* 5, and multiply this by itself, giving 25. From this subtract 21, giving 4. Take the root of this, 2, and subtract it from half of the roots, leaving 3, which represents one of the parts.

Second Problem

I divide ten into two parts in such a way that each part being multiplied by itself, the product of the smaller part taken from the product of the larger part leaves 40.[3]

Explanation. You multiply $10 - x$ by itself, giving $100 + x^2 - 20x$. You multiply x by itself, giving x^2, which you take from $100 + x^2 - 20x$, leaving $100 - 20x$ equal to 40 units. Therefore by adding $20x$ to the 40 units complete 100 units by $20x$. This gives 100 equal to 40 units $+ 20x$. Therefore you take 40 from 100, leaving 60 units equal to $20x$. Three is then the value of x and represents one part.

Third Problem

I divide ten into two parts in such a way that when to the sum of the products of each part by itself is added the difference between the two parts the sum total will be 54 units.[4]

---

[1] The sixteen problems which follow are selected by Robert of Chester from twice that number in the Arabic text. The Boncompagni version presents nine, including the first, second, third, and fifth of this list.

The Boncompagni version interjects (*loc. cit.*, pp. 45–46) before these problems: "pro multitudine data assignatur horum muttatione quaslibet questiones secundum restauracionem propositas in predictos modos solubiles esse palam est. Cuius utilitas ad documentum libri elementorum precipua est, in inveniendis scilicet lineis alogis et medialibus binomiis et residuis sive reccisis que per notum numerum assignari non possunt. In practica quoque geometrie et universis questionibus ignotorum secundum arismeticam formatis certissima via est."

[2] $x(10 - x) = 21$, which leads to the type equation given earlier in the work, $x^2 + 21 = 10x$, and indeed, it incidentally appears a second time in the preceding set of problems. The Arabic text precedes the statement of this problem, Rosen, *op. cit.*, p. 41, as follows: "If a person puts such a question to you as . . ."; the Boncompagni version, *loc. cit.*, p. 46, precedes: "Igitur sub formas precedencium et alias questiones proponimus. Queritur. . . ."

[3] $(10 - x)^2 - x^2 = 40$, whence $100 - 20x = 40$; $20x = 60$, $x = 3$, which is the value of the root.

[4] $x^2 + (10 - x)^2 + 10 - 2x = 54$, whence $2x^2 - 22x + 110 = 54$; $2x^2 + 110 = 54 + 22x$; $x^2 + 55 = 27 + 11x$; $x^2 + 28 = 11x$; $\frac{1}{2}$ of 11 is $5\frac{1}{2}$, $(5\frac{1}{2})^2 = 30\frac{1}{4}$, $30\frac{1}{4} - 28 = 2\frac{1}{4}$, $\sqrt{2\frac{1}{4}} = 1\frac{1}{2}$, $5\frac{1}{2} - 1\frac{1}{2} = 4$, which is the value of the unknown quantity.

producentur 100 ex numero et vna substantia, absque 20 rebus. Rem etiam cum re multiplices, et producetur substantia, quam 100 ex numero et substantiae, absque 20 rebus iunge, et venient 100 ex numero et 2 substantiae, absque 20 rebus. Iam quantitatem quae inter vtramque partem est, 10 sine 2 rebus, toti summae adde, atque hoc totum ad 110 ex numero et 2 substantias, duabus et 20 rebus abiectis, 54 drachmas coaequantes, exuberando peruenit. Compleas ergo, et dic: 110 drachmae et 2 substantiae 54 drachmas et 22 res coaequant. Hoc autem ad vnam conuertas substantiam, et dic: 55 drachmae et vna substantia coaequant 27 drachmas et 11 res. Igitur 27 ex 55 subtrahe, et manebunt 28 drachmae et vna substantia, 11 res coaequantia. Res igitur per medium diuide, et venient 5 et medium; haec cum suo aequali multiplica, et producentur 30 et quarta, ex his 28 subtrahe, et radicem ex residuo, vnum scilicet et $\frac{1}{2}$ accipe, quae simul ex medietate rei auferas, et manebunt 4. Haec igitur vnam partem diuisionis adimplent.

Quaestio quarta

Denarium numerum sic in duo diuido, vt vna eius parte cum seipsa multiplicata, numerus inde productus alteram octagesies et semel comprehendat.

Expositio est, vt 10 sine re cum suo aequali multiplices, et producentur 100 ex numeris et substantia, absque 20 rebus, 81 res coaequantia. Comple igitur 100 ex numeris et substantiam, et adde 20 res numero 81, et venient 100 ex numeris et substantia, 100 res et vnam coaequantia. Mediabis ergo res, et venient 50 et dimidium. Haec cum suo aequali multiplica, et producentur 2550 et quarta. Ex his nunc 100 subtrahe, et manebunt 2450 et quarta. Horum radicem accipe, id est 49 et dimidium, atque hanc ex radicum medietate subtrahe, et manebit vnitas, vnam partem numeri decem ostendens.

Quaestio quinta

Duas substantias duabus drachmis differentes, diuido, maiorem scilicet in

---

1. erit. vna *om.*

1–4. Rem . . . rebus^{1} *om.* V; Rem quoque in re multiplicata, et erit substantia. Hec insimul iunge et erunt c et due substantie absque xx rebus D.

4. Et iam. vtramque + substantiam D. super totam summam addidisti. Dicas ergo quantitas que inter utramque est diuisionem 10 absque duabus rebus designat. Hoc igitur totum ad 100 et 10 et (*om.* D) duas *pro* 10 sine . . . et 2.

5. absque duabus rebus in (et D) *pro* duabus et.

6. Tunc ergo compleas.

8. conuerte. vna *om. et sic infra.*

10. erunt.

11. haec + igitur. consimili. erunt.

12. his + ergo. subtrahas, et radicem (radices D) remanentis (remanentes D) id est duorum et 4^{e} accipias (+ Accipe radicem D) id est unum et medium que simul.

13. radicis V; radices D *pro* rei. igitur + 4.

16. et *pro* vt D. in se ipsam semel deducta (deducto D), 81 vicibus alteri (iubet altera D) comprehendetur.

18. Expositio + huius rei. duo *pro* suo D. consimili. ex numeris *om. saepe.*

19. minus.

20. et substantiam *om.* et^{2} *om.* D. radices super (*om.* D). 81, et erunt. 100 *om.* D.

21. et substantia *om.* D. 100 et unam radicem. radices. erunt.

22. Has ergo. suo consimili, sibi aequali = cum seipsis: se *in marg.* C; in suo consimili V D. erunt 2000,500 et 50 et 4^{a}.

23. ergo *pro* nunc. 2000 et 450 (400 D). Horum + ergo.

24. et ipsam *pro* atque hanc. medietate + id est 50 et medio. subtrahas.

25. unum, unam denarii ostendens diuisionem.

27. diuido *om.* V. id est earum minorem super maiorem.

The explanation of this problem is of the following nature: you multiply $10 - x$ by its equal, which gives $100 + x^2 - 20x$. Also you multiply x by x, giving x^2, which you add to $100 + x^2 - 20x$. This gives $100 + 2x^2 - 20x$. To the sum total now add the difference between the two parts, $10 - 2x$, and this total amounts to $110 + 2x^2 - 22x$, which equals 54 units. You complete therefore (by adding $22x$), and you obtain 110 units $+ 2x^2$ equal to 54 units $+ 22x$. This, moreover, you reduce to one square, giving 55 units $+ x^2$ equal to 27 units $+ 11x$. So subtract 27 from 55, giving 28 units $+ x^2$ equal to $11x$. Halve the unknowns, giving $5\frac{1}{2}$. Multiply this by itself, giving $30\frac{1}{4}$. From this subtract 28 and the root of the remainder, $1\frac{1}{2}$, taken from one-half of the roots, will leave 4. Therefore this represents one part.

Fourth Problem[1]

Divide ten into two parts in such a way that the product of one part by itself equals 81 times the other part.[2]

Explanation: you multiply $10 - x$ by itself, giving $100 + x^2 - 20x$, which equals $81x$. Complete $100 + x^2$ by adding $20x$ to $81x$. This gives $100 + x^2$ equal to $101x$. You halve the unknowns, giving $50\frac{1}{2}$. Multiply this by itself, giving $2550\frac{1}{4}$. Now subtract 100 from it, leaving $2450\frac{1}{4}$. Take the root of this, *i.e.* $49\frac{1}{2}$, and subtract it from the half of the roots. This will give unity, representing one part of ten.[3]

Fifth Problem

Two squares (*i.e.* two quantities or numbers) being given with a difference of two units, I divide the smaller by the larger in such a way that the fraction resulting from the division shall equal one-half.[4]

---

[1] The order of the problems is from this point different from the order of the corresponding set of problems as given in the Libri, Arabic, and Boncompagni versions, but the variations are due to the omission of problems, as will be noted below, rather than to actual changes in order. See chapter V of our Introduction. The Boncompagni text is the least complete, giving only ten problems, which follow, in the main, those of the Libri text.

[2] This problem $(10 - x)^2 = 81x$, appears on pages 47–48 of Rosen's book, and appears twice in the Libri text, pp. 282–283 and pp. 289–290.

In modern notation, the solution proceeds:

$100 - 20x + x^2 = 81x$; $x^2 + 100 = 101x$; $\frac{1}{2}$ of 101 is $50\frac{1}{2}$, $(50\frac{1}{2})^2 = 2550\frac{1}{4}$; $2550\frac{1}{4} - 100 = 2440\frac{1}{4}$; $\sqrt{2440\frac{1}{4}} = 49\frac{1}{2}$; $50\frac{1}{2} - 49\frac{1}{2} = 1$, which is the value of one root. The other root would be $50\frac{1}{2} + 49\frac{1}{2}$, or 100.

[3] The second root, 100, is not given, since it leads to 100 and negative 90 as the two parts into which 10 is divided. This was rejected by the Arabic writer as an impossible solution, nor, indeed, was such a solution regarded as possible for centuries after the time of the Arab.

[4] Rosen, pp. 50–51, and again pp. 62–63; Libri, p. 283 and p. 295. By error Scheybl makes this one-half of the larger square.

minorem, sic, vt vna diuisionis particula, quae est exiens, medietatem substantiae maioris compleat.

Rem ergo et duas drachmas cum medietate quam diuisionis particulam diximus, multiplico, et veniet rei medietas et drachma, rem coaequans. Propter medietatem ergo rei vtrinque medietatem rei abiicio, et manebit drachma medietatem rei coaequans. Hanc igitur duplico, et venient duae drachmae, quae ipsam rem, constituunt, vnde quatuor deinde alteram.

Quaestio sexta

Substantiam et eius radicem sic multiplico, vt multiplicationis productum tres similitudines substantiae, adimpleat.

Expositio est, quoniam quando radicem cum tertia substantiae multiplicaueris, tota producitur substantia. Tria igitur huius substantiae radicem adimplent, et 9 ipsam substantiam.

Quaestio septima

Tres radices substantiae cum quatuor eius radicibus ita multiplico, vt tota multiplicationis summa ipsi substantiae et quadraginta quatuor drachmis coaequetur.

Expositio talis est, vt cum quatuor radicibus substantiae tres eius radices multiplices, et producentur 12 substantiae, substantiam vnam et 44 drachmas coaequantes. Minue igitur substantiam ex 12 manebunt 11 substantiae, 44 drachmas coaequantes. Substantia ergo vna 4 aequat.

Quaestio octaua

Quatuor substantiae radices cum quinque eiusdem substantiae radicibus sic multiplico, vt tota multiplicationis summa ipsius substantiae duplo et triginta sex drachmis coaequetur.

Quatuor radices substantiae cum suis quinque radicibus multiplico, produco autem sic 20 substantias, duas substantias et 36 drachmas coaequantes. Ex 20 igitur substantiis duas substantias aufero, et manebunt 18 substantiae 36 drachmas coaequantes. Igitur 36 in 18 diuido, et exeunt 2, quae ipsam substantiam adimplent.

1. sic diuido, ut unam diuisionis particulam compleat medietas.

4. fiet. coequantes D. Cum medietate; C *corr. in* P C.

5. vtrinque *om.* rei^{2} *om.* V. proicio. drachma *om.* V.

6. Hoc. dico *pro* venient. quae *om.*; *in marg.* C. rem + hoc est vnam substantiam vel numerum vnum C.

7. et alteram 4.

9. in *pro* et.

10. substantiae + hoc tres substantias propositas C.

11. Expositio + huius. substantiam *ante* substantiae D.

12. totam D. oriatur. igitur *om.*

13. ipsam *om.*

15. et *pro* vt C.

16. ipsa D.

18. Huius est expositio, ut 4 eius radices in suis tribus radicibus.

20. 12 + et. substantiae + et V.

21. coequans V. Substantiam V; Substantie D. coequant.

23. eius. substantiae *om.*

24. vt + et. ipsi V; ipse D. duplo *om.*

25. xxxvi radicibus D.

26. Expositio est, vt quatuor C. Quatuor + ergo. substantiae *om.* multiplica V. et erunt 20 substantie.

28. xi *pro* 18 *et* xliiii *pro* 36 D.

29. coaequantes + ex xx ergo substantiis duas substantias aufero et remanebunt xviii substantie D. 36 super (sunt D) 18. erunt. substantiam *om.* D.

I multiply $x + 2$ units by $\frac{1}{2}$, representing the quotient.[1] This gives $\frac{1}{2}x$ and a unit equal to x. On account of the one-half x I take one-half x from both sides, leaving a unit equal to $\frac{1}{2}x$. I double this, giving two units, which equal the unknown. Whence four is the other.

Sixth Problem

I multiply a square by its root in such a way that the product equals three similar squares.[2]

Explanation. From what is given it follows that when you multiply the root by one-third of the square, the total gives the square.[3] Therefore 3 represents the root of this square, and 9 the square.

Seventh Problem

I multiply three roots of a square by four of its roots in such a way that the sum total of the multiplication will equal the same square and 44 units.[4]

Explanation. Multiplying four roots of a square by three of its roots, we have twelve squares, which are equal to one square and 44 units.[5] Take therefore the one square from the 12 squares, leaving 11 squares equal to 44. Hence one square equals 4.

Eighth Problem

I multiply four roots of a square by five roots of the same square in such a way that the sum total of the multiplication will equal double the square and 36 units.[6]

Explanation. I multiply four roots of a square by five of its roots and I have as a product 20 squares, which are equal to 2 squares and 36 units. Therefore I take the two squares from the 20 squares, leaving 18 squares equal to 36 units. I divide 36 by 18, and obtain 2, which represents the square.[7]

---

[1] $\frac{x}{x + 2} = \frac{1}{2}$, whence $x = \frac{1}{2}(x + 2)$; $x = \frac{1}{2}x + 1$; $\frac{1}{2}x = 1, x = 2$.

[2] Rosen, pp. 54–55; Libri, p. 291.

Al-Khowarizmi carefully avoids the term for the cube of the unknown, with which he was certainly familiar, for Diophantus and other Greek writers employed the term. His continuator, Abu Kamil, discussed not only cubics which, like this one, are reducible immediately (to equations of lower degree), but also equations in quadratic form. However, systematic discussion of the general cubic was first attempted by Omar Khayyam.

[3] $x^2 \cdot x = 3x^2$.

[4] Rosen, p. 55; Libri, p. 291.

[5] $3x \cdot 4x = x^2 + 44$.

[6] Rosen, p. 55; Libri, p. 284.

[7] $4x \cdot 5x = 2x^2 + 36$; $18x^2 = 36$; $x^2 = 2$.

Quaestio nona

Radicem substantiae cum eius quatuor radicibus ita multiplico, vt tota multiplicationis summa tribus substantiis et quinquaginta drachmis coaequetur.

Expositio est, vt radicem cum quatuor radicibus multiplicem, et 4 substantias ita productas tribus substantiis et 50 drachmis coaequem. Ex 4 igitur substantiis tres substantias aufero, et manebit substantia, 50 drachmas coaequans. Igitur radix substantiae 50 est radix numeri 50.

Quaestio decima

Ex substantia eius partem tertiam et tres drachmas aufero, et quod residuum fuerit cum suo consimili multiplico, et oritur ex multiplicatione ipsa substantia.

Expositio est, vt quando tertiam et tres drachmas subtraxero, duae tertiae absque tribus drachmis maneant. Sunt autem res. Vnde duas tertias rei absque 3 drachmis cum suo aequali multiplicamus, et producuntur quatuor nonae substantiae et 9 drachmae, absque quatuor radicibus, radicem coaequantia. Adde igitur absque 4 radicibus vni radici, et venient quatuor nouenae substantiae et 9 drachmae quinque radices coaequantes. Oportet ergo vt quatuor nouenas substantiae compleas, vt ipsa substantia perficiatur, et hoc quidem vt singula cum 2 et quarta multiplices, et venient 11 res et $\frac{1}{4}$, vnam substantiam et 20 drachmas cum quarta coaequantes. Operare igitur cum eis quemadmodum in medietatione radicis tibi diximus.

Quaestio vndecima

Tertiam substantiae cum eius quarta sic multiplico, vt tota multiplicationis summa ipsi substantiae coaequetur.

Expositio huius est, vt tertiam rei cum quarta multiplicem, et producetur medietas sextae vnius substantiae, rem coaequans. Radix igitur substantiae in 12 terminatur, et substantia 144 in se continet.

---

3. tribus + similitudinibus.   substantie.   coequatur V; adequatur D.

4. Huius est expositio.   et erunt 4 substantie, 3 substantias.

5. coequantes; C *add. in marg.* aequales eodem preferam.   4 + ex D.   igitur *om.*

6. tollo.   coaequans + et ipsa et radix l' in iiii^{or} radicibus l' D.   Radix ergo 50 in 4 radicibus 50 ducta (sunt D) ducenta que tribus similitudinibus substantie et 50 dragmatibus equantur constituit (manet equalis D).

7. C *add.*: Sequitur examen.

Radix $\sqrt{50}$.   Substantia 50.

$\sqrt{50}$ vt vna radix, cum $\sqrt{800}$, quatuor radicibus, multiplicata, producit 200. Atque tantum sunt etiam 50 ter, hoc est tres substantiae, et quinquaginta.

9. partem *om.*   tollo.

10. et *om.*   ex + qua videlicet.

11. Huius est (*om.* D) expositio; Huius C *sed del.*

12. remanebunt, et ipse erunt radix. Unde (+ et V).

13. multiplico (multiplicamus D) + Due ergo 3^{e} in duas 3^{as} ducte 4 substantiae nouenas componunt et cum (+ et cum D) tribus in duabus rei 3^{iis}multiplicate, duas res componunt similiter (+ duas D) diminutiuas et sine 3^{bus} in 3^{bus} 9 dragmata constituunt adiectiua.   Erunt ergo.   nouene.

14. 8 *pro* 9.

15. sine 4 radicibus super radicem, et fient.   nouenas.

16. coequans.

17. vt^{1} + et.   compleatur et (*om.* D) hoc est ut illas in duabus (duobus D) et (aut D) 4^{ta}.

18. multiplices (+ vel, si placet, singula in 4/9 diuidas C). Multiplica ergo 9 in duo et 4^{a} et erunt xx (11 res V) et 4^{a} (+ et multiplica v radices in duo et 4^{ta} et erunt 11 res et 4^{ta} D).   Habebis ergo substantiam et 20 dragmata et quartam, 11 res et 4^{am} rei coequancia. Oppone ergo.

20. radicum.

24. tertiam + et D.   deducam V; deductam D.

25. sextae + hoc est vna duodecima C.   rei *add. super* substantiae *et* numerum *super* rem C.

Ninth Problem

Multiply the root of a square by four of its roots in such a way that the sum total of the multiplication shall equal three squares and 50 units.[1]

Explanation. I multiply the root by 4 roots and I obtain four squares which are equal to three squares and 50 units. I take the 3 squares from the 4 squares, leaving a square equal to 50 units.[2] Therefore the root of this square is the root of 50.[3] The root of 50 multiplied by 4 roots of 50 gives 200, which is equal to three of the squares and 50 units.

Tenth Problem

I take from a square one-third of it and three units, and multiply the remainder by itself; the product is the square.[4]

Explanation. When I have subtracted one-third and three units, two-thirds less three units remain. Now let the square be represented by x; then we multiply $\frac{2}{3}x - 3$ units by itself and have $\frac{4}{9}x^2 + 9$ units $- 4$ roots equal to one root. Add the one root to the four roots, giving $\frac{4}{9}x^2 + 9$ units equal to 5 roots. It is now necessary to complete the four-ninths of a square, so as to make it a whole square. You multiply each side by $2\frac{1}{4}$,[5] giving $11\frac{1}{4}x$ equal to $x^2 + 20\frac{1}{4}$ units. You perform the operations with these, then, in the manner which we have explained to you in the sections on the halving of the root.

Eleventh Problem

I multiply one-third of a square by one-fourth of it in such a way that the sum total of the multiplication will give the square.[6]

Explanation. I multiply $\frac{1}{3}x$ by $\frac{1}{4}(x)$ and I obtain $\frac{1}{2}$ of $\frac{1}{6}x^2$ equal to x. The root of the square, then, is 12, and the square is 144.[7]

---

[1] Rosen, p. 56; Libri, pp. 291–292.

[2] $x \cdot 4x = 3x^2 + 50$.

[3] Scheybl adds: "The root of fifty multiplied by four roots of 50 gives 200, which is equal to three of the squares and 50 units." Robert of Chester's text is more closely followed by the Vienna and Dresden manuscripts.

The symbol for square root √ used by Scheybl was introduced by Adam Riese in his *Coss* written in 1524, but not printed until recently (Berlet, Leipzig, 1892); for the word *Coss* see page 38.

[4] Rosen, pp. 56–57; Libri, pp. 284–285. In modern notation: $(x - \frac{1}{3}x - 3)^2 = x$. Between this and the preceding problem the Arabic text includes a problem, leading to the equation:

$$x^2 + 20 = 12x.$$

[5] Scheybl adds that you may, if you prefer, divide both sides by $\frac{4}{9}$.

[6] Rosen, p. 58; Libri, p. 292. $\frac{1}{3}x \cdot \frac{1}{4}x = x$.

[7] In the Arabic text this problem follows: $(\frac{1}{3}x + 1)(\frac{1}{4}x + 2) = x + 13$.

Quaestio duodecima

Drachmam et medium ita in duo diuido, vt maior pars duplex minori habeatur.

Expositio huius est, vt maiorem partem ad minorem vnum et rem constituam; vnde etiam dicam drachmam et dimidium super drachmam et rem diuisa, et duae res drachmam constituunt. Duas igitur res cum drachma et re multiplico, producuntur autem duae substantiae et duae res, drachmam et dimidium coaequantes. Eas igitur ad vnam conuerto substantiam, hoc est, vt ex omni re suam auferam medietatem. Dico ergo, substantia et res tres quartas drachmae coaequant. Operare nunc cum his quemadmodum tibi iam diximus.

Quaestio decima tertia

Substantiam cum eius duabus tertiis multiplico, et fiunt quinque.

Expositio talis est, vt rem cum duabus tertiis rei multiplicem, et producuntur duae tertiae substantiae quinque coaequantia. Comple igitur $\frac{2}{3}$ substantiae cum similitudine earum medietati, et veniet substantia. Similiter comple 5 cum sua medietate, et habebis $7\frac{1}{2}$ quae substantiam coaequant. Ipsam igitur substantiae radicem quae cum suis duabus tertiis multiplicata quinarium numerum producit.

Quaestio decima quarta

Inter puellas drachmam sic diuido, vt vnicuique earum aequalis particula rei contingat; quibus etiam si vnam insuper puellam adhibuero, illis omnibus particula primae particulae minus vna sexta aequalis contingit.

Expositio huius est, vt ipsas puellas cum minutia particulae qua differunt multiplicem, postea quod ex multiplicatione excreuerit cum numero puellarum postremarum multiplicem atque tandem productum hoc in id quo puellae priores a posterioribus differunt, diuidam, et complebitur substantia. Puellas igitur priores, vnam rem scilicet, cum sexta, quae est inter eas, multiplico, et producitur $\frac{1}{6}$ rei. Deinde multiplico eam cum numero puellarum postremarum, quae sunt res et vnum, et producitur sexta substantiae et sexta rei in drachmam diuisa, drachmae aequalis. Substantiam igitur compleo, id est, substantiam cum senario numero multiplico, et sic substantiam et radicem habebo. Drachmam etiam cum sex drachmis multiplico, et venient substantia et radix, sex drachmas coaequantes.

---

2. per 2 V; in duo inequaliter D.

3. constituas V.

4. et *pro* etiam. diuisi V; diuisum D.

5. et re *om.* V. et (*om.* V) fiuntque.

6. autem *om.*

7. conuerte. et hoc.

8. Dicam. tres *om.*

9. Oppone ergo. hiis D.

11. sic multiplico, ut fiant.

12. talis *om.*

13. unius substantie 5.

14. erit. Et similiter. unum *pro* v D *del.*

15. substantia *pro* medietate D. quae *om.* substantias D. coequantem. Eius ergo radix est res que quando in.

16. fuerit ad quinarium excrescet numerum. C *add.*: radix numeri quae est $\sqrt{7\frac{1}{2}}$ componet.

18. unum dragma. uniuscuiusque.

19. insuper *om.* His *pro* illis D.

20. minus sexta tocius. contingat.

21. ut primas puellas V; *om.* D. cum *om.* V. que D.

22. Ac postea. numerum V.

23. posteriorum D. multiplicabo (multitudo D). Postea quod ex hac multiplicatione collectum fuerit super illud quod (+ quod V) inter puellas primas est (*om.* D) et posteriores diuido.

24. diuidunt C. et + tunc. Tunc etiam puellas primas que sunt res in sexta.

25. multiplica.

26. radicis.

27. 5 *pro* vnum. radicis.

28. coequalis. ipsam *pro* substantiam.[2]

29. ergo in 7 *pro* etiam cum sex.

30. erunt.

Twelfth Problem

I divide a unit and one-half in such a way that the larger part shall be double the smaller.[1]

Explanation. I let the greater part be to the lesser as one is to x; whence I say divide $1\frac{1}{2}$ units by one unit $+ x$, giving $2x$. Therefore I multiply $2x$ by one unit $+ x$, giving $2x^2 + 2x$, which is equal to $1\frac{1}{2}$ units. I reduce this, therefore, to one square, that is, of each thing I take the half. I say then that $x^2 + x$ is equal to $\frac{3}{4}$ of a unit. You now proceed in the manner which we have explained.

Thirteenth Problem [2]

I multiply a square by two-thirds of itself and have five as a product.[3]

Explanation. I multiply x by two-thirds x, giving $\frac{2}{3}x^2$, which equals five. Complete $\frac{2}{3}x^2$ by adding to it one-half of itself, and one x^2 is obtained. Likewise add to five one-half of itself, and you have $7\frac{1}{2}$, which equals x^2. The root of this, then, is the number which when multiplied by two-thirds of itself gives five.

Fourteenth Problem

I divide a unit among girls in such a way that each one receives the same fractional part of the thing. Now if I add one girl to the number, each receives for her part one-sixth (of a unit) less than before.[4]

Explanation. I multiply the number of girls at the first by the fractional part representing the difference. Then I multiply this product by the second number of girls, and finally I divide this product by the difference between the first and second number of girls. This completes the given quantity.[5] Hence I multiply in this instance one x, representing the first number of girls, by the difference between the two amounts, $\frac{1}{6}$, and $\frac{1}{6}x$ is obtained. Then I multiply this product by the second number of girls, which is $1 + x$, and $\frac{1}{6}x^2 + \frac{1}{6}x$ is obtained; this being divided by a unit equals a unit. I complete the square, that is, I multiply the square by six, and I have $x^2 + x$. Likewise I multiply the unit by six units, giving $x^2 + x$

---

[1] Strictly, $1\frac{1}{2} - x = 2x$, whence $x = \frac{1}{2}$. The English translation of this problem is adapted by Rosen (p. 59) to conform to the solution in the explanation, and this follows closely our explanatory text. The equation which is given by Rosen is: $\frac{1\frac{1}{2}}{1 + x} = 2x$, and to this our text leads.

I am indebted to Professor W. H. Worrell for the following precise translation of the Arabic text of the passage: "If it is said to divide a unit and a half between a man and a part of a man, then the man has received the like of the fraction." The Libri text, pp. 285–286, varies from this only in stating that the man receives double that which the fractional part (of a man) receives.

[2] The following problem precedes in the Arabic text: $(x - \frac{1}{3}x - \frac{1}{4}x - 4)^2 = x + 12$.

[3] $x \cdot \frac{2}{3}x = 5$, whence $x = \sqrt{15/2}$.

[4] Rosen, pp. 63–64; Libri, p. 286. Our text is faulty. The problem is $\frac{1}{x} - \frac{1}{x + 1} = \frac{1}{6}$.

[5] Latin *substantia*, 'square,' obviously an error.

Radices igitur per medium diuido, et earum medium cum suo aequali multiplico, et quod producitur ad 6 adiicio, atque huius aggregati radicem accipio, vnde tandem medietatem radicum subtraho. Nam hoc quod residuum fuerit, numerum puellarum priorum designabit, et ipsae sunt duae.

Quaestio decima quinta

Si ex substantia quatuor radices subtraxero, ac postea tertiam residui accepero, et haec ipsa tertia quatuor radicibus aequalis fuerit, tunc substantia in ducenta quinquaginta sex terminatur.

Expositio huius est vt scias, quod cum tertia residui, posterioris scilicet, aequalis sit quatuor radicibus, residuum prius duodecim radicibus aequale erit. Adde igitur illas super quatuor, et venient 16 radices, quae sunt radix substantiae.

Quaestio decima sexta

Ex substantia tres radices subtraho et postea quod residuum est cum suo aequali multiplico, sitque tota multiplicationis summa aequalis substantiae.

Manifestum est, quòd residuum sit radix, quam quaternarius adimplebit numerus; substantiam verò numerus 16 component.

Hae igitur sunt sedecim quaestiones quae ex prioribus nasci videntur, vt diximus. Quicquid igitur iuxta artem restaurationis et oppositionis multiplicare volueris, facile per ea quae tradita sunt expedies.

CAPUT RERUM VENALIUM

Res autem venales et omnia, quae ad ipsas attinent, duobus modis et quatuor numeris disponuntur. Horum igitur numerorum primus iuxta Arabes, Almuzahar, qui et primus propositus nominatur. Secundus vero, Alszian, id est secundus per primum dinotus, appellatur. Tertius Almuhen id est ignotus. Quartus Alchemon, id est per primum et secundum dinotus. Hi porro quatuor numeri sic disponuntur, vt eorum primus, qui est Almuzahar vltimo, qui est Alchemon, opponatur. Horum autem quatuor numerorum tres semper noti ac certi ponuntur, quatuor vero numerus ignotus ponitur, et is ipse est cum quo quantum inquiritur.

---

2. atque eam multiplicationem super 6 adicio, et huius summe. ex qua *pro* vnde tandem.
3. abstraho. Nam + et.
4. et sunt due V; *om.* D.
6. ex *om.* D. et postea; et C *in ras.* tertiam *om.* D.
7. si *pro* et haec. et *pro* in D.
9. quoniam *pro* quod cum. posterioris scilicet *om.*
10. et quam residuum simile sit (est D) 12 radicibus.
11. quatuor + radices.
13. ac *pro* et. est + et D.
15. Unde manifestum est, quod illud residuum similiter sit radix et quod substantia sit 4 et fiunt (fuerit D) 16 dragmata.
17. Hec. ex 6 primis. vt + iam.
18. Quotquot V; Quidquid D.
19. per earum aliquam illud multiplicatum reperies.
20. *Titulum om.*
21. Item res omnis venales D. ad *om* ipsis.
22. vero *pro* igitur. Almuzarar *siue* Almusarar *ubique* V; Almuzaar *siue* Almusaar D.
23. id est *pro* qui et. nuncupatur D. Alter Alszarar *ubique* V; Alzazar *siue* Alszazar D.
24. Almuthemen *ubique* V; Almute *siue* Almuthemon D. Althemen *ubique* V; Altemon *siue* Althemon D.
25. Sed et hii 4.
27. eciam.
28. numerus *om.* ponitur + et incertus. ipse est ille.

equal to 6 units. I take one-half of the roots and I multiply the half by itself. I add the product to 6, and of this sum I take the root. The remainder obtained after subtracting one-half of the roots will designate the first number of girls, and this is two.

Fifteenth Problem

If from a square I subtract four of its roots and then take one-third of the remainder, finding this equal to four of the roots, the square will be 256.[1]

Explanation. Since one-third of the remainder is equal to four roots, you know that the remainder itself will equal 12 roots. Therefore add this to the four, giving 16 roots. This (16) is the root of the square.

Sixteenth Problem

From a square I subtract three of its roots and multiply the remainder by itself; the sum total of this multiplication equals the square.[2]

Explanation. It is evident that the remainder is equal to the root, which amounts to four. The square is 16.

These now are the sixteen problems which are seen to arise from the former ones, as we have explained. Hence by means of those things which have been set forth you will easily carry through any multiplication that you may wish to attempt in accordance with the art of restoration and opposition.

CHAPTER ON MERCANTILE TRANSACTIONS[3]

Mercantile transactions and all things pertaining thereto involve two ideas and four numbers.[4] Of these numbers the first is called by the Arabs Almuzahar and is the first one proposed. The second is called Alszian, and recognized as second by means of the first. The third, Almuhen, is unknown. The fourth, Alchemon, is obtained by means of the first and second. Further, these four numbers are so related that the first of them, the measure, is inversely proportional to the last, which is cost. Moreover, three of these numbers are always given or known and the fourth is unknown, and this

---

[1] Rosen, p. 66; Libri, p. 296. $\frac{1}{3}(x^2 - 4x) = 4x$.

In the Arabic text these two problems precede: $x^2 \,.\, 3x = 5x^2$ and $(x^2 - \frac{1}{3}x^2) \,.\, 3x = x^2$.

[2] Rosen, p. 67; Libri, p. 296. $(x^2 - 3x)^2 = x^2$, whence $x^2 - 3x = x$.

The problem, $x + \sqrt{x^2 - x} = 2$, precedes. This is one of two problems given in the German excerpt of 1461 from the algebra of Al-Khowarizmi (Gerhardt, *Monatsbericht d. Königl. Akad. der Wissenschaften zu Berlin*, 1870, pp. 142–143).

[3] The famous 'rule of three' is the subject of discussion in this chapter.

[4] The two ideas appear to be the notions of quantity and cost; the four numbers represent unit of measure and price per unit, quantity desired and cost of the same. These four technical terms are *al-musa' 'ir*, *al-si'r*, *al-thaman*, and *al-muthammin*, and further *al-maqūl;* see p. 44.

Talis igitur ad hanc artem regula datur, vt in omni huius inquisitione tres numeri, qui noti ac certi positi sunt, considerantur, quoniam eorum duo semper inter se oppositi inueniuntur. Horum igitur duorum vnus cum altero multiplicandus, atque multiplicationis productum in notum tertium ac certum positum, qui ignoto opponitur, diuidendum erit. Nam quod ex diuisione exierit, erit numerus de quo dubitatur, et ipse ei numero opponitur in quem facta est diuisio. Sed ne hanc artem aliquis error incurrere arbitretur, tale damus exemplum.

De primo modo

Decem pro sex, quot pro quatuor?

Vide nunc, quo modo, pro eo vt diximus, praefati numeri disponuntur. Nam quando 10 dixisti, numerum Almuzahar pronunciasti; et quando pro 6 dixisti, Alszian protulisti; et quando quot dixisti, numerum Almuhen siue Magol, id est ignotum, pronunciasti; et quando pro 4 dixisti, numerum Alchemon edidisti. Vides igitur quòd eorum tres, id est 10, 6 et 4, noti et certi sint, de quarto verò adhuc ignoto, dubitetur. Si igitur ad regulam prius datam respexeris, primum cum vltimo, id est 10 cum 4, multiplicabis, sunt etenim oppositi numeri, noti quoque ac certi, et quod ex multiplicatione excreuerit, id est 40, in alterum numerum notum ac certum, qui est Alszarar, id est in 6 oportet diuidere, et exeunt 6 et $\frac{2}{3}$ vnius, numerum ignotum designantes. Et hic numerus numero senario, qui Arabice Alszarar nominatur, est oppositus.

De secundo modo

Secundus modus huius artis est, vt dicas, decem pro 8, pro quot quatuor?

Decem igitur sunt Almuzahar, qui videlicet numero Almuhen ignoto, cum quo quantum acquiritur, est oppositus, et 8 designat numerum Alszarar, qui numero Alchemon, qui sunt 4 opponitur. Vnum igitur duorum numerorum cognitorum atque oppositorum cum altero multiplica, id est 4 cum 8, et producentur 32. Haec ergo 32 in tertium cognitum numerum 10, qui est Almuzahar, diuide, et exeunt $3\frac{1}{5}$, qui numerum Alchemon designant, quique ei numero in quem diuiditur est oppositus.

---

1. quoque *pro* igitur. hac D.

2. certe V. quoque *pro* quoniam D. adinuicem *pro* inter se.

3. inuenientur. unus est; unius C.

4. et eorum *pro* atque. per. atque V, certe D.

5. diuidenda V; diuidendus D.

6. dubitabatur. per.

7. errorem.

8. *Titulum om. et sic infra.*

9. Secundum ergo primum modum, sic dicas, 10 pro 6, quot pro 4?

10. secundum quod diximus.

11. quoniam *pro* quando.

12. Alszaran C. Magul V; Magulum D.

13. nunciasti. Almuhemen C.

14. qualiter *pro* quod. 10 + et D. ac certi ponuntur et quomodo de quarto, adhuc incognito, dubitatur.

16. omnino *pro* etenim D. numeri *om.* quoque *om.* D.

17. per alterum (alium D). numerum *om.* D.

18. per. erunt (+ 7 siue D).

19. numeri (*om.* D) almagul (maghulis D) id est incognitum. Et huius (+ -modi D).

23. Almuszarar C. videlicet *om.* D. quo *om.*

24. inquiritur V.

25. Primum D. duorum *om.* D.

26. in alterum (alium D). fient. Hunc ergo numerum per alterum (alium D) cognitum; Hunc C *sed del.*

27. Almuzahar + id est 32 per 10 diuide. Almuszarar C. erunt V; exeund D.

28. qui *om.* numeri. designantes, qui videlicet eo numero per quem. est *om.* D.

is indicated by the question as to the quantity. The rule of this kind of problem is to consider the three quantities which are given or known, of which two are always found to be inversely proportional to each other. These two are to be multiplied one by the other and the product of the multiplication is to be divided by the third number, likewise known, which is inversely proportional to the unknown. Now the quotient of this division will be the number which is sought, and it is inversely proportional to the number by which you divide. But lest some error be made in this type of problem we give an example of it.[1]

A Problem of the First Type

Ten for six, how many for four?

Observe now in what manner the given numbers are related, according to what we have said. For when you say "ten," you give the measure, and when you say "for six," you state the price. When you ask, "how much?" you give the unknown, called Almuhen or Magul, and saying "for four," you mention the cost. You note further that three of these, that is, 10, 6, and 4, are known and definite numbers, and the question is concerning the fourth or unknown number. If now you take account of the rule given, you multiply the first by the last, that is, 10 by 4, for they are the known and definite numbers which are inversely proportional to each other. It is necessary to divide the product, that is, 40, by the other known and definite number, that is, the measure, which is 6. This gives $6\frac{2}{3}$, designating the unknown number. This number is inversely proportional to the number six, which in Arabic is called Alszarar.

A Problem of the Second Type

An example of the second type of such problems is given by the question, "ten for eight, what is the cost of four?"

Ten now is the measure which is inversely proportional to the unknown cost, and eight designates the price which is inversely proportional to the quantity, 4. Therefore multiply one of the two known and inversely proportional numbers by the other, that is, 4 by 8, and you will have 32. Divide 32 by the third known number, 10, which is the measure. This gives $3\frac{1}{5}$, designating the cost which is inversely proportional to the number by which

---

[1] As we have noted in the introduction, page 44, this paragraph is not carefully translated by Robert of Chester, and he added the last sentence. The corresponding passage in the Libri version (*op. cit.*, pp. 268–269) is entitled, *Capitulum conventionum negociatorum*, and begins as follows: Scias quod conventiones negotiationis hominum, que sunt de emptione et venditione et cambitione et conductione et ceteris rebus, sunt secundum duos modos, cum quattuor numeris quibus interrogator loquitur.

Leonard of Pisa (*Liber abbaci*, p. 2 and p. 83) follows somewhat the terminology of this version in his discussion. The title (p. 2) is given: *De emptione et venditione rerum uenalium et similium.* The section opens (p. 83), as follows: 'In omnibus itaque negotiationibus quattuor numeri proportionales semper reperiuntur, ex quibus tres sunt noti, reliquus uero est ignotus.'

His igitur duobus modis omnia quae venalia dicuntur, absque omni errore possunt tractari, si deus voluerit.

Quaestio seu interrogatio vltima

Homo in vinea 30 diebus pro decem denariis conducitur, ex quibus operatus est sex diebus, quantum ergo precii totius debet accipere?

Expositio huius est, quoniam manifestum est quòd sex dies quintam partem totius temporis adimplent, et quòd hoc quod ex precio ei contingerit, sit secundum quod ipse ex toto tempore, scilicet ex 30 diebus, sit operatus. Quod autem diximus, sic exponitur. Quoniam quando mensem id est 30 dies dixisti Almuzahar protulisti; et quando 10 dixisti, Alszarar; quando verò 6 dies, Almuhen pronunciasti, quando deinde dixisti quantum precii ei contingerit, Alchemon nunciasti. Multiplica ergo Alszarar, qui sunt 10, cum Almuhen qui ei opponitur, id est cum 6, et quod ex multiplicatione excreuerit, 60 scilicet, in 30 Almuzahar diuide, et exeunt 2 denarii, et ipsi erunt Alchemon, id est pars quae homini contingerit.

Hoc igitur modo quicquid huius tibi propositum fuerit, ex rebus venalibus siue ponderibus, seu ex omnibus quae ad haec attinent, agendum erit.

Laus deo praeter quem non est alius.

Finis libri restaurationis et oppositionis numeri quem Robertus Cestrensis de Arabico in latinum in ciuitate Secobiensi transtulit, [Era] anno millesimo centesimo octogesimo tertio.

---

1. omni *om.* D. 4. dragmata.
5. precium.
6. quod iam *pro* quoniam. quoniam *pro* quòd. partem *om.*
7. mensis *pro* totius temporis. quòd^{1} *om.* D. eius *pro* ei V.
8. ex mense. scilicet ex 30 diebus *om.*
9. Almuszarat C.
10. dixisti 10 + dixisti. et quando 6 dixisti dies.
11. Et quando dixisti.
12. id est 10 V; 10 D. 6 + et fient 60.
13. 60 scilicet *om.* super 30, id est super Almusarar (Almuzaar D) et erunt duo dragmata et ipsa.
14. erunt *om.* D. contingit D.
15. Hoc ergo modo quotquot tibi huius positum V; Huius ergo modi quicquid nisi positum D. venalibus + ac aliis.
16. his V; hiis D *pro* ad haec. est.
17–20. *om.* D.
17. Deo gracias V.
18. Explicit V.
19. Sectobiensi V. Era M C lxxxiii V.

you divided. According to these two methods it is possible to treat all commercial problems, without error if God will.

The Last Problem or Question

A man is hired to work in a vineyard 30 days for 10 pence. He works six days. How much of the agreed price should he receive?

Explanation. It is evident that six days are one-fifth of the whole time; and it is also evident that the man should receive pay having the same relation to the agreed price that the time he works bears to the whole time, 30 days. What we have proposed, is explained as follows. The month, *i.e.* 30 days, represents the measure, and ten represents the price. Six days represents the quantity, and in asking what part of the agreed price is due to the worker you ask the cost. Therefore multiply the price 10 by the quantity 6, which is inversely proportional to it. Divide the product 60 by the measure 30, giving 2 pence. This will be the cost, and will represent the amount due to the worker.

This, then, is the method by which all proposed problems concerning commercial transactions or weights and measures and all related problems are to be solved.

Praise be to God, beside whom there is no other.

Here ends the book of restoration and opposition of number which in the year 1183 (Spanish Era) Robert of Chester in the city of Segovia translated into Latin from the Arabic.

REGULE 6 CAPITULIS ALGABRE CORRESPONDENTES

Prima. Quando numeris assimilantur ℛ committatur φ per ℛ et productum ostendit quesitum.

2^a. Quando φ assimilantur ʒ committatur φ per ʒ et radix propositi (producti) ostendit quesitum.

3^a. Quando ℛ assimilantur ʒ committatur [per φ] per ʒ et productum ostendit quesitum.

4^a. Quando φ assimilantur ℛ ⁊ ʒ, φ ⁊ ℛ debent per ʒ committi; radix mediari, medium in se duci, productum numero addi. Radix tocius aggregati minus medietate ℛ ostendit quod queritur.

5^a regula. Quando ℛ assimilantur φ ⁊ ʒ, ℛ ⁊ φ debent per ʒ committi; ℛ mediari, medium in se duci, a producto φ subtrahi, ℛ residui a medietate ℛ tolli et hiis residuum ostendit quesitum. Quod si ℛ residui a medietati ℛ subtrahi non potest, addere licet eandem.

6^a. Quando ʒ assimilantur φ ⁊ ℛ. Hec debent per ʒ committi; ℛ mediari, medium in se duci, productum φ addi. Radix aggregati plus medietate ℛ ostendit quod queritur.

1–17. Regule . . . queritur *add* V.

RULES CORRESPONDING TO THE SIX CHAPTERS OF ALGEBRA

First. When roots are equal to a number, divide the number by (the number of) the roots, and the quotient represents the desired quantity.[1]

Second. When squares are equal to a number, divide the number by (the number of) the squares, and the root of that which you obtain represents the desired quantity.[2]

Third. When roots are equal to squares, divide (the number of) the roots by (the number of) the squares, and the quotient represents the desired quantity.[3]

Fourth. When a number is equal to the sum of squares and roots, divide by (the number of) the squares. Take one-half of (the number of) the roots after the division and multiply it by itself. To this product add the number. The root of this sum less one-half of the number of roots, represents that which is sought.[4]

Fifth. When roots are equal to a number and squares, divide the roots and the number by (the number of) the squares. Take one-half of (the number of) the roots after the division, and multiply it by itself. From this product subtract the number; the root of the remainder subtracted from one-half of (the number of) the roots is the desired quantity.[5] But if it is not possible [6] to subtract the root of the remainder from one-half of (the number of) the roots, it is permissible to add the same.

Sixth. When squares are equal to a number and roots, on this side divide by (the number of) the squares. Take one-half of (the number of) the roots after the division, and multiply it by itself. To this product add the number; the root of this sum plus one-half of (the number of) the roots represents that which is sought.[7]

---

[1] $ax = n;\ x = a/n.$

[2] $ax^2 = n;\ x^2 = a/n;\ x = \sqrt{a/n}.$

[3] $ax^2 = bx;\ x = b/a.$

[4] $ax^2 + bx = n;\ x = \sqrt{(b/2a)^2 + n/a} - b/2a.$

[5] $ax^2 + n = bx;\ x = b/2a \pm \sqrt{(b/2a)^2 - n/a}.$

[6] It is always possible to subtract the root of the remainder here from one-half of the number of the roots, even from the standpoint of the mathematician of the fourteenth century who made this table of rules, for the remainder would always be positive, since the square root of $(b/2a)^2 - n/a$ is always less than $b/2a$.

[7] $ax^2 = bx + n;\ x = b/2a + \sqrt{(b/2a)^2 + n/a}.$

The six rules with similar algebraic symbolism are found also in Codex Dresdensis, C. 80^{m}; see Wappler, *Programm*, *loc. cit.*, p. 11.

ADDITA QUAEDAM PRO DECLARATIONE ALGEBRAE

Combinationem triplicem in censibus reperimus, aut enim census et radices aequantur numeris, aut census et numeri aequantur radicibus, aut verò radices et numeri censibus aequantur.

Exemplum combinationis primae

Census et octo radices eius 33 denariis aequipollent. Quaeritur ergo, quantus est census?

Respondetur: 9.

Regula est, vt productum ex medietate radicum cum seipsa multiplicata, scilicet 16, ad numerum, 33, addamus et resultant 49, cuius radix est 7. Atque ab hac radice medietatem radicum, 4 scilicet, subtrahamus, et manebunt 3, quae sunt radix census. Est igitur census 9.

Item si dicatur, duo census et octo radices sunt aequales quadraginta duobus denariis. Reduc quaestionem ad vnum censum sic. Si 2 census et 8 radices sunt 42 denarii, igitur vnus census et 4 radices sunt 21. Operando igitur per illud ex duplicatione et peruenies in fine ad intentum.

Item si dicas, medietas census et 4 radices sunt aequales $16\frac{1}{2}$ denariis.

Reduc quaestionem ad integrum censum, et dimidia totum, et patebit in fine intentum.

Exemplum combinationis secundae

Census et 15 denarii valent octo radices, quantus igitur est census?

Respondetur: 9, vel 25.

Regula. Duc medietatem radicum in se, et fiunt 16. Ex quibus subtrahe 15 denarios et manet vnum, cuius radicem, scilicet vnum, subtrahe a medietate radicum, et manent 3, quae sunt radix census. Vel illud vnum adde ad medietatem radicum, et veniunt 5 quae sunt radix census. Patet igitur, quaestionem secundum vtranque partem esse veram, ideo oportet vt eas determines distinctiue.

Corollarium

Notandum si quadratum medietatis radicum minus fuerit quam est vltimus numerus propositus, vt si dicatur, census et 15 numeri sunt aequales 6 radicibus, tunc quaestio impossibilis erit. Si verò fuerit vltimo proposito numero aequalis vt si dicatur, census et 9 denarii valent 6 radices, tunc medietas radicum est radix census. Si autem quaestio venerit cum pluribus censibus, aut paucioribus vno censu, reduc ad vnum censum, et operare vt supra dictum est.

Exemplum combinationis tertiae

Quatuor radices et 12 denarii censui aequipollent.

Reduc, hoc est multiplica, medietatem radicum in se, et fiunt 4. Quibus adde

SOME ADDITIONS IN EXPLANATION OF THE ALGEBRA

We find a triple combination of squares,[1] namely squares and roots equal to numbers, squares and numbers equal to roots, and finally roots and numbers equal to squares.

An Illustration of the First Type

A square and 8 of its roots equal 33 units. The question is, what is the square?[2] Answer: 9.

The rule is to add to the number the product of half of the root multiplied by itself, that is 16 to 33, and 49 is obtained, of which the root is 7. Now from this root we subtract one-half of the number of the roots, namely 4, which leaves three. This is the root of the square, and the square is 9.[3]

Likewise if you are given that two squares and eight roots are equal to 42 units. Reduce in the following manner to one square. If two squares and eight roots are equal to 42 units, then one square and four roots are equal to 21. You operate therefore by duplication and arrive in the end at the desired result.[4]

Likewise if you are given one-half a square and four roots equal to $16\frac{1}{2}$ units. Reduce the problem to a whole square, take one-half of the whole, and in the end you arrive at the desired result.[5]

An Illustration of the Second Type

A square and fifteen units equal eight roots. What then is the square? Answer: 9 or 25.

Rule. Multiply one-half of the roots by itself, and 16 is obtained. From this subtract the 15 units and one is obtained, of which you subtract the root, namely one, from one-half of the roots. This gives 3 as the root of the square. Or add that one to one-half of the roots and 5 is obtained as the root of the square. It is clear then that the problem is solved by each value, and hence it is necessary to determine separately both solutions.[6]

Corollary

It should be noted that if the square of half of the roots is less than the proposed number, as, for example, a square and the number 15 equal to 6 roots, then the problem is not solvable.[7] If the square of one-half of the roots is equal to the given number, as, for example, a square and 9 units equal to 6 roots, then the half of the roots is the root of the square. Moreover, if a problem is proposed involving more than one square or less than a whole square, reduce to one square, and operate as indicated above.

---

[1] Scheybl changes from *substantia* to *census;* see p. 69, n. 1.

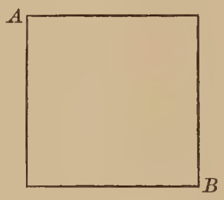
[2] *Textus* in the margin precedes the statement of each problem, and *Minor* the solution.

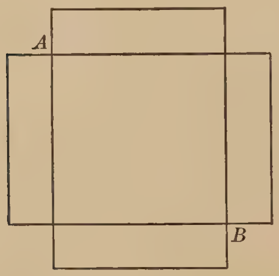
12 denarios et veniunt 16, cuius radicem 4 adde ad medietatem radicum, et resultant 6, quae sunt radix census. Omne etiam, quod aut maius est vno censu aut minus, reducas ad vnum censum, atque deinde operare vt dictum est.

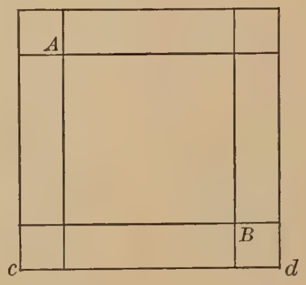
Proponit causas trium combinationum

Causa combinationis primae est haec:

Sit quadratum *a b* quod censum significet, sitque huius quadrati latus ipsius census radix. Et quoniam ductus vnius lateris in numerum radicum, quantitas radicum sumptorum existit. Applicetur igitur ad vnumquodque latus quadrati quarta pars numeri radicum, scilicet duae radices lateri vni, et duae alii, et ita deinceps. Resultat autem sic quadratum aliud, praeter quatuor eius angularia quadrata, quorum singulorum vnumquodque latus duo, hoc est quartam partem numeri radicum, continebit. Ex ductu igitur quartae partis numeri radicum in se quater, resultabunt illa quatuor parua angularia quadrata, atque haec eadem resultant ex multiplicatione medietatis numeri radicum in se semel. Si igitur productum ex medietate numeri radicum in se multiplicata, addatur ad censum et ad radices, quadratum cuius vnius lateris quantitas aequalis est lineae *c d* describetur. Hoc autem latus excedit latus census in medietate numeri radicum, quia per quartam partem in vno extremo, et per quartam partem in altero huius census latus auctum est. Ideo subtracta medietate numeri radicum a tota linea *c d* manebit quantitas radicis census quae quaerebatur.







Illa probatio procedit ex propositione quarta secundi Euclidis.

An Illustration of the Third Type

Four roots and twelve units are equal to a square.

Find, that is to say multiply, half of the roots by itself, and four is obtained. To this add the 12 units, and 16 is obtained. Add the root p. 130 of this, 4, to one-half of the roots and 6 appears as the root of the square. Also whatever is given either greater or less than one square, reduce to one square and operate as indicated above.[1]

The Explanation of the Three Types

The explanation of the first type is as follows: let the square AB represent x^2. Then the side of this square will represent the root of x^2, or x. When one side of this square is multiplied by the number of the roots, the quantity of the assumed roots is represented. Hence let there be applied to each side of the square the fourth part of the number of the roots, namely two roots to one side, two to another, and so on. Thus another square appears, lacking only four small corner squares of which each has every side equal to two, that is the fourth part of the number of the roots. Therefore by multiplying the fourth part of the number of the roots by itself four times, these four small corner squares are obtained, and this same result is obtained by the multiplication of half of the roots by itself once. If then the product of half the roots multiplied by itself be added to the square and roots, the square is formed of which one side is equal in value to the line cd. Moreover, this side exceeds the side of the unknown square by the half of the roots, since the side of the unknown square has been extended by the fourth part at one extremity and by the fourth part at the other. Hence by subtracting the half of the number of the roots from the whole line cd the value appears of the root of the square which is sought.

The proof of this follows from the fourth proposition of the second book of Euclid.[2]

---

[3] $x^2 + 8x = 33$; $x^2 + 8x + 16 = 49$; $x + 4 = 7$; $x = 3$; $x^2 = 9$.

[4] $2x^2 + 8x = 42$; $x^2 + 4x = 21$; $x + 4x + 4 = 25$; $x + 2 = 5$; $x = 3$.

[5] $\frac{1}{2}x^2 + 4x = 16\frac{1}{2}$; whence $x^2 + 8x = 33$, as above.

[6] $x^2 + 15 = 8x$; $x^2 - 8x + 16 = 1$; $x - 4 = \pm 1$; $x = 3$ or 5. Both roots are positive, and so two solutions are given.

[7] $x^2 + n = bx$; $x = b/2 \pm \sqrt{(b/2)^2 - n}$.

The roots are imaginary if $(b/2)^2$ is less than n, as in the illustration which Scheybl gives; similarly, the two roots are equal to each other and each equal to one-half the coefficient of the roots if the square of this quantity is the same as the given constant, that is $(b/2)^2 = n$.

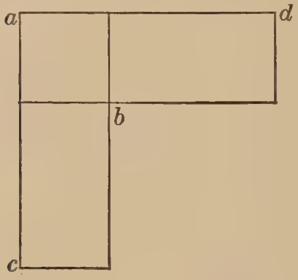
[1] $4x + 12 = x^2$; $x^2 - 4x + 4 = 16$; $x - 2 = 4$; $x = 6$. p. 130

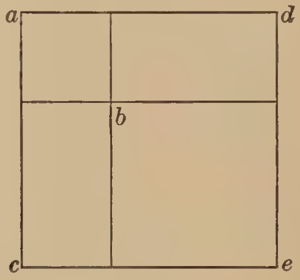
[2] Euclid II, 4, following Heath, *The Thirteen Books of Euclid's Elements:* "If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments." In modern notation, $(a + b)^2 = a^2 + b^2 + 2ab$.

Aliter sic

Sit census quadratum *a b*, cuius vni laterum applicetur medietas numeri radicum, et alteri eius lateri applicetur medietas numeri radicum altera, sit autem primum additum *a c*, secundum vero *a d*. Ad perficiendum igitur quadratum *c d*, deficit quadratum quod vocetur *b e*, cuius latus medietati numeri radicum est aequale. Patet igitur causa operationis; nam ex ductu medietatis numeri radicum in se, resultat quadratum *b e*, et residuum est census cum adiectis radicibus. Vnde cum illud magnum quadratum notum sit, et eius radix nota, per subtractionem radicis quadrati *b e* à radice quadrati *a e*, necessario radix census manebit.

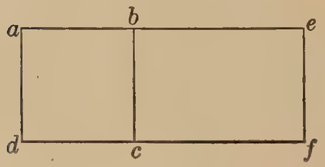
Sequuntur figurae geometricae.

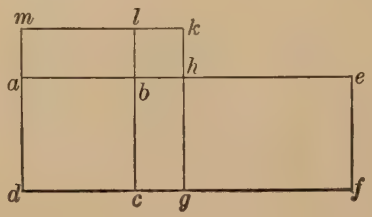




Exemplum combinationis secundae

Sit census quadratum *a b c d*, cuius lateri *b c* applicabo parallelogrammum *c b e f*, et ponam ipsum 15. Totum igitur parallelogrammum *d a e f* est census et 15 denarii, et continet octo radices census. Diuidam ergo lineam *d f* aequaliter in punctum *g* et erigam super latus *g d* quadratum *g k m d*, et protraham *c b* vsque ad *l* et ponatur litera *h* in locum vbi *g k* secat *a e* lineam. Et quoniam *d b* est quadratum, cum ideo ex structura et propositione sexta secundi libri Euclidis, *b k* quadratum sit, duo parallelogramma *g b* et *b m* inter se aequalia erunt; mox deinde, per communem quandam noticiam, *g a* et *c m* aequalia. Atque tandem cum *g a* ex propositione 36 libri primi Eucli[dis], *g e* parallelogrammo aequale sit, *c m* eodem parallelogrammo *g e* aequale erit. Subtractis igitur quindecim, hoc est gnomone *l d h* à quadrato *g m* quod est sedecim, manet vnitas quadratum *b k*. Et quia notum, latus igitur vel radix, quae est linea *l b*, erit nota. Sed et tota *l c* nota est, manet igitur, post subtractionem *l b* ab *l c* linea, et *b c* linea nota. Et in hoc casu, quo punctus *g* cadit in parallelogrammum applicatum censui, sequuntur figurae geometricae.





Another Demonstration

Let the square *ab* represent x^2. To the side of it apply one-half the given number of roots and to another side of it apply the other half of the number of the roots. Let the first addition be represented by *ac* and the second by *ad*. To complete the square *cd* the square called *be* is lacking and the side of this is equal to one-half the number of the roots. The reason for our procedure (in halving the roots) is now apparent; for by multiplying the half of the number of roots by itself we obtain the square *be*, and the remainder is x^2 together with the added roots. Now since the larger square is known, and the root of it is known, then by subtracting the root of the square *be* from the root of the square *ae*, necessarily the root of our unknown square remains.

The geometrical figures follow.[1]

An Illustration of the Second Type

Let the square *abcd* represent x^2, and to its side *bc* apply the parallelogram *cbef*, which we take to be equivalent to 15 units. Then the whole rectangle *daef* is equivalent to x^2 and 15 units, thus containing 8 roots of x^2 ($8x$). Divide now the line *df* into two equal parts by the point *g* and erect a square *gkmd* upon the side *gd*. Extend *cb* up to a point *l* (the point of intersection with the upper side of the square upon *gd*). Mark the point of intersection of *gk* with *ae* by the letter *h*. Since *db* is a square, it follows by our construction and by proposition 6 of the second book of Euclid[2] that *bk* is a square and further that the two parallelograms *gb* and *bm* are equal to each other.[3] Further, then, by a well-known axiom[4] *ga* and *cm* are equal. Also finally, since *ga* is equal to the parallelogram *ge* by the thirty-sixth proposition of the first book of Euclid,[4] it follows that *cm* is equal to the same parallelogram *ge*. Hence subtracting 15, that is the gnomon *ldh*, from the square *gm*, which is 16, one is left as the area of the square *bk*. Since the area is known, the side or root, which is the line *lb*, is also known. But the whole line *lc* is known, whence it follows by subtraction of *lb* from *lc* that the line *bc* is known. In this instance it is to be noted that the point *g* falls within the parallelogram which is applied to the square.

The geometrical figures follow.

---

[1] Compare these and subsequent figures with the corresponding figures on pages 77–89.

[2] Euclid II, 6, following Heath, *The Thirteen Books of Euclid's Elements*, p. 1. "If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole with the added straight line and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line." In modern algebraic notation, $(a + x)x + (a/2)^2 = (a/2 + x)^2$.

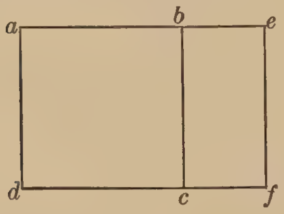
[3] By Euclid I, 43, following Heath, *loc. cit.*, "In any parallelogram the complements of the parallelograms about the diameter are equal to one another."

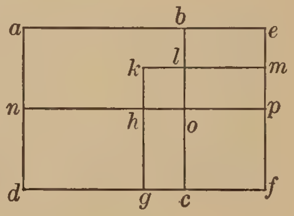
[4] Euclid I, 36, following Heath, *loc. cit.*, "Parallelograms which are on equal bases and in the same parallels are equal to one another."

Sed esto iam, quod punctus *g* cadat in latus census vel quadrati.

Sit census vt prius *a b c d*, et parallelogrammum, numero quindecim, lateri *b c* applicatum *c b e f*. Tunc super medietatem lineae *d f*, erigo quadratum *g k m f* et accipio lineam *c o* aequalem lineae *c f* et protraho lineam *n h o p* aequedistantem lineae *d f*. Et quoniam lineae *d c* et *c b* inter se aequales sunt, lineae quoque *c o* et *c f* aequales; et parallelogrammum igitur *d o* propter aequalitatem linearum, parallelogrammo *c e* aequale erit. Sed quia aequalia etiam inter se sunt ex propositione 43 primi libri Euclidis, duo supplementa *g o* et *o m*, cum per subtractionem aequalium ab aequalibus, ex communi quadam noticia, aequalia relinquantur, parallelogrammum *d h* parallelogrammo *l e* cum quadrato *c p* aequale erit. Sed quia *d h* aequale etiam est, ex propositione 36 primi, parallelogrammo *g o* cum quadrato *c p*; haec igitur duo, *g o* parallelogrammum et *c p* quadratum, prioribus duobus *l e* parallelogrammo et *c p* quadrato ex communi quadam noticia, aequalia. Atque mox, per ablationem quadrati *c p* communis, *g o* ipsi *l e* parallelogrammo aequale. Sunt vero *c o* et *l m* lineae inter se aequales. Ergo et linea *m e* lineae *g c* aequalis erit, atque ita aequalis etiam lateri quadrati *h l*. Cum ergo quadratum *g m* sit notum, eo quòd eius latus sit medietas numeri radicum, si subtrahantur ab eo 15, quae gnomonem valent, quadrato *h l* circumiacentem et cui etiam parallelogrammum *d o* aequale est, manebit quadratum *h l* notum. Vnde et latus eius notum. Sed quia illud est aequale lineae *g c* vel *m e*, latere igitur quadrati *h l* ad medietatem numeri radicum *m f*, latus quadrati *g m* addito, constituitur linea *e f* nota. Atque haec est radix census. Patet ergo propositum.

Sequuntur figurae geometricae.





Exemplum combinationis tertiae

Quatuor radices et duodecim denarii censui aequipollent.

Causa huius combinationis est haec. Sit census quadratum *a b c d* ignotum continens quatuor radices et 12 denarios. Ex isto igitur quadrato resecabo parallelogrammum *b c e f* continens quatuor radices; manebit ergo parallelogrammum *a e* continens precise duodecim denarios. Deinde diuidam lineam *e c*, numerum radicum, aequaliter in puncto *g*, et super latus *e g* erigam quadratum *g e h k*. Super latus etiam *d g* erigam quadratum *g d m l*, et secet linea *m l* lineam *e f* in puncto *n*. Et quoniam lineae *a d* et *d c* inter se aequales sunt, linea quoque *m d* et *d g* aequales, cum per subabiectionem aequalium ab aequalibus, linea *a m* lineae *g c* ex communi quadam noticia, aequalis sit. Eadem *a m* linea propter aequalitatem, ex altera quadam noticia, lineae *e g*, atque mox etiam lineae *h k*, aequalis erit. Item, quia lineae *h e* et *e g* inter se aequales sunt, lineae quoque *n e* et *d g*

Now let the point *g* fall within the side of the square.

Let the square as before be represented by *abcd* and the parallelogram *cbef*, equivalent to the given number 15, be applied to the side *bc*. Then upon half of the line *df* erect the square *gkmf*. Construct the line *co* equal to the line *cf* and draw the line *nhop* everywhere equally distant from the line *df*. Since the lines *dc* and *cb* are equal to each other (being sides of a square), and further, the lines *co* and *cf* equal (by construction), it follows that the parallelogram *do* is equal to the parallelogram *ce* on account of the equality of the sides. But further, since the two supplementary rectangles *go* and *om* are equal to each other by the forty-third proposition of the first book of Euclid, by subtracting equals from equals according to the well-known axiom, the remainders are equal, giving the parallelogram *dh* equal to the parallelogram *le* together with the square *cp*. But also since *dh* is equal to the parallelogram *go* plus the square *cp* by the thirty-sixth proposition of the first book (of Euclid), it follows that these two, the parallelogram *go* and the square *cp*, are equal to the preceding two, the parallelogram *le* and the square *cp* by another well-known axiom.[1] Whence by subtracting the common part, the square *cp*, *go* is equal to the parallelogram *le*; and indeed the lines *co* and *lm* are also equal to each other. Further, the line *me* will be equal to *gc*, and so also equal to the side of the square *hl*. Since the square *gm* has a known area, by the fact that its side is one-half of the given number of roots, if from it there be subtracted 15 which is represented by the gnomon *gfm*, circumscribed about the square *hl* and equal, as we have just shown, to the parallelogram *do*, the square *hl* remains known in area; whence also the side of it is known. But since this side of the square *hl* is equal to the line *gc* or *me*, when added to the half of the number of the roots *mf*, a side of the square *gm*, the line *ef* is then known. And this is the root of the unknown square. The proposed question is solved.

The geometrical figures follow.

An Illustration of the Third Type

Four roots and twelve units are equal to x^2.

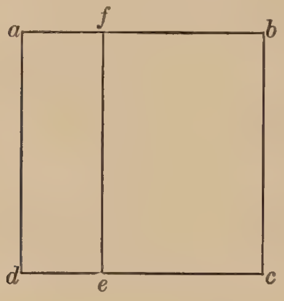
The explanation of this type is as follows. Let x^2 be represented by the unknown square *abcd* which contains 4 roots and 12 units. From this square cut off the parallelogram *bcef* containing four roots. The parallelogram *ae* consequently will contain precisely 12 units. Then divide the line *ec*, representing the number of the roots, into two equal parts by the point *g* and upon the side *eg* erect the square *gehk*. Also upon the side *dg* erect a square *gdml*, and let the line *ml* cut the line *ef* in the point *n*. Since the lines *ad* and *dc* are equal to each other, and further the lines *md*

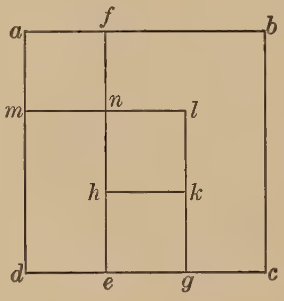
---

[1] Following Heath, *loc. cit.*, "Things which are equal to the same thing are also equal to one another."

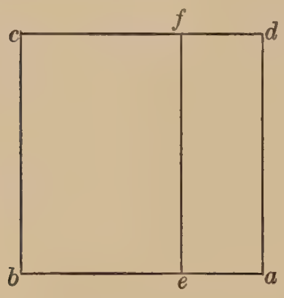
aequales, cum per subtractionem aequalium ab aequalibus, linea $n h$ lineae $d e$ aequalis sit. Eadem $n h$ linea, propter aequalitatem, lineae $m n$ aequalis erit. Igitur superficies $h l$ et $m f$ aequales. Duae igitur superficies $d n$ et $h l$ simul sumptae, ex communi illa noticia, si aequalibus aequalia adiiciantur, vni superficiei $d f$ quae 12 continet, aequales erunt. Vnde si numerus 12 addatur ad quadratum $e k$, cuius latus vel radix est linea $e g$, medietas numeri radicum, resultabit quadratum $d l$, cuius latus vel radix est linea $d g$. Si igitur illi radici addatur medietas numeri radicum, quae est linea $g c$, linea $d c$, quae est latus census, proueniet. Patet ergo propositum.

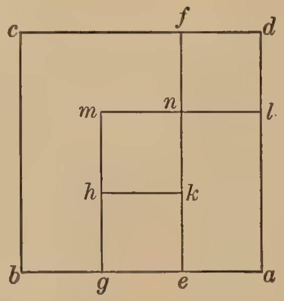
Sequuntur figurae geometricae.





Vel sub alia forma, sic:





Sequuntur multiplicatio cum additis et diminutis

Sicut idem est multiplicare compositum cum composito, et multiplicare vtramque partem compositi cum vtraque parte compositi, vt sic fiat quadruplex multiplicatio, scilicet articuli cum articulo et digiti cum articulo, deinde articuli cum digito, et quarto digiti cum digito. Comsimiliter potest fieri quadruplex multiplicatio, vbi articulus et digitus cum articulo praeter digitum multiplicare debet, et similiter vbi articulus praeter digitum cum articulo et digito, vel contrà, multiplicari debet. Vnde indicimus quadruplicem multiplicationem, atque in omni multiplicatione tali praedicta haec regula notanda est.

Si digiti tam in multiplicando quam in multiplicante additi cum articulis aut

and *dg* are equal, then by subtracting equals from equals the line *am* will be equal to the line *gc* by this well-known axiom. On account of this equality the same line *am* will, by another axiom,[1] be equal to the line *eg* and hence also to *hk*. Further, since the lines *he* and *eg* are equal to each other and also the lines *ne* and *dg* are equal, then by subtracting equals from p. 136 equals the line *nh* is equal to the line *de*. On account of this equality the same line *nh* will be equal to the line *mn*. Hence the areas *hl* and *mf* are equal. Therefore the two areas *dn* and *hl* taken together, by the well-known axiom "if equals be added to equals," will be equal to the single area *df*, which contains 12 units. Whence if the number 12 be added to the square *ek*, whose side or root is the line *eg*, the half of the number of the roots, the square *dl* is obtained whose side or root is the line *dg*. If, then, the half of the number of the roots, which is represented by the line *ge*, be added to that side (*dg*) the line *dc* will be obtained, which is the side of the unknown square. The proposed problem is solved.

The geometrical figures follow.

[Or in another form, thus]

Multiplication with Positives and Negatives [2]

Since the multiplication of a composite[3] number by a composite number is the same as the multiplication of each part of the one composite by each part of the other, so it follows that the multiplication is fourfold, namely article by article, digit by article, then article by digit, and fourthly digit by digit. Similarly, you may have a fourfold multiplication when an article and a digit is to be multiplied by an article less[4] a digit, or the reverse.[5] It seems desirable to indicate the fourfold nature of this multiplication (by some examples), and in every multiplication of this kind this rule is to be followed.

If the digits in the multiplicand as well as in the multiplier are added to

---

[1] The halves of equals are equal, and the axiom of the preceding note.

[2] Attention is called to a similar use of this expression in the text (p. 32). Johannes de Muris in the third book of the *Quadripartitum numerorum*, doubtless familiar to Scheybl, entitles a similar chapter, *De multiplicatione et diuisione additorum et diminutorum* (Cod. Pal. Vind. 4770, fol. 230^b); see also Karpinski, *The "Quadripartitum numerorum" of John of Meurs, loc. cit.*, p. 110.

[3] In this discussion Scheybl uses the more common terms such as "composite numbers," "articles," and "digits," instead of "nodes" and "units" as in the corresponding section of Robert of Chester's text (pp. 88–96).

[4] *praeter* is used to suggest the idea of negative, illustrating in fact the Arabic conception of negative, namely that the "article" or ten (in this instance) is incomplete by the digit which is subtracted from it.

[5] Probably including all four types, $(x+a)(x+b)$, $(x+a)(x-b)$, $(x-a)(x+b)$, and $(x-a)(x-b)$. That multiplication is commutative was doubtless felt, even though not expressed.

diminuti fuerint in vtroque, tunc multiplicatio quaelibet debet addi. Si autem vnus fuerit additus et alter diminutus, tunc ista multiplicatio subtrahi debet a productis.

Sequitur huius rei vel regulae exemplum

Idem est multiplicare 8 cum 17, et multiplicare 10 praeter 2 cum 20 praeter 3, et hoc fiet sic. Multiplica 10 cum 20, et resultabant 200. Deinde multiplica praeter 2 cum 20, et resultabunt 40, subtrahenda à 200 et manent 160. Tertio multiplica 10 cum praeter 3, et resultabunt 30, subtrahenda a 160 et manent 130. Quartò multiplica praeter 2 cum praeter 3 et resultabunt 6 addenda, et veniunt 136. Igitur octo decies septies sunt 136.

Sequitur huius rei calculus.

10 praeter 2
20 praeter 3
200 praeter 40
praeter 30 et 6
136 et caet.

Aucta minuta simul minues; sed caetera iunges.
Aequantur numero radices censibus ambo,
In medio minues, alibi quod colliges addes.

Alia tria carmina, quae trium aequationum exempla proponunt

Census et 8 res 30 valent simul et tres.
Census cum seno, res quinque valere notato.
Aequiualent censum res 4 et duodenus.

Sequuntur nunc in declaratione exempla multiplicationis alia

Si cum 10 praeter rem debeas multiplicare 10.

Multiplica 10 cum 10, et fiunt 100. Deinde multiplica 10 cum re, et fiunt 10 res diminuendae. Dic ergo quòd resultent 100 denar. praeter 10 res.

Item secundò: si cum 10 et re multiplicares deberes 10.

Multiplica 10 cum 10 et re, et resultabunt 100 denarii et 10 res.

Item tertiò. Si cum 10 et re multiplicare deberes 10 et rem.

Multiplica 10 cum 10, et fiunt 100. Deinde multiplica rem cum 10, et fiunt 10 res addendae. Tertiò multiplica 10 cum re, et fiunt 10 res addendae. Quartò multiplica rem cum re, et fit census addendus. Erit ergo totum 100 dena., 20 res et census.

Item quartò. Si cum 10 praeter rem multiplicare debes 10 praeter rem.

Multiplica 10 cum 10, et fiunt 100. Deinde multiplica rem diminutam cum 10, et fiunt 10 res diminuendae. Manent autem 100 praeter 10 res. Tertiò multiplica 10 cum re diminuta, et fiunt 10 res diminuendae. Manent autem 100 praeter 20 res. Quartò multiplica rem diminutam cum re diminuta, et fit census addendus. Totum igitur erunt 100 denarii, census praeter 20 res.

the articles, or both negative, then the fourth product is positive. If, however, one term in one binomial is positive and the other corresponding term is negative then this product should be subtracted from the other products.

There Follows an Illustration of this Rule

To multiply 8 by 17 is the same as to multiply 10 less 2 by 20 less 3, and it will be done in this way. Multiply 10 by 20, giving 200; then multiply negative 2 by 20, giving 40, which subtracted from 200 leaves 160; in the third place multiply 10 by negative 3, giving 30, which subtracted from 160 leaves 130; in the fourth place multiply negative 2 by negative 3, giving positive 6, which being added gives 136. Therefore 8 times 17 gives 136.

The calculation follows:

$$\begin{array}{lrr} & 10 & -2 \\ & \underline{20} & \underline{-3} \\ 200 & -40 & \\ & -30 & +6 \\ \hline & & 136, \text{ etc.} \end{array}$$

Positives by negatives you subtract, but other products you add. When roots are equal to both number and squares, the half (is multiplied by the half) and you subtract, otherwise you add that which you obtain.[1]

Three other lines of verse which illustrate the three types of equations.

A square and 8 roots equal 33.
A square together with 6 equals 5 roots.
Four roots and 12 are equal to a square.

In further explanation: other examples of multiplication.

Suppose you are to multiply 10 by 10 less x.

Multiply 10 by 10, giving 100. Then multiply 10 by (negative) x, giving negative 10 x. Hence the product is 100 units less 10 x.

Second. Suppose you are to multiply 10 by 10 plus x.

Multiply 10 by 10 and by x, giving 100 units and 10 x.

Third. Suppose you are to multiply 10 plus x by 10 plus x.

Multiply 10 by 10, giving 100. Then multiply x by 10, giving positive 10 x. Thirdly, multiply 10 by x, giving positive 10 x. Fourthly, multiply x by x, giving positive x^2. Hence the product is 100 units, 20 x and x^2.

Fourth. Suppose you are to multiply 10 less x by 10 less x.

Multiply 10 by 10, giving 100. Then multiply negative x by 10, giving negative 10 x. The total, so far, is 100 less 10 x. Thirdly, multiply 10 by negative x, giving negative 10 x, or, in all, 100 less 20 x. Fourthly, multiply negative x by negative x, giving x^2 positive. The final product will be 100 units, and x^2 less 20 x.

---

[1] These verses are somewhat obscure. The meaning probably is that in the type $x^2 + n = bx$ the number is subtracted from the square of half the coefficient of x, whereas in the other two types of complete quadratics you subtract the number from this square.

Item quintò. Si cum 10 praeter rem debes multiplicare 10 cum re.

Multiplica 10 cum 10, et fiunt 100. Deinde multiplica rem cum 10, et fiunt 10 res addendae. Tertiò multiplica 10 cum praeter rem, et fiunt 10 res diminuendae. Quartò multiplica rem adiectam cum re diminuta, et sit census diminutus. Est ergo totum 100 dena. praeter censum.

Item sextò. Si cum 10 praeter rem deberes multiplicare rem.

Multiplica rem cum 10, et fiunt 10 res; deinde multiplica etiam rem cum re diminuta, et fit census diminutus. Est igitur totum, 10 res praeter censum.

Item septimò. Si cum 10 et re deberes multiplicare rem praeter 10.

Multiplica rem cum 10, et fiunt 10 res. Deinde multiplica praeter 10 cum 10, et fiunt 100 diminuenda. Tertiò multiplica rem cum re, et fit census addendus. Quartò multiplica praeter 10 cum re, et fiunt 10 res diminuendae. Est igitur totum, census praeter 100 denar.

Item octauò. Si cum 10 et medietate rei multiplicare deberes medietatem denarii praeter quinque res.

Multiplica medietatem denarii cum 10, et fiunt 5. Deinde multiplica praeter 5 res cum 10, et fiunt 50 res diminuendas. Tertiò multiplica medietatem denarii cum medietate rei, et fit quarta rei addenda. Quartò multiplica praeter 5 res cum medietate rei, quod est multiplicare duas res et dimidiam cum vna re, et fiunt duo census et $\frac{1}{2}$ diminuendi. Est igitur totum, 5 denarii praeter 2 census et $\frac{1}{2}$ et praeter 49 res et $\frac{3}{4}$ rei.

Item nono. Si cum vno denario praeter sextam denarii, multiplicare deberes denarium praeter sextam.

Multiplica denarium cum denario, deinde praeter sextam denarii cum denario, et fit denarius praeter sextam denarii. Tertiò multiplica denarium cum praeter sextam denarii, et fit vna sexta minuenda; manent ergo quatuor sextae. Quartò duc vel multiplica praeter sextam cum praeter sextam denarii, et fit vna trigesima sexta addenda. Est igitur totum viginti quinque trigesimae sextae vnius denarii.

Item decimò. Si cum 10 et re multiplicare deberes 10 et rem praeter 10. Idem est ac si cum 10 et re multiplicare deberes rem.

Et tunc fit totum, census et 10 res.

De radicum duplatione, triplatione et quadruplatione

Notandum quod cum census radicem, siue notam siue surdam duplare volueris, multiplica duo cum duobus, et cum producto multiplica censum. Et erit huius producti radix, dupla radix census propositi quae quaerebatur. Consimiliter si eius triplum habere volueris, multiplica ternarium cum ternario. Et si eius quadruplum, multiplica quatuor cum quatuor, et ita deinceps, et cum producto multiplica censum propositum, et erit producti radix, census propositi radix duplata, triplata vel quadruplata et caet.

Idem in fractionibus obseruandum

Vt si medietatem radicis habere consideras, multiplica medietatem cum medietate, et cum producto deinde censum, et tunc radix producti ostendit quaesitum. Similiter si tertiam partem habere volueris, multiplica tertiam partem cum tertia parte. Et ita deinceps.

Fifth. Given to multiply 10 plus x by 10 less x.

Multiply 10 by 10, giving 100. Then multiply x by 10, giving positive 10 x. Third, multiply 10 by negative x, giving negative 10 x. Fourth, multiply the positive x by the negative x, giving negative x^2. The sum total is 100 units less x^2.

Sixth. Given to multiply x by 10 less x.

Multiply x by 10, giving 10 x; then multiply x by negative x, giving negative x^2. The total is therefore 10 x less x^2.

Seventh. Given to multiply x less 10 by 10 plus x.

Multiply x by 10, giving 10 x. Then multiply negative 10 by 10, giving negative 100. Third, multiply x by x, giving positive x^2. Fourth, multiply negative 10 by x, giving negative 10 x. The total is therefore x^2 less 100 units.

Eighth. Given to multiply one-half a unit less 5 x by 10 plus $\frac{1}{2}x$.

Multiply the half unit by 10, giving 5. Then multiply negative 5 x by 10, giving negative 50 x. Third, multiply the half unit by $\frac{1}{2}x$, giving $\frac{1}{4}x$ positive. Fourth, multiply negative 5 x by $\frac{1}{2}x$, which is the same as multiplying $2\frac{1}{2}x$ by x, giving $2\frac{1}{2}x^2$ negative. The total therefore is 5 units less $2\frac{1}{2}x^2$, and less $49\frac{3}{4}x$.

Ninth. Given to multiply a unit less $\frac{1}{6}$ by a unit less $\frac{1}{6}$ of a unit.

Multiply a unit by a unit, then negative $\frac{1}{6}$ of a unit by a unit, giving a unit less $\frac{1}{6}$ of a unit Third, multiply a unit by negative $\frac{1}{6}$ of a unit, giving negative $\frac{1}{6}$ of a unit. These give $\frac{4}{6}$ of a unit. Fourth, multiply negative $\frac{1}{6}$ by negative $\frac{1}{6}$ of a unit, giving positive $\frac{1}{36}$. The total is therefore twenty-five thirty-sixths of a unit.

Tenth. Given to multiply 10 plus x less 10 by 10 plus x. This is the same as to multiply x by 10 plus x. Hence the product is x^2 plus 10 x.

Concerning the doubling, tripling, and quadrupling of radicals

It should be noted that when you wish to double the root of a square, either a definite root or a surd, you multiply 2 by 2 and multiply the given square by this product. The root of this product will be the double which you seek of the root of the proposed square. Similarly if you wish its triple, you multiply 3 by 3. And if you wish the quadruple of it, you multiply 4 by 4, and so on; and finally you multiply the proposed square by the product; the root of the product thus obtained will be the double, triple, or quadruple, etc., of the proposed square.

A similar note on fractions

If you have it in mind to obtain half of the root, multiply one-half by one-half, then the given square by the product. The root of this final product gives the desired result. Similarly if you wish to have a third part, multiply a third part by a third part, and so on.

De radicum diuisione

Nota, si radicem nouenarii in radicem numeri 4 diuidere volueris, diuide 9 in 4 et erit exiens 2 et quarta, atque exeuntis huius radix, quae est vnum et semis, erit numerus exiens diuisionis radicis in radicem. Quòd si duas radices nouenarii in radicem numeri 4 diuidere volueris, quaeras primò duplum radicis nouenarii secundum quod docuimus, et illud diuide in radicem numeri quatuor.

De radicum multiplicatione

Si radicem nouenarii cum radice numeri 4 multiplicare volueris, multiplica 9 cum 4, et producti radix est quod quaeris. Ita cum aliis.

Quòd si radicem tertiae cum radice medietatis multiplicare volueris, multiplica tertiam cum medietate, et producitur sexta, cuius radix est quod quaeris.

Quod si duas radices nouenarii cum tribus radicibus quaternarii multiplicare volueris, inquiras primò secundum quod supra docuimus censum cuius radix est duplum radicis numeri 9. Consimiliter inquiras censum cuius radix est triplum radicis numeri 4, et multiplica vnum censum cum altera et radix producti erit quaesitum.

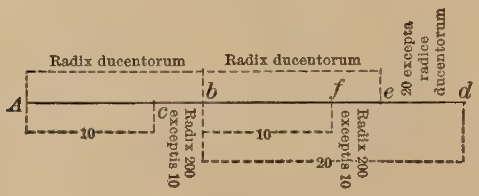
Sequuntur nunc quatuor aenigmata

Primum

Radix ducentorum subtractis 10, addita ad duplum subtracti, scilicet ad 20, subtracta ducentorum radice aequaliter erit 10.

Dicit aenigma, si a radice numeri 200 subtrahantur 10, id deinde quod relinquitur ad 20 addatur, ab hoc collecto postea radix numeri 200 auferatur, quòd subtractum tandem, hoc est 10, aequaliter maneant. Hoc autem sic probabitur. Sit linea *a b* radix ducentorum, a qua resecabo lineam *a c*, quae sit 10. Deinde lineae *a b* adiungam *b d*, lineam quae sit 20, a qua resecabo lineam *b e*, aequalem lineae *a b*, et a linea *b e* resecabo lineam *b f* aequalem lineae *a c*. Erit igitur linea *c b* aequalis lineae *f e*. Est autem linea *c b* radix ducentorum exceptis 10. Ac linea *e d*, 20 excepta radice ducentorum, et *c b* linea est aequalis *f e* lineae, igitur linea *f d* radix ducentorum erit exceptis 10, addita ad 20 excepta radice ducentorum. Quòd autem haec linea *f d* sit praecise 10, probabo. Linea *b d* est 20 et cum *b f* linea aequalis sit lineae *a c*, quae 10 posita est, oportet igitur quod et *f d* linea propter aequalitatem 10 sit, quod erat probandum.

Sequitur figura.



On the division of radicals

Notice, if you wish to divide the root of 9 by the root of 4, divide 9 by 4, giving $2\frac{1}{4}$. Of the result take the root, which is $1\frac{1}{2}$, and the resulting number will be the quotient of the root divided by the root. But if you wish to divide two roots of 9 by the root of the number 4, you seek first the double of the root of 9 according to that which we have explained, and divide the product by the number 4.

On the multiplication of radicals

If you wish to multiply the root of the number 9 by the root of the number 4, multiply 9 by 4 and the root of the product is that which you seek. Other multiplications are similar.

Thus if you wish to multiply the root of $\frac{1}{3}$ by the root of $\frac{1}{2}$, you multiply $\frac{1}{3}$ by $\frac{1}{2}$, giving $\frac{1}{6}$, and the root of this is that which you seek.

In the same way if you wish to multiply two roots of 9 by three roots of 4, you first try to find, as we have explained above, the square whose root is twice the root of the number 9. Similarly you try to find the square whose root is three times the root of the number 4, and you multiply the one square by the other. The root of the product will be that which you seek.

Now follow four problems[1]*: First problem*

The root of 200 less 10, added to the double of that which is subtracted, in other words to 20, will be equal to 10 when the root of 200 is subtracted.[2]

The problem says that if from the root of the number 200, 10 is subtracted, and if then 20 is added to that which remains, and if from this sum the root of the number 200 is taken away, we finally have left that which was subtracted, namely 10. This will be proved in the following manner:

Let $a\,b$ represent the root of 200, and from it cut off the line $a\,c$, representing 10. Then to the line $a\,b$ join $b\,d$, a line which is 20 (units in length). From this cut off $b\,e$, equal to the line $a\,b$, and from $b\,e$ cut off the line $b\,f$ equal to the line $a\,c$; the line $c\,b$ will be equal to the line $f\,e$. Moreover the line $c\,b$ is the root of 200 less 10, and the line $e\,d$ is 20 less the root of 200. The line $c\,b$ is equal to the line $f\,e$, whence the line $f\,d$ equals the root of 200 less 10 plus 20 less the root of 200. Moreover I shall prove that this line $f\,d$ is exactly 10. The line $b\,d$ is 20, and as $b\,f$ is equal to the line $a\,c$, which was taken as 10, it follows then that the line $f\,d$, on account of this equality, is 10, which was to be proved.

---

[1] This and the following three problems are not given by Robert of Chester, although the Libri and Arabic versions give the first, second, and fourth problems, and the Boncompagni text the first and second. Scheybl was probably familiar with the other Latin text, the one published by Libri. The geometrical figure in the Libri text is an L-shaped figure, and the same reversed in the Arabic text. In each of these the vertical line represents $\sqrt{200}$.

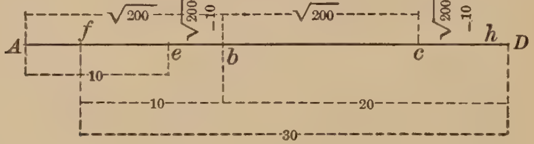
[2] The geometrical demonstration of this simple algebraic addition is relatively complicated; in algebraic symbolism, $\sqrt{200} - 10 + 20 - \sqrt{200} = 10$, and similarly below.

Secundum aenigma

Radix ex 200, subtractis 10, diminuta a duplo subtracti, scilicet a 20 excepta radice ex 200 est triplum subtracti, scilicet 30, praeter duas radices ducentorum.

Dicit aenigma, si a radice numeri 200 subtrahantur 10, id deinde quod relinquitur a 20 subtrahatur, atque ex hoc residuo postea radix numeri 200 auferatur, quod tandem triplum subtracti, hoc est 30, et duae radices numeri 200 maneant. Hoc autem sic probabitur. Sit linea *a b* radix ducentorum, et sit *b c* sibi aequalis, *b d* verò 20, atque postea *a e* 10. Secabo autem de linea *b a* lineam *b f* aequalem *a e*. Erit igitur tota linea *d f* 30. Porro ex linea *c d* secabo lineam *c h* aequalem lineae *e b*. Et quoniam linea *e b* est radix ducentorum exceptis 10, cum linea *c d* sit 20 excepta radice ducentorum, facta subtractione *e b* lineae a linea *c d*, manet linea *h d* quae erit 30 exceptis duabus radicibus ducentorum. Quòd autem haec eadem linea *h d* praecise tantum sit, probabo. Linea *f d* est 30 et cum linea *b c* radix sit ducentorum, linea deinde *a e* lineae *f b*, linea etiam *e b* lineae *c h* aequalis; et aggregatum ergo ex lineis *f b* et *c h* radix ducentorum erit. Manet autem subtractione facta linea *h d*. Probatum ergò quod probandum erat.

Sequitur figura.



Tertium aenigma

Duae radices alicuius numeri sunt vna sui quadrupli.

Hoc satis patet ex eo quòd quadratum est quadruplum ad aliud quadratum, cuius costa est dupla ad costam alterius.

Quartum aenigma

Centum et census exceptis 20 radicibus adiuncti ad 50 et ad 10 radices exceptis duobus censibus, sunt 150 exceptis censu et 10 radicibus.

Probatio. Vbi enim subtrahuntur 20 radices, et adduntur 10 radices, idem est ac si solum 10 radices subtraherentur, et vbi additur census et subtrahuntur duo census, idem est ac si solum modo vnus census subtraheretur, ex quo patet propositum.

Sequuntur nunc harum regularum exercitii maioris causa quaestiones decem et octo

Prima

Diuisi 10 in duas partes et multiplicaui vnam partem cum altera, et postea vnam cum seipsa, et produxit haec multiplicatio partis cum seipsa tantum quantum multiplicatio vnius partis cum altera quater: quae igitur fuerunt partes?

Second problem

The root of 200 less 10, taken from twice that which is subtracted, namely from 20, less the root of 200, is the triple of that which was first subtracted, namely 30, less two roots of 200.

The problem states that when you subtract 10 from the root of the number 200, and this in turn from 20, and from the remainder you take the root of the number 200, then the triple, 30, of the quantity originally subtracted, 10, less [1] two roots of the number 200 remains. This will be proved in the following manner:

Let the line *a b* represent the root of 200, and let *b c* be equal to it. Let *b d* be 20, and further *a e* be 10. Cut off from the line *b a* the line *b f* equal to *a e*. Then the whole line *d f* is 30. Further, from the line *c d* cut off the line *c h* equal to the line *e b*. Since the line *e b* is the root of 200, less 10, and the line *c d* is 20 less the root of 200, making the subtraction of the line *e b* from the line *c d*, you obtain the line *h d* which is then 30 less two roots of 200.

Moreover that this line *h d* is exactly of that length, I will prove. The line *f d* is 30, and since the line *b c* is the root of 200, the line *a e* equals *f b*, and also the line *e b* equals *c h*. Hence by adding the two lines *f b* and *c h* we shall obtain the root of 200. The subtraction being made there remains the line *h d*. That therefore which was to be proved, has been proved.

Third problem

Two roots of any number make one of the quadruple.

This is sufficiently evident from the fact that any square is the quadruple of another square if the side of the first is double the side of the other.

Fourth problem

One hundred and x^2 less $20\,x$, added to 50 plus $10\,x$ less $2\,x^2$, gives 150 less x^2 and less also $10\,x$.

Proof. Where 20 roots are subtracted and 10 roots are added, the result is the same as if only 10 roots were subtracted. Also where a square is added and two squares are subtracted, the result is the same as if only one square were subtracted. From this follows that which was proposed.

There follow eighteen questions for greater practice in these rules [2]

First question

I divided 10 into two parts and multiplied one part by the other, then I multiplied one part by itself. This product of one part by itself gave as much as four times the product of one part by the other. What were the two parts? The answer by the rule is 8 and 2.

---

[1] Not "and," as in the text.

[2] The eighteen problems are all, with minor changes, from Al-Khowarizmi's algebra, but problems 10, 11, and 14, which follow, Robert did not include in his text:

(10) $\frac{x}{10 - x} + \frac{10 - x}{x} = 2\frac{1}{6}$; (11) $\frac{1}{2} \cdot \frac{5\,x}{10 - x} + 5\,x = 50$; (14) $\frac{x\,(10 - x)}{10 - 2\,x} = 5\frac{1}{4}$.

Respondetur ex regula: 8 et 2.

Regula. Vnam sectionem ponas rem et alteram 10 praeter rem. Multiplica deinde rem cum 10 praeter rem, et fiunt 10 res praeter censum. Postea quadrupla hoc totum et producuntur 40 res praeter 4 census, et illud aequatur productum ex multiplicationis rei cum re, quod est census. Ex hypothesi ergo quaestionis census aequalis est 40 rebus praeter 4 census. Vnus igitur census aequatur 8 rebus. Sed ille est 64, radix vero numerus 8. Vna igitur sectio est 8, et per consequens reliqua 2. Ista autem quaestio reducitur ad caput vbi census radicibus aequatur.

Quaestio secunda

Diuisi 10 in duas partes, et multiplicaui, 10 cum seipso, fuitque resultans ex tali multiplicatione aequale vni duarum partium multiplicatae cum seipsa bis et septem nonis multiplicationis vnius: quae igitur fuerunt partes?

Respondetur ex regula: 6 et 4.

Regula. Ponatur vna duarum partium res, quae multiplicetur cum seipsa, et fit census. Is census dupletur et addantur septem nonae census et resultant 2 et $\frac{7}{9}$, hoc est viginti quinque nonae, census. Igitur si census esset nouem partes, istae essent quintae pars et quatuor quintae vnius quintae totius, quod esset viginti quinque nonae. Sed ex hypothesi totum multiplicatum vel productum est 100. Accipiatur igitur quinta pars et $\frac{4}{5}$ quintae partis ex numero 100, erunt autem haec 36. Atque tantus est census, cuius radix scilicet 6 vna diuisionis pars in proposita quaestione. Reducitur aut haec quaestio ad caput in quo census numero aequatur.

Quaestio tertia

Diuisi 10 in duas partes, et diuisi vnam partem in aliam et exiuerunt 4. Quaeritur, quae sint partes.

Respondetur ex regula: 8 et 2.

Regula. Sit vna pars res, et altera 10 praeter rem. Si igitur ex diuisione 10 praeter rem in rem exierint 4. Igitur ex multiplicatione 4 cum re resultabunt 10 praeter rem. Quatuor igitur res sunt 10 praeter rem, atque ita quinque res decem praecise. Ergò vna res 2. Reducitur autem haec quaestio ad illud caput vbi res numero aequatur.

Quaestio quarta

Multiplicaui tertiam partem census et denarium vnum cum quarta parte census et cum denario vno, et prouenerunt 63: quantus igitur est census?

Respondetur ex regula: 24.

Regula. Multiplica tertiam partem cum quarta, et producetur pars duodecima census; deinde multiplica vnum denarium cum $\frac{1}{4}$ et producetur $\frac{1}{4}$ rei; multiplica etiam tertiam cum denario et producetur $\frac{1}{3}$ rei, atque tandem denarium cum denario, et producetur denarius. Est autem totum multiplicationis productum $\frac{1}{12}$ census, quarta rei et tertia rei atque denarius, aequantes 63 denarios. Demas igitur primò denarium vtrique, casum deinde hunc, vt quidem semper

Rule. You would let one part equal x and the other $10 - x$; then multiply x by $10 - x$, giving $10x - x^2$. Afterwards take the quadruple of the total, which gives $40x - 4x^2$. This equals the product of x multiplied by x, which is x^2. Now under the supposition of our question, x^2 is equal to $40x - 4x^2$. Hence x^2 equals $8x$, whence x^2 is 64, and the root of it is the number 8. One of the parts of 10 is then 8, and consequently the remaining part is 2. This problem, then, is reduced to the chapter wherein a square is equal to roots.

Second question

I divided 10 into two parts, and multiplied 10 by itself. The result of this multiplication was equal to the product of one of these parts multiplied by itself and by two and seven-ninths. What are the parts? The answer in accordance with the rule is 6 and 4.

Rule. Let x represent one of the two parts, which multiplied by itself becomes x^2. Double this square and add to it seven-ninths of one square, giving $2\frac{7}{9}x^2$ or $\frac{25}{9}x^2$. Hence if the whole square consists of nine parts (ninths) that will be the fifth part and four-fifths of one-fifth of the whole, which is twenty-five ninths. But under our supposition the product is equal to 100. Therefore take one-fifth, and $\frac{4}{5}$ of one-fifth, of the number 100, namely 36, which is the value of x^2. The root of this, namely 6, is one part of 10 in the proposed problem. This question then is referred back to the chapter in which a square is equal to a number.

Third question

I divided 10 into two parts in such a way that the one part divided by the other equaled 4. The question is, what are the parts? The answer in accordance with the rule is 8 and 2.

Rule. Let x represent one part, and $10 - x$ the other. Now if by the division of $10 - x$ by x there results 4, it follows that by the multiplication of 4 by x you will obtain $10 - x$; $4x$, then, equals $10 - x$, and hence 10 precisely equals $5x$. The value of x is 2. This question is referred back to that chapter in which a root is equal to a number.

Fourth question

I multiplied one-third x^2 plus one unit by one-fourth x^2 plus one unit and the result was 63. What is the value of x^2? The answer in accordance with the rule is 24.

Rule. Multiply one-third (x) by one-fourth (x), and one-twelfth x^2 will be the result; then multiply one unit by $\frac{1}{4}$ (x), and $\frac{1}{4}x$ will be the result; again multiply $\frac{1}{3}$ (x) by one unit, giving $\frac{1}{3}x$, and finally a unit by a unit, giving a unit. The sum total of this multiplication is $\frac{1}{12}x^2 + \frac{1}{4}x + \frac{1}{3}x + 1$ which equals 63 units. Take one unit from both sides, and then, according to the rule that it is always necessary to reduce to an integral square, the whole

necesse, ad censum integrum reducas, totum scilicet, vel singula, cum 12 multiplicando, et venient 1 census et 7 res aequales 744 denariis. Per primam igitur regulam dimidia radices, et medietatem cum seipsa multiplica et producentur $12\frac{1}{4}$; quibus additis ad 744 resultabunt $756\frac{1}{4}$. Hinc elice radicem quadratam, veniunt $27\frac{1}{2}$, atque ab his medietatem radicum 3 scilicet et $\frac{1}{2}$ subtrahe et relinquuntur 24, quantitas census. Haec autem quaestio ad caput in quo census et res numero aequales sunt, reducitur.

Quaestio quinta

Diuisi 10 in duo, et multiplicaui vtrumque cum seipso, et aggregatum quadratorum fuit 58: quae igitur sunt partes?

Respondetur ex regula: 7 et 3.

Regula. Multiplica 10 praeter rem cum seipsa, et producentur 100 den. et census, exceptis 20 rebus; deinde multiplica rem cum seipsa, et producetur census. Habes igitur 100 den. et 2 census, exceptis 20 rebus, quae omnia aequantur 58. Restaurando igitur 20 res diminutas, patet quòd 100 denarii et 2 census valent 58 den. et 20 res, atque reducendo ad vnum censum, veniunt 50 den. et 1 census aequales 29 den. et 10 rebus. Postea vero subtrahendo 29 a 50 relinquuntur 21 den. et vnus census aequales 10 rebus. Operare igitur per caput secundum, et patet quòd altera pars 7 et altera 3 sint. Haec autem quaestio reducitur ad caput in quo census et numeri rebus aequantur.

Questio sexta

Multiplicaui tertiam census cum quarta eiusdem, et prouenerunt census et 24 denarii: quantus igitur est census?

Respondetur ex regula: 24.

Regula. Quia satis nosti, quòd ex multiplicatione tertiae partis rei cum quarta parte rei, duodecima proueniat pars census, quae in hoc casu vni rei et 24 denariis aequalis est. Integra igitur censum, multiplicando totum cum 12, et perficias quod census vnus 12 rebus et 288 denariis aequetur. Age igitur, multiplicando medietatem radicum cum seipsa, et addendo productum seu quadratum ad 288, et prouenient 324, cuius radix est 18, quae addita ad medietatem radicum, veniunt 24, census propositae quaestionis. Reducitur autem haec quaestio ad caput in quo radices et numeri substantias coaequant.

Quaestio septima

Diuisi 10 in duo, vt multiplicatio vnius cum altero producat 24: quae igitur sint partes, quaeritur.

Respondetur ex regula: 6 et 4.

Regula. Scias vnam partem esse rem, et alteram 10 praeter rem. Multiplica igitur vnam cum altera, et producuntur 10 res praeter censum, quae valent 24 denarios. Restaurando igitur dic, quòd 10 res valeant 24 den. et vnum censum. Age nunc per caput quo census et numeri rebus aequantur, et patebit quòd vna pars diuisionis 6 et altera 4 sint.

or each part is now multiplied by 12; you will obtain $x^2 + 7x$ equal to 744 units. By the first rule you take one-half of the roots and multiply the half by itself, obtaining $12\frac{1}{4}$, which being added to 744 will make a total of $756\frac{1}{4}$. Take the square root of this and you obtain $27\frac{1}{2}$. Now from $27\frac{1}{2}$ subtract the half of the roots, namely 3 and $\frac{1}{2}$, and 24 remains as the value of the square. This question is referred back to the chapter in which squares and roots are equal to a number.

Fifth question

I divided 10 into two parts and I multiplied each part by itself, and the sum of the squares was 58. What are the two parts? The answer in accordance with the rule is 7 and 3.

Rule. Multiply $10 - x$ by itself, and you obtain 100 units $+ x^2 - 20x$; then multiply x by itself, and x^2 is obtained. You have then 100 units $+ 2x^2 - 20x$, a total equal to 58. By restoration then of the negative $20x$, it follows that 100 units $+ 2x^2$ equal 58 units $+ 20x$, and reducing this to one square, 50 units $+ x^2$ are found equal to 29 units $+ 10x$. Accordingly, by subtracting 29 from 50, 21 units $+ x^2$ remain equal to $10x$. Proceed therefore according to the second chapter, and it becomes clear that the parts are 7 and 3. This problem is referred back to the chapter in which squares and numbers are equal to roots.

Sixth question

I multiplied $\frac{1}{3}x$ by $\frac{1}{4}$ of it and obtained $x + 2$ units. What is the value of x? The answer in accordance with the rule is 24.

Rule. Since you well know that the product of $\frac{1}{3}x$ by $\frac{1}{4}x$, is $\frac{1}{12}x^2$, it follows that $\frac{1}{12}x^2$, in this instance, equals $x + 24$ units. Make the square whole by multiplying all by 12, and you find that $x^2 + 12x$ is equal to 288 units. Treat this (equation), then, by multiplying the half of the roots by itself and adding the product, or square, to 288. You obtain 324, whose root is 18; this added to the half of the roots gives 24, the square which was sought in the proposed question. This problem is referred back to the chapter in which roots and numbers are equal to squares.

Seventh question

I divided 10 into two parts in such a way that the product of one by the other is 24. The question is, what are the parts? The answer in accordance with the rule is 6 and 4.

Rule. You know that you can let the one part equal x, and the other $10 - x$. Then multiply the one by the other and you obtain $10x - x^2$, which equals 24 units. Now by restoration say that $10x$ amounts to 24 units $+ x^2$. Treat this then according to the chapter in which squares and numbers are equal to roots, and it will be clear that one part is 6 and the other 4.

Quaestio octaua

Diuisi 10 in duo, atque vtroque multiplicata cum seipso, subtraxi minus de maiori et manserunt 40. Quaeritur de duobus.

Respondetur ex regula: 7 et 3.

Regula. Multiplica rem cum re, et proueniet census. Deinde multiplica etiam 10 praeter rem cum 10 praeter rem, et prouenient 100 dena. et census, exceptis 20 rebus. Subtrahe igitur censum a 100 et censu exceptis 20 rebus, et manent 100 exceptis 20 rebus quae aequantur 40 dena. Restaurando igitur dic, quòd 100 denarii aequentur 40 denariis et 20 rebus. Subtrahendo deinde 40 a 100 patet quòd 60 denarii aequantur 20 rebus; tres igitur denarii rei vni; estque ternarius vna diuisionis pars, quare 7 altera.

Quaestio nona

Diuisi 10 in duo et vtramque multiplicaui cum seipso, adiunxi deinde producta simul et insuper addidi differentiam horum duorum antequam multiplicarentur cum seipsis, et prouenerunt 54. Quaeritur.

Respondetur ex regula: 6 et 4.

Regula. Multiplica rem cum re, deinde etiam 10 praeter rem cum 10 praeter rem, et adde simul producta, et veniunt 100 denarii et 2 census, exceptis 20 rebus. Cum igitur superfluum inter partes, vel partium differentia, sit 10 praeter 2 res, adiecto hoc superfluo, erit totum 110 dena. et 2 census exceptis 22 rebus, quae omnia aequantur 54 denariis. Dic igitur, res integrando, quòd 110 den. et 2 census aequentur 54 denariis et 22 rebus. Reducendo deinde ad vnum censum, dic, quòd 55 denarii et 1 census aequentur 27 denariis et 11 rebus; subtrahendo verò 27 a 55, dic quòd 28 denarii et vnus census aequentur 11 rebus. Age igitur per caput quo census et numeri rebus aequantur, et patebit quòd vna pars diuisionis 6 et altera 4 erunt.

Quaestio decima

Diuisi 10 in duo et vtrunque diuisi in alterum, et aggregatum ex diuisionibus, id est exeuntium, fuit 2 et vna sexta. Quaeritur.

Respondetur ex regula: 4 et 6.

Regula. Aggregatum ex multiplicatione vtriusque cum seipso, aequum est multiplicationi vnius cum altero et producti cum aggregato ex diuisionibus vtriusque in alterum quod in hoc casu est 2 et vna sexta. Ideo multiplica 10 praeter rem cum seipso, et re cum re, et prouenient 100 et 2 census, exceptis 20 rebus, et hoc totum aequatur multiplicationi rei cum 10 praeter rem, et producti cum 2 et vna sexta. Sed multiplicatio rei cum 10 praeter rem producit 10 res praeter censum, quibus cum 2 et $\frac{1}{6}$ multiplicatis 21 res et $\frac{2}{8}$ rei praeter $2\frac{1}{6}$ census resultabunt, quae aequantur 100 dena. et 2 censibus exceptis 20 rebus. Restaurando igitur census et res, veniunt 41 res et $\frac{2}{3}$ rei aequales 100 denariis et 4 censibus cum sexta parte census. Reduc igitur totum ad vnum censum sic. Consideretur censum esse 6, et erunt 4 census et sexta, 25. Atque huius vnus census, 6 scilicet, est vna quinta et quinta quintae. Totius igitur quod habes

Eighth question

I divided 10 into two parts, and each being multiplied by itself, I subtracted the smaller from the larger, and 40 remained. The question is as to the parts? The answer in accordance with the rule is 7 and 3.

Rule. Multiply x by x, and you obtain x^2. Then multiply also $10 - x$ by $10 - x$ and you obtain 100 units $+ x^2 - 20x$. Subtract, therefore, x^2 from $100 + x^2 - 20x$, and you have $100 - 20x$, which equals 40 units. By restoration say, then, that 100 units are equal to 40 units and $20x$. Then by subtracting 40 from 100 it is plain that 60 units are equal to $20x$, and hence 3 units to one x. Three is one part; hence 7 is the other.

Ninth question

I divided 10 into two parts, and multiplied each part by itself; then I added these products together, and the difference between these two, before each was multiplied by itself, and the result was 54. The question is stated. The answer in accordance with the rule is 6 and 4.

Rule. Multiply x by x, and also $10 - x$ by $10 - x$; add the products and you obtain 100 units $+ 2x^2 - 20x$. Since the excess of one part over the other, or the difference of the parts, is $10 - 2x$, when this excess is added we shall have 110 units $+ 2x^2 - 22x$ as the sum total, which equals 54 units. Say therefore, by adding the $22x$, that 110 units $+ 2x^2$ equals 54 units $+ 22x$. Then by reduction to one square, you say that 55 units $+ x^2$ equal 27 units and $11x$. By subtracting 27 from 55, you say that $x^2 + 28$ units equals $11x$. Treat this by the chapter in which squares and numbers equal roots, and it will be plain that one part is 6, and the other 4.

Tenth question

I divided ten into two parts, and I divided each of these by the other; the sum of the two quotients, that is to say, the result, is two and one-sixth. The question is stated. The answer in accordance with the rule is 4 and 6.

Rule. The sum of the products of each (of the parts) multiplied by itself is equal to the product of the one by the other when this product is multiplied by the sum of the quotients of each of the divisions, which in this case is $2\frac{1}{6}$. Hence you multiply $10 - x$ by itself, and x by x, obtaining $100 + 2x^2 - 20x$. This total is equal to the product of x by $10 - x$, multiplied by $2\frac{1}{6}$. But the product of x by $10 - x$ gives $10x - x^2$, which being multiplied by $2\frac{1}{6}$ there results $21x + \frac{2}{3}x - 2\frac{1}{6}x^2$ equal to 100 units $+ 2x^2 - 20x$. By restoration, then, of the squares and the roots, $41x + \frac{2}{3}x$ are obtained equal to 100 units $+ 4x^2 + \frac{1}{6}x^2$. Reduce the whole then to one square in the following manner: If a square is supposed to be 6, then $4\frac{1}{6}$ squares would be 25; of this, one square, namely 6, is $\frac{1}{5}$ and $\frac{1}{5}$ of $\frac{1}{5}$. Take therefore of everything which you have $\frac{1}{5}$ and $\frac{1}{5}$ of $\frac{1}{5}$, and it will be plain that $10x$ is equal to $x^2 + 24$ units. Proceed then by that chap-

accipe quintam partem et quintam quintae, et patebit, quòd 10 res vni censui et 24 dena̋riis aequentur. Age igitur per caput, quo census et numeri rebus aequantur, multiplicando medietatem radicum cum seipsa, et producuntur 25; a quibus subtrahe 24 et manet vnitas, cuius radix est vnitas. Hanc radicem subtrahe a medietate radicum, et manent 4, quae sunt vna diuisionis pars. Et nota, quòd cum illud quod resultat ex diuisione primae partis in secundam alicuius totius, multiplicetur cum illo quod resultat ex diuisione secundae partis in primam, illud quod prouenit semper idem sit.

Quaestio vndecima

Diuisi 10 in duas partes et multiplicaui vnam illarum cum 5 et productum diuisi in reliquam partem, et exeuntis medietatem addidi ad productum ex multiplicatione primae partis cum 5, et totum aggregatum fuit 50. Quaeritur.

Respondetur ex regula: 8 et 2.

Regula. Ex 10 accipias rem; hanc multiplicabis cum 5, et fient 5 res. Deberes diuidere 5 res in 10 praeter rem, et addere medietatem exeuntis ad 5 res. Sed hoc idem est, ac si diuideres medietatem 5 rerum in 10 praeter rem, et adderes exeuntem totum ad 5 [res]. Vtraque enim operatio producitur 50. Si ergo diuidas 2 res et semissem in 10 praeter rem, exeunt 50 praeter 5 res, eo quod addito producto ad 5 res, prouenirent 50. Cum igitur constet quod multiplicato illo quod prouenit ex diuisione cum diuisore redeat census tuus, qui est 2 res et semis. Multiplica igitur 10 praeter rem cum 50 exceptis 5 rebus, prouenient 500 et 5 census exceptis 100 rebus, quae omnia aequantur duabus rebus et semissi. Reduc igitur totum ad vnum censum accipiendo quintam partem totius, et patebit, quòd 100 et census, exceptis 20 rebus aequentur medietati rei. Restaurando igitur, dic quod 100 et census aequentur 20 rebus et medietati rei. Age igitur per caput quo census et numeri rebus aequentur, multiplicando medietatem rerum cum se, et prouenient 105 et $\frac{1}{16}$; a quibus subtractis 100 manent $5\frac{1}{16}$, cuius radix est $2\frac{1}{4}$; quibus subtractis a medietate radicum, et manent 8 vna diuisionis pars.

Quaestio duodecima

Diuisi 10 in duos partes, et multiplicatio vnius partis cum seipsa produxit numerum continentem alteram partes octogesies semel. Quaeritur de partibus.

Respondetur ex regula: 9 et 1.

Regula. Multiplica 10 praeter rem cum se, et fient 100 et census, praeter 20 res quae aequantur 81 rebus. Restaurando igitur dic, quòd 100 et census aequentur 101 rebus. Age nunc per caput quo census et numeri rebus, aequantur, et veniet tandem vnitas, vna diuisionis pars.

Quaestio decima tertia

Duo sunt census, quorum maior excedit minorem in duobus, diuisi autem maiorem in minorem, et exibat medietas maioris. Quaeritur.

Respondetur ex regula: 2 census minor et 4 maior.

Regula. Pone rem pro censu, et dic, quia res minor diuidens maiorem producit

ter in which a square and numbers are equal to roots. Multiplying one-half of the roots by itself you have 25; from this subtract 24 and there remains one, of which the root is one. Subtract this root from the half of the number of the roots, and four remains as the value of one part. Now note that when the quotient obtained by dividing the first part by the second part of any whole is multiplied by the quotient of the second part by the first, that which is obtained is always the same.

Eleventh question

I divided ten into two parts and I multiplied one of these by five, and the product I divided by the other part; one-half of this result I added to the product of the first part multiplied by 5, and the sum total was fifty. The question is stated. The answer in accordance with the rule is 8 and 2.

Rule. You may take x as one part of 5; this you will multiply by 10, giving $5x$. You should divide $5x$ by $10-x$, and add $\frac{1}{2}$ of the quotient to $5x$. But this is the same as if you should divide $\frac{1}{2}$ of $5x$ by $10-x$ and add the total result to $5x$; either operation gives 50. If therefore you divide $2\frac{1}{2}x$ by $10-x$, $50-5x$ is obtained, since when $5x$ was added to the quotient, the sum was given as 50. Moreover it should be evident that the product of the result of any division multiplied by the divisor gives your quantity (the dividend), which is $2\frac{1}{2}x$. Therefore multiply $10-x$ by $50-5x$, obtaining $500+5x^2-100x$, all of which is equal to $2\frac{1}{2}x$. Reduce the whole then to one square by taking the fifth part of the whole, and it will be clear that $100+x^2-20x$ equals $\frac{1}{2}x$. By restoration then say that $100+x^2$ equals $20x$ and $\frac{1}{2}x$. Operate then by the chapter in which squares and numbers are equal to roots. Multiplying one-half of the roots by itself, 105 and $\frac{1}{16}$ is obtained; from this you subtract 100, leaving $5\frac{1}{16}$, of which the root is $2\frac{1}{4}$, and this being subtracted from one-half of the roots 8 remains as the value of one part.

Twelfth question

I divided 10 into two parts, and the product of one of these parts by itself contained the other part 81 times. The question is as to the parts. The answer in accordance with the rule is 9 and 1.

Rule. Multiply $10-x$ by itself, giving $100+x^2-20x$, which is equal to $81x$. Then by restoration, say that $100+x^2$ equals to $101x$. Operate now by the chapter in which squares and numbers are equal to roots, and unity will finally appear as the value of one part.

Thirteenth question

There are two quantities of which the greater exceeds the less by two. I divided the greater by the less and the quotient was one-half the greater quantity. The question is stated. The answer in accordance with the rule is 2, for the smaller quantity, and 4 for the larger.

medietatem rei maioris, ideo econtra res minor multiplicata cum medietate rei maioris producit rem maiorem, et duo multiplicata cum medietate rei maioris, producit rem maiorem. Binarius igitur est res minor, et quaternarius maior.

Quaestio decima quarta

Diuisi 10 in duas partes, et multiplicaui vnam partem cum altera, et productum diuisi in differentiam inter partes, et resultarunt $5\frac{1}{4}$. Quaeritur, quae sint partes.

Respondetur ex regula: 3 et 7.

Regula. Multiplica rem cum 10 praeter rem, et fient 10 res excepto censu; deinde diuide 10 res excepto censu in 10 exceptis 2 rebus, quae sunt differentia inter partes, et exeunt $5\frac{1}{4}$. Si igitur econtrà multiplicaueris $5\frac{1}{4}$ cum 10, exceptis 2 rebus, prouenient 10 res excepto censu. Multiplica igitur $5\frac{1}{4}$ cum 10, exceptis 2 rebus, et producuntur 52 den. et semis praeter 10 res et semissem. Atque haec omnia aequantur 10 rebus, excepto censu. Dic igitur, restaurando res et denarios, quòd $20\frac{1}{2}$ res aequentur $52\frac{1}{2}$ denariis et vni censui. Age igitur per caput quo census et numeri rebus aequantur, multiplicando medietatem radicum in se, et prouenient $105\frac{1}{16}$ et quae sequuntur et caet.

Quaestio decima quinta

Quatuor radices census multiplicatae cum quinque radicibus eiusdem census, producunt duplum census et 36. Quaeritur de censu.

Respondetur ex regula: 2.

Regula. Multiplica 4 res cum 5 rebus, et fiunt 20 census qui aequantur 2 censibus et 36 denariis. Diuide ergo 36 in 18 et exeunt 2. Atque tantus est census, quod examinari poterit.

Quaestio decima sexta

Subtraxi a censu eius vnam tertiam et tres denarios, multiplicaui deinde residuum cum seipso, restituit haec multiplicatio ipsum censum. Quantus igitur census sit, quaeritur.

Respondetur ex regula: 9.

Regula. Subtracta tertia et tribus a tribus tertiis rei, manent $\frac{2}{3}$ rei praeter 3 dena. quae sunt radix census. Multiplica igitur $\frac{2}{3}$ rei praeter 3 den. cum se, et producentur $\frac{4}{9}$ census et 9 den. praeter 4 res, et illud aequatur radici. Ergo $\frac{4}{9}$ census et 9 denarii valent 5 res. Reducas $\frac{4}{9}$ ad vnum censum, eundem, denarios etiam et res cum duobus et quarto multiplicando, et inuenies, quòd census et $20\frac{1}{4}$ denarii aequantur 11 rebus et $\frac{1}{4}$. Age igitur per caput quo census et numeri rebus aequantur. Accipiendo medietatem radicum quae est $5\frac{5}{8}$ et multiplicando eam cum seipsa, et fiunt $31\frac{41}{64}$, de quibus subtrahe $20\frac{1}{4}$ et manent $11\frac{25}{64}$, cuius radix est $3\frac{3}{8}$, quam ad medietatem radicum adde, quia non per subtractionem non deunies ad intentum, et veniunt 9, census qui quaerebatur.

Rule. Let x represent the one quantity, and say, since the lesser quantity divided by the greater gives one-half of the greater, that consequently the lesser multiplied by one-half of the greater gives the greater quantity. But two times one-half of the greater quantity gives also the greater quantity. Therefore 2 is the value of the lesser quantity, and 4 is the greater.

Fourteenth question

I divided 10 into two parts, and I multiplied one by the other and divided the product by the difference between the two, obtaining $5\frac{1}{4}$ as the result. The question is, what are the parts? The answer is 3 and 7.

Rule. Multiply x by $10 - x$, giving $10x - x^2$; then divide $10x - x^2$ by $10 - 2x$, which is the difference between the parts, and $5\frac{1}{4}$ is obtained. Now, on the other hand, if you multiply $5\frac{1}{4}$ by $10 - 2x$, you will obtain $10x - x^2$. Hence multiply $5\frac{1}{4}$ by $10 - 2x$, which gives $52\frac{1}{2}$ units $- 10\frac{1}{2}x$, all of which is equal to $10x - x^2$. Observe, then, that by restoring to the $10x$ and to the units (the quantities, x^2 and $10\frac{1}{2}x$ respectively, which are subtracted from them) $20\frac{1}{2}x$ is equal to $52\frac{1}{2}$ units $+ x^2$. Operate therefore by the chapter in which squares and numbers are equal to roots, multiplying the half of the roots by itself, and there will result $105\frac{1}{16}$, etc.

Fifteenth question

Four roots of a square multiplied by five roots of the same square give double the square and 36. The question is as to the square. The answer in accordance with the rule is 2.

Rule. Multiply $4x$ by $5x$, giving $20x^2$, which equals $2x^2 + 36$ units. Hence divide 36 by 18, giving 2 as the result. And this amount is the square, which may be tested.

Sixteenth question

I subtracted from a quantity one-third of it and three units, then I multiplied the remainder by itself, restoring the quantity itself by this multiplication. The question is, how great is the quantity? The answer in accordance with the rule is 9.

Rule. Subtracting $\frac{1}{3}x + 3$ (units) from $\frac{3}{3}x$, there remain $\frac{2}{3}x - 3$ units, which is the root of the quantity (x). Therefore multiply $\frac{2}{3}x - 3$ units by itself, giving $\frac{4}{9}x^2 + 9$ units $- 4x$, and that is equal to the root (x). Hence $\frac{4}{9}x^2 + 9$ units equals $5x$. You reduce the $\frac{4}{9}$ to a whole square by multiplying it, and also the units and the $5x$, by $2\frac{1}{4}$, and you will find that $x^2 + 20\frac{1}{4}$ units is equal to $11\frac{1}{4}x$. Operate then by the chapter in which squares and numbers are equal to roots. Taking the half of the roots, $5\frac{5}{8}$, and multiplying it by itself, you have $31\frac{41}{36}$: from this subtract $20\frac{1}{4}$, there remains $11\frac{25}{64}$, of which the root is $3\frac{3}{8}$. Add this to the half of the roots, since by subtraction you will not arrive at the desired result, and 9 appears as the quantity which you seek.

Quaestio decima septima

Diuisi drachmam et semissem inter homines et partem hominis, et contigit homini duplum eius quod parti. Quanta igitur fuerit pars, quaeritur.

Respondetur ex regula: $\frac{1}{2}$.

Regula. Idem est homo et pars, ac si diceres, vnum et res. Diuidatur ergo drachma et semis in vnum et rem, et venient 2 res. Multiplica deinde 2 res cum drachma et re et fient 2 census et 2 res, quae aequantur drachmae et semissi. Reducendo igitur ad vnum censum, dic quòd census et res, aequentur $\frac{3}{4}$ drachmae. Age igitur per caput, quo census et res numero coaequantur, multiplicando medietatem rei in seipsa, et fit quarta, quae addita ad $\frac{3}{4}$ facit vnum, cuius radix est vnum, a qua subtrahe medietatem rei et manet medietas, pars quae quaeritur.

Quaestio decima octaua

Diuisi drachmam inter homines et prouenit simul res, deinde addidi eis hominem et postea diuisi drachmam inter eos, et cuilibet contigit minus quod prius sexta parte drachmae. Quot igitur fuerint homines, quaeritur.

Respondetur ex regula: 2.

Regula. Huius quaestio consideratio est vt multiplices homines primos cum diminuto inter diuisiones; deinde multiplices aggregatum cum hominibus primis et cum homine addito, et proueniet census tuus. Scias autem, quòd hoc non sit vniuersaliter verum vt credo, sexta census et sexta radicis, quae aequantur drachmae. Dic ergo, res integrando, quòd census et res aequentur 6 drachmis. Age igitur per caput quo census et res numeris coaequentur. Multiplicando medietatem radicum cum seipsa, et fit quarta, quam adde ad 6 drachmas et veniunt $6\frac{1}{4}$. Inde radix quadrata erunt 2 et semis a qua subtracta medietate radicum, manent 2, qui est numerus hominum.

Sequitur vltimo de rebus venalibus

Sunt autem conuentiones negociationum quae fiunt in venditione, emptione, permutatione, et caeteris rebus, secundum duos modos.

Primus est modus, vt si dicatur, decem res venditae sunt 6 drachmis, quot igitur veniunt 4 drachmis?

Secundus modus est, vt si dicatur, decem res venditae sunt 6 drachmis, quantum igitur est precium 4 rerum?

In primo casu, 10 res est numerus appreciati secundum positionem, et 6 drachmae est praecium secundum positionem; quaestio quot, est numerus ignotus appreciati secundum quaerentem, et 4 res precium secundum quaerentem. In secundo casu, precium et appreciatum secundum positionem, sunt vt prius, et quaestio et precium secundum quaerentem et 4 res est appreciatum secundum quaerentem. Vnde precium secundum positionem dicitur opponi appreciato secundum quaerentem et appreciatum secundum positionem dicitur opponi precio secundum quaerentem. Multiplica igitur inter se, et productum diuide in tertium modum et exhibit quartus ignotus per regulas quatuor proportionalium quantitatum.

Finis annotationum pro declaratione regularum Algebrae.

Seventeenth question

I divided a drachma and one-half between a man and a part of a man, and to the man there fell the double of that which fell to the part (of a man). The question is, how large was the part? The answer is $\frac{1}{2}$.

Rule. A man and part of a man is the same as $1 + x$. Hence $1\frac{1}{2}$ is divided by $1 + x$, giving $2x$. Then multiply $2x$ by $1 + x$, giving $2x^2 + 2x$, which is equal to $1\frac{1}{2}$. Therefore by reduction to one square, say that $x^2 + x$ is equal to $\frac{3}{4}$ of a unit. Operate now by the chapter in which squares and roots equal number, multiplying one-half of the roots by itself, obtaining $\frac{1}{4}$; this added to $\frac{3}{4}$ makes 1, of which the root is 1; from this subtract the half of the number of the roots, giving $\frac{1}{2}$, the value of the part that is sought.

Eighteenth question

I divided a drachma among some men, and each one obtained an unknown amount (x); I then added one man to the group and again I divided a drachma among them; to each man there now fell $\frac{1}{6}$ drachma less than before. The question is, how many men were there? The answer is 2.

Rule. In considering this problem you multiply the first number of men by the decrease; then you multiply the product by the number of men + 1, and the quantity will be obtained. However you should note that this is not a general rule; you have $\frac{1}{6}x^2 + \frac{1}{6}x$, which is equal to 1. Hence by completing the quantity you obtain $x^2 + x$ equal to 6 units. Operate therefore by the chapter in which squares and roots equal numbers. Multiplying one-half of the roots by itself, $\frac{1}{4}$ is obtained. Add this to the 6 units, giving $6\frac{1}{4}$; then the square root will be $2\frac{1}{2}$. From this one-half of the roots is subtracted, leaving 2, which is the number of men.

The last section, on commercial transactions

There are certain customs of business which hold in buying, selling, exchange, and the like, according to two methods. The first method is illustrated: 10 things are sold for 6 drachmas, how many are sold for 4? The second: 10 things are sold for 6 drachmas, what is the price of 4?

In the first case 10 things is the number priced, according to that which is given, and 6 drachmas is the price, as given; the question, how many, represents the unknown number of things, according to the question, and 4 drachmas the price, according to the question. In the second case, the price and the quantity, as given, are the same as before; the question represents the price, according to the problem, and 4 is the corresponding quantity. Whence the price given is said to be in opposition to the price sought, and similarly the quantity given to that sought. Multiply therefore among themselves, and divide the product by the third kind, and the fourth unknown will appear by the rules of four proportionals.

End of the annotations to explain the rules of algebra.

LATIN GLOSSARY

ABBREVIATIONS

adj.,	adjective.
adv.,	adverb.
comp.,	comparative.
conj.,	conjunction.
demonstr.,	demonstrative.
f.,	feminine noun.
indef.,	indefinite.
indecl.,	indeclinable.
m.	masculine noun.
n.,	note.
neut.,	neuter noun.
num.,	numeral.
prep.,	preposition.
pron.,	pronoun.
R,	characterizing a usage of the *Regule*.
S,	characterizing a usage peculiar to Scheybl as opposed to Robert of Chester.
subst.,	substantive.
super.,	superlative.
v.,	verb.

The first appearance in the original text of each Latin word listed is recorded by reference to page and line. In a few instances other references are added to indicate differences in meaning. In cases of variation in spelling the catchword presents the form probably used by Robert of Chester; the current form is generally added in parentheses. The limits of the volume preclude a complete study of the Latinity.

abicio, v., *disregard* (76, 2); with **ex**, *subtract from* (108, 9).

ablatio, f., S, *subtraction* (134, 14).

absque, prep., *minus* (96, 4, n); as adjective, S, *negative* (116, 15).

abstraho, v., with **ex**, *subtract* (120, 3, n).

accipio, v., *take* (70, 32); *extract* root (72, 1).

addendus (**addo**), adj., *to be added, positive* (90, 13).

addicio (-tio), f., *addition* (74, 22, n).

additum, neut., S, *addition* (128, 1).

addo, v., *add* (110, 27); with **ad**, S, *add* (128, 10); with **super**, *add to* (74, 2, n).

adequo (adaequo), v., *equal* (106, 18, n).

adhibeo, v., *add* to (118, 19).

adicio (adiicio, S), v., *add* to (72, 1); with **super**, *add to* (72, 1, n).

adiectio, f., *addition* (74, 22).

adiectiuus, adj., *added, positive* (90, 13, n.).

adiectus, adj., *added, positive* (96, 23, n).

adimpleo, v., *fill up, complete* (74, 21).

aditus, m., *approach* (88, 25).

adiungo, v., *add* (70, 30).

adnullo, v., *reduce to nothing* (74, 29, n).

ae-, see also **e-**.

aenigma, neut., S, *problem* (142, 17).

aequalitas, f., S, *the equal* (80, 12).

aequatio, f., S, *equation, equality* (66, 2).

aequipolleo, v., S, *equal* (128, 6).

aequus, adj., S, *equal* (150, 31.)

aggregatum, m., S, *sum* (72, 20).

ago, v., *operate* (70, 11).

algaurizim, Arabic, *Al-Khowarizmi* (66, 9, n).

algebra, f., S, *algebra* (66, 1).

aliqui, **aliqua**, **aliquod**, indef. pron. adj., *some, any* (74, 21).

aliquot, indef. indecl. num., *some, a number* (90, 9).

almucabola, f., S. A transliteration of part of the Arabic title *al-jebr w'al-muqabala*, for algebra (66, 1).

almuthemen, Arabic, *quantity desired* (120, 24, n).

almuzarar, Arabic, *unit of measure, quantity* (120, 22, n).

alszarar, Arabic, *price per unit* (120, 23, n).

althemen, Arabic, *amount, payment* (120, 24, n).

amplector, v., *embrace, surround, include* (90, 23, n).

angularis, adj., S, *corner* (130, 10).

angulus, m., *an angle, corner* (78, 11).

annotatio, f., S, *annotation* (156, 43).

appareo, v., S, *appear, be evident* (84, 8).

applico, v., *apply, place* (78, 4).

appono, v., *place by, set in apposition* (96, 23, n).

appreciatum, neut., S, *that priced* (156, 36).

arbitror, *consider, judge* (122, 7).

area, f., *area, figure* (78, 4).

arithmeticus, adj., S, *arithmetical* (66, 7).
ars, f., *type* (70, 28).
articulus, m., S, *multiple of ten* (136, 15).
assigno, v., *mark out*, *indicate* (104, 6).
assimilo, v., R, *make like*, *be equal* (68, 12, n and 126, 2).
assumo, v., *take*, *extract* square root (72, 1, n).
attineo, v., *pertain* (120, 21).
auctus, v., S, *positive* (138, 17).
aufero, v., with **ex**, *take away* (84, 22; 110, 21).
augeo, v., S, *increase* (130, 19).

binarium, neut., *two* (98, 1).
binarius, adj., (consisting of) *two* (82, 16).
bis, adv., *twice* (98, 16).
breuis, adj., S, *short* (82, 23).

cado, v., S, *fall* (*upon*) (132, 25).
calculus, m., S, *calculation* (90, 19, n).
capitulum, neut., *chapter* (74, 21, n).
caput, n., S, *chapter* (74, 21).
casus, m., S, *event*, *instance* (74, 14).
causa, f., S, *the reason*, *the explanation* (130, 4).
census, m., S, *the second power of the unknown* (128, 2); *quantity* (146, 34).
centenarius, adj., *one hundred* (66, 17).
centenus, adj., *one hundred* (66, 17, n).
certus, adj., *definite* (120, 27).
circumdo, v., *encompass*, *surround* (78, 12, n).
circumduccio (-tio), f., *perimeter* (78, 13).
circumiaceo, v., S, *surround* (134, 18).
coequacio (coaequatio), f., *equality* (70, 2).
coequalis (coae-), adj., *equal* (88, 1, n).
coequo (coae-), v., *equal* (68, 7).
cognitus, adj., *known* (122, 25).
collectio, f., *sum* (70, 30).
colligo, v., *combine*, *sum up* (68, 6).
combinatio, f., S, *combination* (128, 2).
committo, v., R, *compare*, *divide* (126, 2).
compendiose, adv., *briefly* (88, 23).
complector, v., S, *include*, *embrace* (90, 24).
compleo, v., *fill up*, *complete* (72, 28); *complete* by transferring negative term (108, 4).
completus, adj., *complete* (108, 20).
compono, v., *compose*, *make* (66, 12).
compositus, adj., S, *composite*, used of numbers, including both tens and units, such as 16 or 24 (136, 13).
comprehendo, v., *include* (112, 17).
comprobo, v., *prove*, *confirm* (80, 11, n).
concipio, v., *think* (*of*), *conceive* (74, 13).
concretus, adj., *formed*, *complete* (100, 16, n).
conduco, v., *employ*, *hire* (124, 4).
coniungo, v., *connect*, *join*, *add* (68, 2).
considero, v., *consider*, *reflect* (66, 10).
consimilis, adj., *equal* (80, 6, n).
constituo, v., *constitute*, *make* (74, 18).
contineo, v., *contain* (84, 16).
contingo, v., *reach*, *attain* (96, 4, n).
contra, adv., S, *opposite* (78, 25, n).
conuentio, f., S, *custom* (156, 27).
conuersio, f., *reduction* of an equation to simpler form (72, 8).
conuerto, v., *arrive* (66, 21, n); *reduce* (68, 23).
costa, f., S, *side* (144, 21).
cum, prep., S, *by* (68, 3); *with* (68, 23).

decenarius, adj., *ten* (66, 14, n).
decenus, adj., *ten* (66, 14, n).
declaratio, f., S, *exposition* (128, 1).
deduco, v., with **in**, *multiply* (86, 15, n).
deficio, v., S, *fail*, *be lacking* (132, 4).
deleo, v., S, *efface*, *cancel* (80, 10).
demo, v., with **a**, *subtract* (110, 9, n).
demonstratio, f., S, *demonstration* (66, 2).
demonstro, v., *designate*, *represent* (68, 17).
denarius, adj., *ten* (100, 27).
denarius, m., S, *unit of money*, *penny* (124, 4); *unit*, S (128, 6).
descripcio (-tio), f., *description*, *explanation* (88, 22, n).
designo, v., *represent*, *designate* (72, 14).
deuinco, v., S, *surpass*, *exceed* (84, 1).
differentia, f., S, *difference* (150, 19).
differo, v., *differ* (112, 27).
difficultas, f., *difficulty* (88, 24).
digitus, m., S, *digit*, *unit* (136, 15).
dimensio, f., S, *measurement*, *side* (84, 4).
dimidium, neut., S, *half* (76, 12).
diminucio (-tio), f., *diminution*, *subtraction* (74, 23).
diminuo, v., with **ex**, *subtract* (72, 22), see (72, 2, n); with **ab**, *subtract* (72, 2, n).
diminutiuus, adj., *negative*, *subtracted* (90, 13, n).
diminutus, part. and adj., *reduced* (68, 4); *lessened*, *negative* (96, 23, n).
dinosco, v., *distinguish*, *represent* (72, 4).

dinotus, adj., *pointed out*, *distinguished* (120, 24).

disciplina, *discipline*, *art*, *study* (88, 25).

dispono, v., *arrange* (66, 13).

distinctus, adj., *distinct*, *separate* (70, 22).

distinguo, v., *distinguish*, *separate*, *discriminate* (70, 20).

diuido, v., with **in**, S (100, 5), with **per** (100 5, n), and with **super** (100, 8, n), *divide by*; **diuido in duo media**, *bisect* or *take half of* (86, 13); **diuido in duo**, *separate into two parts* (102, 21); with **inter**, *distribute among* (118, 18); **diuido per medium**, *take the half of* (70, 31).

diuisio, f., *division* (84, 4, n); S, *distribution* (156, 18).

diuisor, m., S, *divisor* (152, 20).

do, v., *give* (74, 11).

doceo, v., *teach*, *show* (88, 22, n).

dragma (**drachma**, S), f., *unit* (70, 28); *unit* of money (124, 4, n).

dubito, v., *consider*, *doubt*, *be in doubt* (122, 6).

duco, v., with **in**, *multiply* (80, 8, n); S, *draw* (82, 21).

duplatio, f., S, *doubling* (98, 15).

duplex, adj., *twofold*, *double* (118, 2).

duplicacio (-tio), f., *doubling*, *multiplication* (66, 16).

duplico, v., *double* (66, 15); *multiply* (90, 7, n).

duplo, v., S, *double* (140, 33).

duplum, neut., S, *double* (142, 19).

e contrario, S, *on the contrary*, *in the reverse way* (100, 8).

e conuerso, *in the reverse way* (100, 8, n).

edo, v., *give out*, *publish* (66, 9).

elicio, v., *draw forth*, *solve* (108, 29, n); *take* square root, S (148, 4).

elucesco, v., *shine forth* (102, 18).

equalis (ae-), adj., *equal* (78, 4, n).

equidistans (ae-), adj., *equidistant* (82, 8, n).

equiparo (ae-), v., *equal* (68, 20).

equo (ae-), v., *equal* (70, 18).

ergo, adv., *therefore*, *then*, *consequently* (68, 6, n).

erigo, v., S, *erect* a perpendicular, *raise up* (132, 15).

error, m., *error* (102, 12).

essentialiter, adv., *essentially*, *naturally* (66, 13, n).

et, conj., *and* (66, 7); *plus* (90, 16).

euacuo, v., *cancel*, *vacate* (80, 10).

euenio, v., *come out*, *result* (82, 25).

exaequo, v., S, *equal* (106, 4).

examen, neut., S, *testing* (116, 7, n).

excedo, v., *exceed*, *go beyond* (66, 14).

excresco, v., *grow*, *increase* (66, 19).

exemplar, neut., *model*, *pattern* (102, 12).

exemplum, neut., *example*, *type* (122, 7).

exeo, v., *come out* (104, 30).

exerceo, v., *exercise*, *practise* (102, 18).

exercitium, neut., S, *exercise*, *problem* (144, 29).

exhibeo, v., *represent*, *show* (74, 13).

exigo, v., *require*, *demand* (98, 9).

expedio, v., *carry through* (120, 19).

explano, v., *explain* (86, 1, n).

expono, v., *explain*, *set forth* (88, 22).

expositio, f., *explanation*, *exposition* (104, 16).

exprimo, v., *represent*, *show the form of* (72, 23).

extendo, v., *extend*, *reach* (72, 12).

extraho, v., with **ex**, *subtract* (82, 27).

extremitas, f., *end* of a line (78, 23).

extremum, neut., S, *extremity*, *end* (130, 18).

facile, adv., *easily* (120, 19).

facilis, adv., *easy* (88, 25).

facio, v., *make* (66, 16).

figura, f., *figure* (88, 23, n).

finio, v., *terminate*, *come to* (104, 27).

forma, f., S, *form* (136, 11).

formula, f., *rule*, *formula* (80, 11).

fractio, f., S, *fraction* (92, 7).

fractus, adj., S, *fractional* (100, 3).

genero, v., *generate*, *produce* (90, 23).

genus, neut., *class*, *species*, *kind* (70, 22).

geometrice, adv., *geometrically* (76, 20, n).

geometricus, adj., S, *geometrical* (66, 7).

gnomon, m., S, *gnomon*, or the form of a carpenter's square, consisting of three rectangles lying around any given rectangle and forming with it a larger, similar rectangle (132, 21).

habeo, v., *have*, *constitute* (70, 15).

hypothesis, f., S, *hypothesis* (146, 19).

igitur, conj., *therefore* (68, 2).

ignoro, v., *not to know*, *be ignorant of* (76, 23).

ignotus, adj., *unknown* (78, 11).

imperfectio, f., *incompleteness* (78, 13).
imperfectus, adj., *incomplete, imperfect* (80, 5).
in, prep., *in* (70, 31); with verbs of division or multiplication, *by* (68, 3, n).
inaequalis, adj., S, *unequal* (78, 10).
incertus, adj., *doubtful, unknown* (120, 28, n).
incognitus, adj., *unknown* (122, 14, n).
incurro, v., *incur, run into* (122, 7).
indigeo, v., *need, require* (66, 11).
infinitus, adj., *infinite* (66, 21).
infra, adv. and prep., *below, less than* (68, 4).
inquisicio (-tio), f., *problem* (122, 1).
inscribo, v., *inscribe* (82, 12).
insuper, adv., *in addition, besides* (118, 19).
integer, adj., *integral, whole* (98, 10).
integro, v., S, *make whole* (150, 21).
intelligo, v., *understand, comprehend* (88, 24).
intentum, neut., S, *result proposed* (128, 16).
inter, prep., *between* (84, 17).
interrogacio (-tio), f., S, *question* (70, 31).
inuenio, v., *discover, find* (66, 10).
inuestigacio (-tio), f., *investigation, finding out* (66, 21).
inuestigo, v., *track out, investigate, seek after, look into* (74, 11).
inuicem, adv., *in turn, alternately* (68, 7).
iungo, v., *join, add* (112, 3).
iuxta, prep., *according to, after the manner of, in case of* (68, 29).

lanx, f., *scale* of a balance (96, 23, n).
latitudo, f., *breadth* (78, 5).
latus, neut., *side* (76, 23).
liber, m., *book* (66, 8).
linea, f., *line* (82, 23).
liqueo, v., *appear, be evident* (76, 17).
locus, m., S, *place* (76, 24).
longitudo, f., *length* (78, 5).

magnus, comp. **maior**, **maius**, adj., *great* (76, 3, n and 78, 13).
magul, Arabic, *unknown* (122, 12, n).
maneo, v., S, *remain, abide* (72, 2).
manifestus, adj., *evident* (72, 14).
medietacio (-tio), f., *halving, half* (116, 19).
medietas, f., *half* (70, 8).
medio, v., *halve* (74, 26).
medium, neut., *middle, half* (70, 31).
medius, adj., *middle, mean, half* (70, 18).
mensuro, v., *measure* (68, 27).
millenarius, adj., *thousand* (66, 19).
millenus, adj., *thousand* (66, 19, n).
minor, **minus**, comp. of **parvus**, adj., *less* (68, 22).
minucia (-tia), f., *a small particle; a small part* (118, 21).
minuendus (**minuo**), *to be subtracted, negative* (90, 15).
minuo, v., with **ex**, *subtract from* (74, 14, n).
minutus, adj., S, *negative* (138, 17).
modus, m., *manner, fashion* (66, 15).
multiplicacio (-tio), f., *multiplication* (90, 10).
multiplico, v., *multiply* (66, 17); with **in**, *multiply by* (68, 3, n); with **cum**, S, *multiply by* (68, 3).
multitudo, f., S, *multitude, a great number* (68, 29).

nascor, v., *arise, spring forth* (82, 16).
natura, f., *nature* (98, 9).
naturaliter, adv., *naturally* (96, 29).
necessario, adv., *necessarily* (86, 8).
necesse, adj., *necessary* (78, 25).
negligo, v., S, *neglect, nullify* (96, 21).
nihil, **nil**, neut., *nothing* (66, 12).
nodus, m., *node, a multiple of ten* (90, 8).
nomino, v., *name, call* (122, 20).
nosco, v., *know, recognize* (72, 4, n).
noticia (-tia), f., S, *axiom* (132, 19).
notus, adj., *known* (120, 27); *rational*, S (140, 33).
nullus, adj., S, *not any, null, void* (74, 29).
numerus, m., *number* (66, 9); **numerus diminutus**, *fraction* (98, 10).
nuncio, v., *announce, report* (122, 13, n).
nuncupo, v., *name, call* (120, 23, n).

obtineo, v., *maintain, prove, have, obtain* (78, 6, n).
omnino, adv., *wholly* (102, 17).
omnis, adj., *all, every* (66, 11).
operor, v., *work, operate* (100, 20).
opifex, m., *worker, student* (108, 28).
oppono, v., *set in opposition to, oppose* (96, 4, n).
oppositio, f., *opposition, balancing* (66, 8).
oppositus, adj., *opposed* (122, 16).
ordo, m., *arrangement, denomination* (66, 20).
orior, v., *arise, appear, spring up* (70, 21).
ostendo, v., *represent, show, reveal* (68, 21).

parallelogrammum, neut., S, *rectangle* (82, 8).

pario, v., *produce, obtain* (78, 16, n).

pars, f., *part* (68, 16).

particula, f., *small part, fractional part* (100, 7).

pateo, v., S, *be clear, follow* (128, 26).

paucior, comp. of **paucus**, adj., *fewer, less* (72, 6, n).

paucitas, f., *fewness, scarcity, paucity* (68, 29).

perduco, v., *bring* to, *lead* to (104, 10).

perfectio, f., *perfection, completion* (80, 22).

perfectus, part. and adj., *complete* (72, 28).

perficio, v., *complete, make* (78, 17).

permaneo, v., *remain* (96, 4, n).

perpendicularis, adj., *perpendicular* (82, 21).

perspectus, adj., *evident, clear* (88, 24).

pertineo, v., *concern, relate to* (76, 19).

peruenio, v., *arrive at, come up to* (66, 17).

pluralitas, f., *many, plurality* (68, 29, n).

plures, adj., the plural of **plus**, *more* (70, 2).

plus, S, comp. of **multus**, adj., *plus, more than* (76, 3).

pondus, neut., *weight* (124, 16).

pono, v., *place, assume, assert, propose* (82, 9); with **super**, *place upon* (84, 1).

portio, f., S, *portion, segment* (82, 22).

possum, v., *be able* (102, 12).

postea, adv., *then, afterwards* (66, 22).

posterior, comp. of **posterus**, adj., S, *latter, following* (76, 16).

postremus, super. of **posterus**, adj., *last* (118, 23).

praeter, adv. and prep., S, *minus* (136, 17); *negative* (138, 7).

precium (pretium), neut., *price* (124, 5).

prefatus (prae-), adj., *before-mentioned* (122, 10).

premitto (prae-), v., *place before* (70, 21).

pretaxo (prae-), v., *mention, assign, enumerate* (74, 14, n).

primum, primo, adv., *first* (74, 13).

primus, adj., *first* (76, 21).

principium, n., *beginning, commencement* (76, 15).

prius, adv., *before, previously* (72, 28).

probatio, f., *trial, test, proof* (76, 23).

probo, v., *try, check, test, prove* (76, 21, n).

procreo, v., *create, produce* (74, 12, n).

produco, v., S, *produce, give* (70, 32).

productum, neut., S, *product* (80, 3).

profero, v., *bring forward, produce, give* (122, 12).

progenero, v., *generate, produce* (106, 12, n).

proicio, v., *cast out, take away* (96, 4, n).

pronuncio, v., *mention, relate* (70, 31).

propono, v., *propose, set forth* (68, 26).

proporcio (-tio), f., *connection, proportion, ratio* (68, 2).

proposicio (-tio), f., *proposition* (76, 21).

propositus, part., *proposed* (102, 14, n).

protendo, v., *extend* (70, 30).

protraho, v., S, *draw, extend* (132, 15).

prouenio, v., S, *come forth, appear* (100, 16).

prout, adv., *just as, as* (76, 17).

punctum, neut., *point* (82, 20).

quadratum, neut., S, *square* (76, 23).

quadratus, adj., *square* (82, 14, n).

quadrilaterus, adj., S, *four-sided, quadrilateral* (82, 14).

quantitas, f., *amount, quantity* (78, 11).

quantum, adv., *how much* (76, 19).

quantus, adj., *how great, as great* (76, 8, n).

quaternarius (sometimes **quarternarius**, S), adj., *four* (68, 20).

quero (quae-), v., *inquire, ask* (70, 3).

questio (quae-), f., *question, problem* (72, 15).

quociens (quotiens), adv., *how many times* (90, 7).

quotquot, adj., *however many* (72, 6).

radix, f., *root* (72, 1); *unknown* (68, 1).

rectangulum, neut., S, *rectangle* (82, 10).

rectangulus, adj., S, *rectangular* (82, 8).

rectus, adj., S, *straight* (as noun 82, 23); *right* angled (82, 15).

reddo, v., *give back, make, render* (80, 5).

redeo, v., S, *return, arise* (152, 20).

reduco, v., S, *bring back* (146, 8); *multiply* (128, 37).

refero, v., S, *represent* (82, 10).

regula, f., *rule* (72, 18).

relinquo, v., S, *leave* (84, 5).

remaneo, v., *remain, be left* (72, 2, n).

reperio, v., *find, discover* (66, 13).

repeto, v., *repeat* (90, 10).

res, f., *thing* (68, 3); *unknown* first power (82, 5).

reseco, v., *cut off* (86, 9).

residuus, neut., *remaining* (76, 16).

respicio, v., *look back, refer* (122, 15).

restauracio (-tio), f., *restoration, transference* of negative terms to the other side of the equation (66, 8).

restauro, v., *restore, transfer* (104, 3).

resto, v., *remain* (80, 16, n).
rumbus (for **rhombus**), m., *square* (76, 23, n).

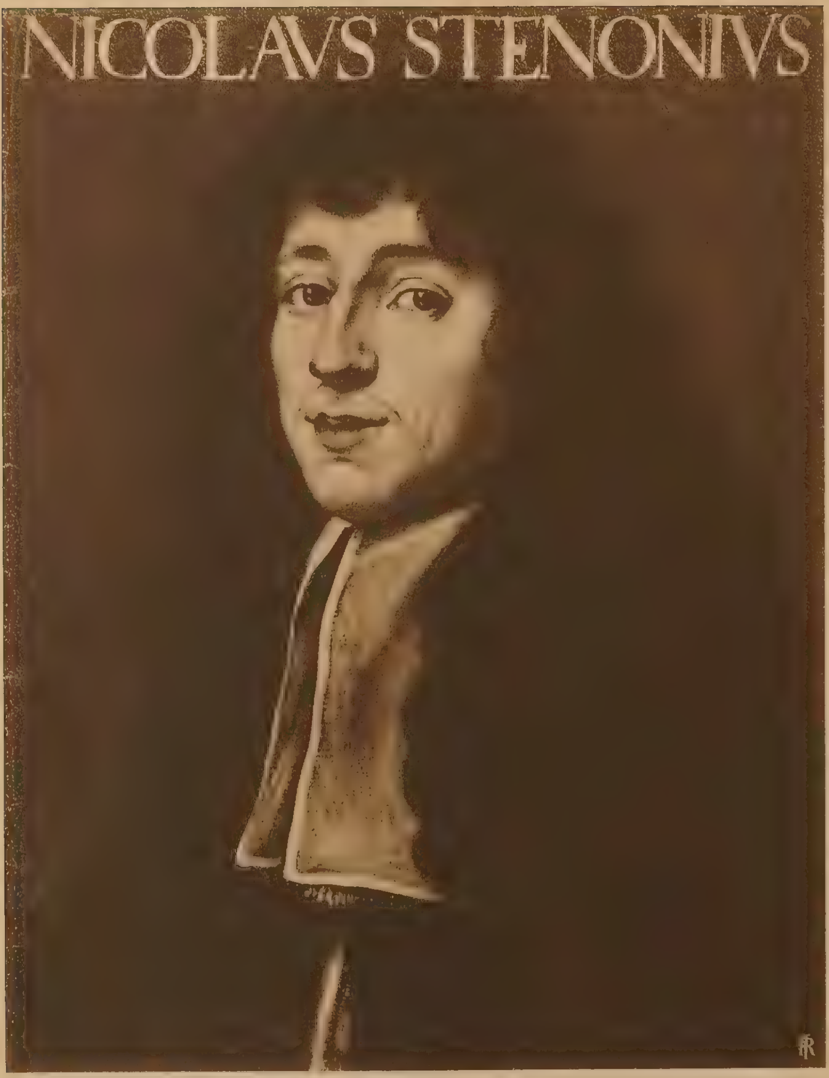
scientia, f., *knowledge*, *science* (66, 10).
scio, v., *know*, *understand* (74, 26).
seco, v., S., *cut* (132, 16).
secundum, prep., *according to*, *following* (66, 20).
secundus, adj., *second*, *following* (120, 25).
semel, adv., *once*, *a single time* (82, 15).
semis, m., S, *one-half* (142, 3).
semper, adv., *always* (68, 6).
significo, v., S, *signify*, *represent* (78, 17).
signo, v., *mark*, *represent*, *signify* (78, 17, n).
similis, adj., *similar*, *equal* (82, 9, n).
similiter, adv., *in like manner*, *similarly* (68, 23).
similitudo, f., *similitude*, *likeness* (70, 29); **ad similitudinem**, *likewise* (68, 18).
simul, adv., *at the same time*, *together*, *also* (74, 24).
sine, prep., *minus*, *negative* (90, 22).
singularis, adj., S, *one by one*, *each* (78, 7).
solucio (-tio), f., *solution* (74, 13).
solus, adj., *alone*, *pure* (68, 1).
structura, f., S, *construction* (132, 17).
studium, neut., *study*, *zeal* (102, 18).
sub, prep., *under* (102, 14).
subabiectio, f., S, *subtraction* (134, 33).
subiectus, adj., *accompanying*, *adjacent* (88, 22, n).
substantia, f., *second power of the unknown* (68, 1); *quantity* (106, 9, n).
subtractio, f., S, *subtraction* (84, 5).
subtraho, v., with **ex**, *subtract from* (78, 23); with **ab**, *subtract from* (90, 9).
sufficienter, adv., *sufficiently* (76, 19).
summa, f., *amount*, *sum* (72, 20, n).
sumo, v., *assume*, *take* (100, 24).
super, adv. and prep., *above* (72, 1, n); in additions. *to*.
superaddo, v., *add* (104, 15).
superficies, f., S, *area* (136, 3).
superfluum, neut., S, *excess* (150, 19).
superius, comp. adv., *above* (76, 17).
supersum, v., *remain* (92, 19).
supplementa, neut., S, *supplementary rectangles* cut off from a larger rectangle by two lines parallel to the sides and intersecting on the diagonal (134, 8).
supra, prep. and adv., *above* (68, 4).
surdus, adj., S, *surd*, *irrational* (140, 33).

tantum, adv., with **quantum**, *as much as* (92, 13).
tempus, neut., *time* (124, 7).
tendo, v., *extend*, *represent* (84, 7).
termino, v., *limit*, *end* (82, 13).
ternarium, neut., *three* (98, 4).
ternarius, adj., (consisting of) *three* (84, 23).
tociens (totiens), adv., *so often*, *so many times as* (90, 7, n).
tollo, v., *take* square root (98, 8); with **ex**, *subtract*, *take away* (110, 13, n).
totus, adj., *whole*, *all* (66, 11).
tracto, v., *treat*, *use*, *handle* (70, 1).
trado, v., *impart*, *set forth* (120, 19).
transfero, v., *translate* (124, 19).
tribuo, v., S, *add* (96, 20).
triplicacio (-tio), f., *tripling* (66, 16).
tripliciter, adv., *triply* (70, 22).
triplico, v., *triple* (66, 15).
tunc, adv., *then*, *immediately* (84, 26).

uinco, v., *exceed*, *surpass* (84, 1, n).
ullus, adj., *any* (68, 2).
ultimus, adj., *extreme*, *last* (120, 26).
unitas, f., *unity*, *unit* (66, 12).
unus, num. adj., *one* (68, 19).
unusquisque, pronominal adj., *each one*, *each* (76, 25), (78, 4).
usque, adv., with **ad**, *up to*, *as far as* (66, 14).

venalis, adj., *salable*, *to be sold* (120, 21).
venio, v., S, *come*, *arrive* (72, 1).
vero, adv., *truly*, *certainly* (68, 5).
versor, v., *be situated*, *lie* (102, 17).
verus, adj., *true* (76, 20).
vis, f., *force*, *significance* (66, 10).

Plate V.


NICOLAVS STENONIVS

Portrait of Steno in the Pitti Palace.

PART II

THE PRODROMUS OF NICOLAUS STENO'S DISSERTATION CONCERNING A SOLID BODY ENCLOSED BY PROCESS OF NATURE WITHIN A SOLID: AN ENGLISH VERSION WITH AN INTRODUCTION AND EXPLANATORY NOTES

WITH A FOREWORD BY WILLIAM H. HOBBS

(*The Index to Part II*
will be found on pages 277–283.)

PREFACE

The task of preparing this translation of Steno's *Prodromus* has been lightened by the generous help of several of my colleagues in the University of Michigan. To Professor W. H. Hobbs I am indebted for suggesting the work and for reading the entire manuscript, as well as for contributing a *Foreword*. Professor E. H. Kraus read in manuscript the section dealing with crystallography, and Professor E. C. Case gave helpful suggestions in questions of palæontology. A point in physics was clarified by Professor W. D. Henderson.

To Professor J. B. Woodworth, of Harvard University, my thanks are due for permission to reprint the section entitled *The Interpreter to the Reader* from his copy of the *H. O.* version, and for verifying certain references. Mr. Bernhard Berenson, of Florence, kindly furnished photographs of the portrait of Steno in the Pitti Palace, and of Duke Ferdinand II in the Uffizi. Dr. Fr. C. C. Hansen, of the University of Copenhagen, generously sent a photograph of the portrait of Steno as Vicar of Schwerin. But from the editorial side my greatest debt, and one I have especial pleasure in acknowledging, is to Dr. Vilhelm Maar, of the University of Copenhagen, whose scholarly edition of Steno's *Opera Philosophica* has been of invaluable service. In addition to furnishing photographs of the portraits of Steno, Dr. Maar has given me, by letter, not only much information, but also warm encouragement.

The recent publication of a facsimile edition of the *Prodromus*, by W. Junk (Berlin, 1904), and of the text, by V. Maar (Copenhagen, 1910), has obviated the necessity of presenting the Latin text in connection with this translation.

I am under much obligation to Mr. William H. Murphy, of Detroit, whose generosity has made possible this publication in the Humanistic Series.

JOHN G. WINTER.

Ann Arbor, Michigan,
March 15, 1916.

CONTENTS

PAGE
FOREWORD 169
INTRODUCTION :
I. Life of Steno 175
II. The Writings of Steno 188
III. Bibliography of the *Prodromus* 194
IV. Selected References 202
TRANSLATION OF THE *PRODROMUS* WITH EXPLANATORY NOTES . . . 205
ATTESTATIONS 271
EXPLANATION OF THE FIGURES 272
INDEX 277

PLATES

V. PORTRAIT OF STENO IN THE PITTI PALACE . . . . *Frontispiece*
FACING PAGE
VI. PORTRAIT OF STENO AS VICAR OF SCHWERIN 184
VII. REPRODUCTION OF ORIGINAL TITLE PAGE 194
VIII. REPRODUCTION OF FIRST PAGE 196
IX. REPRODUCTION OF STENO'S FIGURES, 1–13 272
X. REPRODUCTION OF STENO'S FIGURES, 14–19 274
XI. REPRODUCTION OF STENO'S FIGURES, 20–25 276
PAGE
REPRODUCTION OF TAILPIECE (p. 76 of Original Edition published at Florence in 1669) 270

FOREWORD

The Science of the Prodromus of Nicolaus Steno

In reading the *Prodromus* of Nicolaus Steno one should remember that the essay was written near the middle of the seventeenth century, when scientific observation was hardly thought of. All knowledge concerning the causes of natural phenomena was generally supposed to have been given by God directly to man, and the message was strictly guarded by the church. Giordano Bruno, who denied that there had been a universal deluge, and who had brought forward evidence that a change had taken place in the distribution of land and sea, was burned at the stake for heresy. More than a century later, and a half a century after Steno wrote, de Maillet, in order to express his conviction that the rocks of the earth were marine deposits, thought it necessary to disguise his name in the anagram *Telliamed* and to allow his views to be published only after his death.

In view of these conditions Steno's *Prodromus* is remarkable for its generally untrammelled reasoning, although the concluding pages of the essay are given over to a somewhat labored effort to prove that his views are not incompatible with Scripture, and that the written word has supplemented his observation. It seems doubtful, however, that he would have escaped persecution had he not been a devout Catholic and, moreover, under the protection of a powerful prince, the Grand Duke Ferdinand II. Some indication of the atmosphere of Florence in Steno's time may be gained from Vincentius Viviani's certificate appended to the *Prodromus* and approving it for publication, "since I have recognized in it a perfectly sincere manifestation of the Catholic faith and of good morals, as in the very candid author, I have thought the same worthy of being entrusted to type" (p. 271).

Steno is the pioneer of the observational methods which dominate in modern science, but he was destined to pass away and be almost forgotten before the methods which he used were to be adopted by students of science. If we except Leonardo da Vinci, who like Steno was a Florentine by adoption and who antedated him by a century and a half, there was no writer upon natural science before the eighteenth century that in accuracy of observation, in cogency of reasoning, or in discrimination of judgment might be compared with the "learned Dane." In some measure Steno reflected, of course, the crude notions of his time. Thus we find him adopting, though apparently with some reserve, the doctrine of the four elements, fire, earth, air, and water. In the main, however, if we exclude the prolix introduction addressed "to the Most Serene Grand Duke" and the weak conclusion intended to prove the orthodoxy of his position, the *Prodromus* with but moderate changes may be made to harmonize with the science of the twentieth century. We must attribute it largely to the closeness of his observation of Nature and to his discriminating judgment, that Steno was not lured into wild speculations, as were so many in his time. One of his statements might well be printed in large letters and placed upon the walls of our laboratories and lecture rooms, as a warning to those who undertake scientific investigation. "The nurse of doubts," says Steno, "seems to me to be the fact that in the consideration of questions relating to nature those points which cannot be definitely determined, are not distinguished from those which can be settled with certainty" (p. 213). How much trouble would be saved if to-day scholars had this point oftener in mind!

The form of Steno's essay is geometrical, and this is responsible for the almost unintelligible title and the correlation of subjects which, interpreted in the elaborate differentiation of twentieth century science, appear somewhat incongruous. As stated in the introduction, the *Prodromus* is divided into four parts. The first of these contains among other things an inquiry into the origin of fossils. The second part is stated to be: "Given a substance possessed of a certain figure, and produced according to the laws of nature, to find in the substance itself evidences disclosing the place and manner of its production." In like geometrical form the third part discusses solids which are contained within solids. The concluding portion of the essay is largely a consideration of the prehistoric geological changes which Steno was able to read in the rocks of Tuscany.

The broad outlines of the Cartesian conception of matter were adopted by Steno, who regarded a natural body as an aggregate of imperceptible particles subject to the action of forces such as proceed from a magnet, from fire, or sometimes from light. A fluid differed from a solid in having its particles in constant motion and withdrawing from their neighbors, that is to say, changing their relative positions.

Some of Steno's greatest contributions to science lie in the field of crystallography, for he studied the growth of crystals and showed that those formed in the mountains must have developed in the same manner as crystals of niter separating from solutions in water. These grow, he said, by accretions of substance upon the surface of the crystal nucleus, and not as do plants and animals.

The prevalent columnar form of crystals and the variation of their habit through the occurrence of faces of variable size, Steno explained by the addition of substance on certain sides only of the growing crystal. The force which draws the substance out of the surrounding fluid he recognized to be inherent in the crystal itself, and this crystallizing force he happily likened to what we should to-day call the lines of force about a magnet.

It is hardly to be expected that, great as Steno was, he should in his day have discovered the important fact of the orientation of the molecules of crystals, but he did point to the striking peculiarity of light refraction that distinguishes the crystal from amorphous substances, such as glass. Steno was, however, the discoverer of the fundamental law of crystallography known as the *law of constancy of interfacial angles.* As usually stated, this law affirms that *no matter how much the faces of a crystal may vary in their size or shape, the interfacial angles remain constant, provided they are measured at the same temperature.* The absolutely empirical verification of this law was delayed until the invention of the reflecting goniometer in 1805. Barring the refinement of temperature variations, it was amply verified by Rome de l'Isle with the simple goniometer which he invented in 1783. It is clear, however, that Steno more than a century earlier fully grasped the principle of the law, and gave it some sort of crude experimental verification. In the explanation of his figures, Steno says (p. 272):

"Figures 5 and 6 belong to the class of those crystals which I could present in countless numbers to prove that in the plane of the axis both the number and the length of the sides are changed in various ways without changing the angles."

As a corollary to his deductions concerning the growth of crystals, Steno showed that so-called "phantom crystals" are no product of the action of the larger crystal, but existed first and were enveloped through continued growth of the crystal nucleus.

In the realm of geology we owe to Steno the first clear enunciation of some of those great principles which to-day we assume to be axiomatic only because so much has been built upon them as a foundation. That rocks in the main result from sedimentation in water is thus expressed in the *Prodromus* (p. 219):

" The strata of the earth, as regards the manner and place of production, agree with those strata which turbid water deposits."

The reasons for this belief are most cogent: "The strata of the earth are due to the deposits of a fluid, (1) because the comminuted matter of the strata could not have been reduced to that form unless, having been mixed with some fluid and then falling from its own weight, it had been spread out by the movement of the same superincumbent fluid; (2) because the larger bodies contained in these same strata obey, for the most part, the laws of gravity, not only with respect to the position of any substance by itself, but also with respect to the relative position of different bodies to each other" (p. 227).

It is further clearly shown how marine deposits may be distinguished by their character from those deposits which are laid down in fresh water upon the continents, as well as from the ejectamenta of volcanoes. The origin of variation in the character of strata from place to place, and of the alternation of layers of different characters, are all discussed with a clear understanding of the actual conditions. The great principle that the order of superposition of beds determines the age of formations, is given its first expression (p. 230):

" At the time when any given stratum was being formed, all the matter resting upon it was fluid, and, therefore, at the time the lowest stratum was being formed, none of the upper strata existed."

Likewise it is pointed out that sedimentary formations were either laid down in definite basins of deposition or were universal in their extent. The original horizontality of sedimentary formations is now regarded as one of the great fundamental principles of geology. Steno says of the strata "that the upper surface was parallel to the horizon, so far as possible; and that all strata, therefore, except the lowest, were bounded by two planes parallel to the horizon. Hence it follows that strata either perpendicular to the horizon or inclined toward it, were at another time parallel to the horizon" (p. 230).

If strata are no longer in a horizontal position, it indicates, says Steno, subsequent disturbance of them; and this may be due either to uplift "by violent thrusting up of the strata," or "spontaneous slipping or downfall of the upper strata after they have begun to form cracks, in consequence of the withdrawal of the underlying substance, or foundation" (p. 231).

These changes in position of the strata are according to Steno the chief cause of mountains, and he pretty clearly distinguishes three of the more important mountain types; namely, (1) block or fault mountains, (2) volcanic mountains, and (3) mountains of erosion. The relation of earthquakes to the formation of mountains is indicated with a much nearer approach to present beliefs than is to be found in any save Robert Hooke and comparatively recent writers.

The fissures which form in the strata were recognized by Steno to be the passageways or channels for the movement of underground water, and for subterranean gases as well. These crevices are thus the places where veins of mineral are formed. The storehouses of the precious metals being brought about by natural processes, the foolishness of those who employ the divining rod for the locating of them is pointed out. An imperfect notion of the manner of replacement of one mineral by another seems to have been gained by Steno from his studies.

In the description of the figures — a most important part of the essay — a clear conception is revealed of the relative order of age of strata, of the alternation of transgression and recession of the sea over the same places, and of the nature of a structural unconformity, whereby one set of strata comes to overlie another from which it differs in its lesser angle of inclination. Here Steno gives us the results of his careful field observations in the vicinity of Florence. His figures may, therefore, be regarded as the earliest geological sections ever prepared.

Over the origin of fossils war had long been waged in Steno's time. Like Leonardo, a century and a half before, Steno declared that fossils were petrified remains of plants and animals which had once existed.

Steno's activity in biological studies is brought out in his elaborate examination of the structure of the shells of mollusks. His description of the subdivisions of the shells and the division of these into filaments, and of the various surfaces formed by the aggregation of these filaments, is suggestive of the methods of modern histological science. He shows that the substance of the filaments is developed from a fluid exuded through the outer surface of the animal. The structure of pearls, and their relation to the growing mollusks, is discussed at considerable length.

In treating the length of geological time, Steno was clearly hampered by the church doctrine of the time, to which he himself subscribed. Accepting as correct the Usher chronology of the Scriptures and the Noachian conception of the universal deluge, it is small wonder that Steno fell into error in evaluating geological time. " There are those," he said, " to whom the great length of time seems to destroy the force of the remaining arguments, since the recollection of no age affirms that floods rose to the place where many marine objects are found to-day, if you exclude the universal deluge, four thousand years, more or less, before our time " (p. 258). He thinks it possible to affirm that the shells dug from the hill on which Volterra was built, were formed more than three thousand years ago. The remains of elephants and extinct animals which were found in the valley of the Arno, and which we now know crossed from Africa on a land bridge in Tertiary times, Steno was forced to regard as the mired pack animals which had been brought by Hannibal's army on its way to besiege Rome.

Now that the Latin text of Steno's work has become available through republication, it seems opportune to make his argument accessible in English, and it is believed that Dr. Winter's rendering of the learned Dane's *Prodromus*, with annotations and with a brief account of his life and writings, will be welcomed by students of natural science.

WM. HERBERT HOBBS.

ANN ARBOR, Michigan,
February, 1916.

INTRODUCTION

I. THE LIFE OF NICOLAUS STENO

Nicolaus Steno,[1] the son of a goldsmith, Steen Pedersen, was born in Copenhagen, January 10, 1638.[2]

'From early childhood,' he wrote in 1680,[3] 'the association with those of my own age had little charm for me. For I was constantly in poor health from my third to my sixth year, and was accordingly under the continual care of my parents and older friends. As a result, I grew to prefer the conversation of older people, especially when they spoke of religion, to the frivolous chatter of younger companions. In my journeys, also, I kept away, as much as possible, from idle and dangerous people and sought friendship with those who had won repute through their upright life or their learning.'

After acquiring a thorough training in ancient and modern languages and mathematics in the grammar school of his native city, Steno entered its University in 1656, where he took up the study of medicine and had among his instructors the distinguished scientists Thomas Bartholin, Borrichius (Ole Borch), and Simon

---

[1] Niels Steensen, the Danish form of the name, in accordance with the learned custom of his day was Latinized by its bearer as Nicolaus Stenonis. The current form, Steno, is due to the mistaken idea that Stenonis was a genitive case. The spelling in French is Sténon and in Italian Stenone. Cf. Vilhelm Maar, *Nicolai Stenonis Opera Philosophica* (2 vols., Copenhagen, 1910), Vol. I, p. 1, note 1. According to custom, Steno took his surname from his father's given name; see Plenkers, *Der Däne Niels Stensen, Ein Lebensbild* (Freiburg im Br., 1884), p. 3, note 1.

The sources of Steno's life, consisting chiefly of letters and unpublished manuscripts, are given by Plenkers, *op. cit.*, pp. v, vi. To Plenkers, Wichfeld (*Erindringer om den danske Videnskabsmand Niels Stensen* in *Historisk Tidsskrift*, 3 Raekke, 4 Bind, Kjøbenhavn, 1865, pp. 1–109) and Maar (Vol. I, pp. i–xi) I am chiefly indebted for the biographical material here given.

[2] The *Encyclopædia Britannica*, in its eleventh edition, presents an inadequate biography of Steno in seventeen lines, and incorrectly gives 1631 as the date of his birth. The ninth edition contains no biographical notice. The error appears also in the account by Chéreau in the *Dictionnaire Encyclopédique des Sciences Médicales* (Troisième Série, Tome Onzième, Paris, 1883, pp. 689–691), in which January 1, 1631, is given as the date.

[3] *Defensio et plenior elucidatio epistolae de propria conversione*, Hannover, pp. 18, 19; quoted by Plenkers, *Niels Stensen*, pp. 3, 4.

Paulli. In all probability he took no degree,[1] for the times were troublous, and Steno, like other students, helped defend Copenhagen during its siege by Carl Gustav, king of Sweden.

In 1660 Steno went to Amsterdam to continue his studies and was warmly received by Gerard Blaes, the anatomist, to whom he had been recommended by Bartholin. His stay of four months in Amsterdam was made memorable by the discovery on April 7, 1660, of the parotid duct, which is still known as the *ductus Stenonianus*.[2] Blaes, however, claimed the discovery for his own, and a warm controversy ensued, in which Steno defended himself with ability and dignity. On April 22, 1661, Steno wrote to Thomas Bartholin from Leyden: [3]

'Since you urge me in your letter to publish an account of the exterior salivary duct, I am constrained to explain to you briefly the envy which this bit of a discovery (*inventiuncula*) has won for me, and also the result of this envy; not with the purpose of seeking fame in trifles,[4] but in order to free myself from the hateful charge of stealing what does not belong to me. For I am sorry that the necessity has been laid upon me of being forced to say much upon a subject of no importance, or else to submit to the base brand of shame. A due consideration of the matter will show that it is not worth making much ado about. For a similar duct[5] had been previously discovered, and even the very duct in question had been observed by Casserius[6] although he called it a muscle. . . . Since, however, the charge imputed to me by reason of that duct does not permit me to keep silent, I shall tell you the entire affair, as pupil to preceptor, and shall leave the decision to your judgment. . . .

---

[1] Wichfeld, *op. cit.*, p. 6, says that Steno went to Amsterdam in 1660 as "Dr. physices." He is followed by de Angelis in *Biographie Universelle* (Michaud; *Nouvelle Édition*, *Tome Quarantième*, p. 209), by Chéreau in the *Dictionnaire Encyclopédique des Sciences Médicales* (p. 689), and by Hughes in *Nature* (Vol. 25, 1882, p. 484). Plenkers (*Niels Stensen*, p. 11, note 5) gives good evidence for believing that no degree had been conferred, and Maar (*Opera Philosophica*, Vol. I, p. ii) implies as much.

[2] It appears that the parotid duct was observed independently by Needham in 1655, but his results were not published until 1667 (Maar, *op. cit.*, Vol. I, p. iii). Steno's treatise bears the following title and date: *De Glandulis Oris et Novis Inde Prodeuntibus Salivae Vasis*, Lugd. Batav. Anno 1661. It is printed by Maar, *op. cit.*, Vol. I, pp. 9–51.

[3] *De Prima Ductus Salivalis Exterioris Inventione et Bilsianis Experimentis*, Lugd. Batav. Ao (anno) 1661, 22 ap. (Aprilis). Printed by Maar, *op. cit.*, Vol. I, pp. 1–7.

[4] *In mustaceo laureolam quaeram* means literally 'look for a laurel-wreath in a cake.' Cicero uses the proverb in writing to his friend Atticus, V. 20, 4.

[5] *Ductus Whartonianus*, for which see *Adenographia . . . Auctore Thoma Whartono*, London, 1656, c. XXI, p. 129; Maar, *op. cit.*, Vol. I, p. 222.

[6] *De Vocis Auditusque Organis Historia Anatomica*, Ferrara, 1600, tab. V, p. 27, d, according to Maar, *op. cit.*, Vol. I, p. 222.

'It is a year now since I was hospitably received by Blaes. After waiting three weeks for a chance to secure anatomical material, I asked the distinguished man whether I might be permitted to dissect with my own hand such material as I could buy for myself. He gave his consent, and fortune so favored me that in the first sheep's head, which I had bought on April 7 and was dissecting alone in my room, I found a duct which, so far as I knew, had been described by no one before. I had removed the skin and was preparing to dissect the brain when I decided to examine first the ducts. With this end in view I was exploring the courses of the veins and arteries when I noticed that the point of my knife was no longer closely confined between tissues but moved freely in a large cavity, and presently I heard the teeth themselves resound, as I thrust my knife forward.

'In amazement at the discovery I called in my host (Blaes) that I might hear his opinion. First he ascribed the sound to the violence of my thrust, then resorted to calling it a freak of Nature, and finally referred to Wharton.[1] But inasmuch as that did not help, and the ducts, which had been handled carelessly, allowed no further investigation, I decided to examine them another time more carefully. I succeeded, although not so well, a few days later with a dog's head. Since its affinity to the inferior duct indicated the function of the one I had found, I told Jacob Henry Paulli, my intimate friend, that I had discovered a salivary duct, and I added a description of it. But since I knew that something like it had been discovered before and could not determine whether this identical duct had been examined, I remained silent until I could find opportunity to consult Sylvius about it. After he had heard my account he determined to seek the duct in man, and having found it he demonstrated it to spectators on several occasions.'[2]

Steno then proceeds to show that Blaes's brother, who was in Amsterdam at the time and was thoroughly conversant with the discovery, had accredited it to Steno in letters to Eysson, Professor at Groningen. And furthermore, while Blaes mentioned the duct in his *Medicina Generalis*, which appeared in 1661, a year after the discovery, he could account for neither the beginning nor the end of the duct.[1]

---

[1] See p. 176, note 5.

[2] Maar, *op. cit.*, Vol. I, pp. 3–5. For a convenient description of the duct and the relation of Steno's work to that of Richard Hale see de Angelis, *Biographie Universelle* (Michaud), *Nouvelle Édition, Tome Quarantième*, p. 209.

Meanwhile Steno had gone to Leyden, where he remained from 1660–1664. Here he worked under van Horne, the surgeon, and Franciscus de la Boë Sylvius, the distinguished anatomist who discovered the Sylvian aqueduct. Among his intimate friends were men of widely differing attainments. The brilliant young Dane seems, in fact, to have had a genius for friendship. No fellowship could fail to stimulate reflection which included such men as Jan Swammerdam, the naturalist, whom Steno had previously known at Amsterdam; Borrichius, his old teacher, who had come from Copenhagen; Matthias Jacobaeus, Professor at Copenhagen, and later Bishop of Aarhus in Jutland; Peter Schumacher, who later became Count Griffenfeldt and High Chancellor of Denmark; Jacob Golias, Professor of Arabic at Leyden, and Baruch Spinoza, the philosopher, who was then living at Rijnsburg, a suburb of Leyden. But great as the influence of these men was, it was, perhaps, less telling for his subsequent spiritual development than the religious tolerance which Holland alone of European countries then afforded.[2]

While pursuing his anatomical studies in Leyden, Steno learned of the death of his step-father,[3] and thereupon returned to his native city. Disappointed in his expectation of gaining a professorship, Steno set out in the same year, 1664, for Paris, where he and Swammerdam lived with the naturalist Thévenot. It was in the latter's house that Steno delivered his discourse on the anatomy of the brain.[4] This treatise shares with the *Prodromus* the virtues of

---

[1] Maar, *op. cit.*, Vol. I, p. 223; Plenkers, *Niels Stensen*, pp. 12–14, and Wichfeld, *Erindringer om Niels Stensen*, pp. 7, 8.

[2] Maar, *Opera Philosophica*, Vol. I, pp. iv–v.

[3] Steno's father died in 1644, and his mother, Anna Nilsdatter, had contracted a second marriage with Johannes Stichman. Her death followed closely upon that of the latter. See Plenkers, *Niels Stensen*, pp. 3, 22, 25.

[4] *Discours sur l'anatomie du cerveau*, first printed in Paris in 1669; it is reprinted by Maar, *op. cit.*, Vol. II, pp. 3–35.

Jacques Bénigne Winslow, a grand-nephew of Steno, and himself a scientist of note, was so impressed by the treatise that he printed it in full in his *Exposition Anatomique* (Paris, 1732), pp. 641–659. The preface of the work closes with this remarkable acknowledgment:

"Je finis en avertissant avec une sincere reconnoissance, que le seul Discours de feu M. Stenon sur l'Anatomie du Cerveau, a été la source primitive et le modele general de toute

lucidity and scientific objectivity. At a time when fantastic metaphysics were rife, Steno trusted only to induction based upon experiment and observation. Combating the theories of Descartes and Willis in particular, the author prefaces his discourse with the candid admission:

"Au lieu de vous promettre de contenter vostre curiosité, touchant l'Anatomie du Cerveau; ie vous fais icy une confession sincere et publique, que ie n'y connois rien. Ie souhaiterois de tout mon cœur, d'estre le seul qui fust obligé à parler de la sorte; car ie pourrois profiter auec le temps de la connoissance des autres, et ce seroit un grand bon-heur pour le genre humain, si cette partie, qui est la plus delicate de toutes, et qui est sujette à des maladies tres-frequentes, et tres-dangereuses, estoit aussi bien connuë, que beaucoup de Philosophes et d'Anatomistes se l'imaginent. Il y en a peu qui imitent l'ingenuité de Monsieur Sylvius, qui n'en parle qu'en doutant, quoy qu'il y ait travaillé plus que personne que ie connoisse. Le nombre de ceux à qui rien ne donne de la peine, est infailliblement le plus grand. Ces gens qui ont l'affirmative si prompte, vous donneront l'histoire du cerveau, et la disposition de ses parties, avec la mesme asseurance, que s'ils avoient esté presens à la composition de cette merveilleuse machine, et que s'ils avoient penetré dans tous les desseins de son grand Architecte."[1]

Paris, however, did not hold Steno long. In the summer of 1665 we find him in Florence,[2] where he was soon attached to the court of the Grand Duke Ferdinand II. Upon the recommendation of Thévenot and Viviani, Steno was appointed physician to the Grand Duke, with a house and pension. He was also given a position in the hospital of Santa Maria Nuova. This patronage, gratefully acknowledged in the present treatise, and the opportunity for travel

---

ma conduite dans les travaux Anatomiques. Je l'ai inseré dans le Traité de la Tête, croyant faire plaisir au Public de lui communiquer de nouveau cette Piece, qui étoit devenue rare, et qui renferme beaucoup d'excellens avis, tant pour éviter le faux et l'imaginaire, que pour découvrir le vrai et le réel, non seulement par rapport à la structure et aux usages des parties, mais aussi par rapport à la maniere de faire les Dissections et les Figures Anatomiques."

The first and only complete edition of *L'Autobiographie de Jacques Bénigne Winslow* is that of Maar, Copenhagen, 1912. I am indebted to it for the foregoing passage, p. xxiv.

[1] Maar, *Opera Philosophica*, Vol. II, p. 3.

[2] I have followed Maar, *Opera Philosophica*, Vol. I, p. vi. Plenkers, *Niels Stensen*, p. 30, and Wichfeld, *Erindringer om Niels Stensen*, p. 17, give 1666 as the date of Steno's arrival in Florence.

which his position at court afforded him, made possible Steno's scientific researches.

Now followed the happiest and most productive period of Steno's life. Ferdinand II, although a weak prince, was a generous patron of art and science. The Accademia del Cimento, founded in 1657 by Leopold de' Medici, the brother of Ferdinand, was the center of a learned group including Vincenzo Viviani, the pupil and biographer of Galileo, Francesco Redi, poet and naturalist, Carlo Dati, the scientist, and Lorenzo Magalotti, the versatile secretary of the society. 'I have the honor,' Redi wrote to Athanasius Kircher,[1] 'to serve at a court where distinguished men gather from all parts of the world. In their wanderings they bring and seek in exchange the fruits of high endeavor, and so warm is their welcome that they fancy themselves transported to the mythical gardens of the Odyssey.'

Through the influence of Maria Flavia del Nero, a nun who had long been in charge of the apothecary connected with Santa Maria Nuova, also Lavinia Felice Cenanni Arnolfini, the wife of the ambassador from Lucca, and Emilio Savignani, a Jesuit priest, Steno was induced to embrace Catholicism. He was deeply religious by nature, and there can be no question about the sincerity of his conversion. His seriousness as a lad, and the impression made upon him by the religious tolerance in Holland, have been mentioned. Furthermore, since meeting the eloquent Bossuet in Paris he had been pondering deeply the question of Catholicism versus Protestantism.[2] He was finally received into the Church, December 8, 1667. Five days later Viviani wrote to Magalotti, who was then in Flanders:

'My very dear friend, N. Steno, who lacked only this to make him adorable, so to say, has turned back to life on the day of the dead,[3] in that he has confessed the Catholic faith. His decision to take the final step gave great joy to His Highness (Ferdinand II) and all his friends. On the day of the Immaculate Conception, after he had finally declared his conversion before the Nuntius also, he received a letter from his King, which he called an invitation, with the command to return as soon as possible. An annual pension of four hundred scudi was promised him from the day of his departure. There are no further stipulations, and he can expect an increase of this amount. Still, he is unwilling to begin the journey until he has learned whether His Majesty will support him in this way in spite of his change of belief. Inasmuch as we cannot hope that this will be the case, we have the prospect of keeping him with us.'[1]

---

[1] Quoted by Plenkers, *Niels Stensen*, p. 31.

[2] The question of Steno's conversion is treated at length by Plenkers (*Niels Stensen*, pp. 36–50), who includes in his account many of the letters that passed between Steno and his friends. The reasons which induced Steno to take this step were set forth by him in *Epistola de propria conversione* (Florence, 1677) and *Defensio et plenior elucidatio epistolae de propria conversione* (Hannover, 1680).

[3] All Souls' Day, November 2, is *Giorno de' Morti* in Italian.

According to Blondel[2] Steno wrote to Frederik III informing him of his change of belief. While these negotiations were in progress Steno composed his *Prodromus*[3] (1668). The original plan of writing the treatise in Italian was given up in favor of Latin, and when the remarkable essay appeared it was under the title *Nicolai Stenonis De Solido Intra Solidum Naturaliter Contento Dissertationis Prodromus.* It was printed at Florence, in 1669, with the full sanction of the papal authorities, among whom were his influential friends, Redi and Viviani.

The *Prodromus* was intended as a preliminary statement of principles which the author expected to elaborate more fully in a later, comprehensive work, which is referred to throughout as the Dissertation. The larger work, however, never appeared, probably because

---

[1] Quoted by Plenkers, *Niels Stensen*, p. 51. This order of Frederik III, dated 19 October, 1667, is still preserved in Copenhagen. Compare *op. cit.*, note 1.

[2] *Les Vies des saints pour chaque jour de l'année*, Paris, 1722, p. 738.

[3] The use of the word *Prodromus* to designate a treatise preliminary to a larger work is not found in classical Latin. The *New Oxford Dictionary* amply illustrates its occurrence in English, but the examples are from works subsequent to Steno's time. Francis Bacon (1561–1626) employs the word in the *Instauratio Magna, Prodromi sive Anticipationes Philosophiae Secundae* (edition of Spedding, Ellis, and Heath, Vol. V, p. 182). Larousse, *s.v. Prodrome*, not only gives the best definition of the word as used by Steno, but also cites an excellent example of a writer whose accomplishment, like Steno's, fell short of his original expectation:

"Ce mot a été employé pour désigner une préface, une introduction, un discourse préliminaire; mais, dans sa signification la plus généralement acceptée, il est le titre même d'un ouvrage destiné a préparer d'autres écrits dont il donne l'idée et auxquels il prépare le lecture. Il a été fait des livres de ce genre sur les matières théologiques et philosophiques. Il en existe aussi qui sont relatifs aux sciences exactes et naturelles. L'un des plus remarquables est celui que Candolle a publié sous ce titre: Prodrome du système du regne végétal (Paris, 1824 et suiv. in 8^{vo}). Ce célèbre botaniste avait d'abord conçu le plan d'un ouvrage extrêmement vaste, qu'il intitula: Système naturel du règne végétal, et dont il fit paraître deux volumes (1818–1821, in 8^{vo}); mais, comprenant que la vie d'un homme ne se suffirait pas à remplir ce plan, il y renonça et fit son *Prodrome*, recueil déjà fort vaste, présentant le répertoire des ordres, des genres, des espèces du règne végétal, et qu'il ne put terminer."

Steno's interest in geology had meanwhile given way to his interest in theology.[1] Brief as it is, the *Prodromus* remains one of the most noteworthy contributions to the science of geology, and especially the geology of Italy. Steno's habits of observation, analysis, and induction resulted in an enlightened exposition of geology considered from the petrological, palæontological, and stratigraphical point of view, at a time when many of his contemporaries were still satisfied with some of the absurdities of metaphysical speculation.[2] Steno's work, von Zittel remarks,[3] "already contained the kernel of much that has been under constant discussion during the two centuries which have passed since his death; and if one reads the most recent text-books of geology, it will be evident that science has not yet securely ascertained the share that is to be assigned to subsidence, to upheaval, to erosion, and to volcanic action in the history of the earth's surface conformation in different regions."

The journey to Denmark was not undertaken until a year after Steno's conversion, and then by a circuitous route. After visiting Rome, Naples, and Murano, he reached Innsbruck in May, 1669, Vienna and Prague in the late summer of the same year, and finally Amsterdam in the spring of 1670. Meanwhile Frederik III had died February 2, 1670, and Steno remained in Holland. Upon learning of the serious illness of his patron Ferdinand II he departed at once for Florence. When he arrived (1670), Cosimo III had already succeeded his father as Grand Duke of Tuscany. But the change in rulers brought no change in the warmth of Steno's welcome. Under Cosimo he arranged the minerals in the Pitti Palace, and continued his studies in geology.

In the Pitti Palace is a series of portraits of distinguished men who were associated with the Court of Ferdinand II and Cosimo III.

---

[1] The *Prodromus* was Steno's last scientific work of note. After his conversion (cf. p. 180) his interest in science rapidly waned. Leibnitz, who came to know and esteem Steno later in Hannover, in letters to Conring expresses deep regret that Steno had abandoned his earlier studies. See Gerhardt, *Die philosophischen Schriften von G. W. Leibniz* (Vol. I, Berlin, 1875), p. 185, and especially p. 193: "*Stenonium Episcopum doleo nunc a physiologicis studiis averti ad theologica vel ideo quia in his facilius quam in illis habebit parem.*"

[2] A striking instance of this is Kircher's *Mundus Subterraneus*, Amsterdam, 1665. Compare Maar, *Om Faste Legemer*, Copenhagen, 1902, p. ii ff.

[3] *History of Geology and Palæontology*, Eng. trans. (London, 1901), p. 27. Compare Huxley, *Nature*, Vol. 24 (1881), p. 453; A. von Humboldt, *Essai Géognostique sur le Gisement des Roches dans les deux Hémispheres* (Paris, 1823), p. 38; *Cosmos*, Eng. trans. (London, 1852), Vol. 2, pp. 347–348; M. J. P. Flourens, *De la Longévité humaine et de la Quantité de Vie sur le Globe* (Paris, 1855), pp. 211–215.

Among these is a portrait of Steno, by an unknown painter, evidently made in the period when the *Prodromus* was composed; this is reproduced in our Plate V.

On the recommendation of Count Griffenfeldt,[1] Christian V invited Steno to the Professorship of Anatomy in Copenhagen. The royal order, dated February 13, 1672, is still preserved in Copenhagen, and reads:

'Know that by special royal grace and favor We have allowed you, until further gracious increase, four hundred reichsthaler a year. This pension shall date from the time of your arrival here. For it is our gracious command and will that you undertake at once your journey to our Kingdom of Denmark, in order to be here as soon as possible. You will comply in humble obedience.'[2]

Steno's reply to Count Griffenfeldt is dated April 26:

'I thank your Excellency most humbly for your good will to me and wish with all my heart that God may grant me to prove to you, one day, my gratitude and willing service. I suppose that your Excellency already knows the reason why my answer is so late in arriving. For the letters containing the orders of His Royal Majesty did not reach Holland until April 3. From there they were forwarded to me yesterday, the twenty-fifth of April. I humbly ask you therefore to excuse my delay to our most gracious Lord and King. This morning I went to the Grand Duke to tell him of the command of His Royal Majesty, and I hope to receive leave for my final departure within a few days.'[3]

Steno arrived in Copenhagen July 3, 1672. His notable address on the reopening of the Theatrum Anatomicum[4] was as much a valedictory as an inaugural, for it marks the close of his scientific career. He soon became involved in religious controversies which made his tenure so disagreeable that he yearned for the old life at Florence. Accordingly, he resigned his position in the summer of 1674, and set out for Florence, visiting the Catholic Duke of Hannover, Johann Friedrich, on the way. Thence he came back to Amsterdam before proceeding to Italy. Upon his arrival in Florence, late in the year 1674, he was appointed tutor to the son of Cosimo III, and thenceforth gave up natural science, for which his keen powers of observation and analysis so admirably fitted him, in order to devote himself to questions of education and theology.

---

[1] See p. 178.

[2] Quoted by Plenkers, *Niels Stensen*, p. 91.

[3] Plenkers, *op. cit.*, p. 91.

[4] Printed by Maar, *Opera Philosophica*, Vol. II, pp. 249–256.

In 1675 Steno took Holy Orders; in the following year (September 14, 1676), Pope Innocent XI rewarded his zeal in attempting to convert his former friends[1] and co-religionists by appointing him Bishop of Titopolis,[2] *in partibus infidelium*, and Apostolic Vicar of Northern Germany and Scandinavia. In consequence, toward the end of 1677, he took up his residence in Hannover. An account preserved by Manni[3] gives a graphic picture of the austere life Steno's devotion now induced him to lead:

'The prelate lived and dressed as though he were the poorest person in the world. His position could only be inferred from his ecclesiastical garb, and even this was only serge. For he would not take the robes of his predecessor although they were offered to him at a low price. And notwithstanding the Duke made him an ample allowance to enable him to live as became his rank, he gave everything to the poor. For them he sacrificed everything. And he did this as long as we knew him. He even gave the gold necklace with a medallion containing a portrait of the Duke—he had received it on his second return from Denmark to Rome by way of Hannover[4]—to a friend with the injunction that it be bestowed upon the poor. When he had nothing else he sold his silver crucifix and costly bishop's ring to relieve the distress of others.'

Upon the death of Johann Friedrich, in 1679, and the accession of his Protestant brother, Duke Ernst August, Steno was forced to withdraw to Munster. Here he was appointed Suffragan Bishop to Ferdinand, Baron von Fürstenberg, the Bishop of Munster (1680). The latter died in 1683, and was succeeded by Archbishop Maximilian Heinrich. Steno had opposed his election and refused to celebrate mass in honor of the event. He therefore withdrew to Hamburg, where his self-imposed poverty and his asceticism alienated the Catholics themselves. They threatened to cut off his nose and ears, to drive him from the city, and even to kill him. In his

---

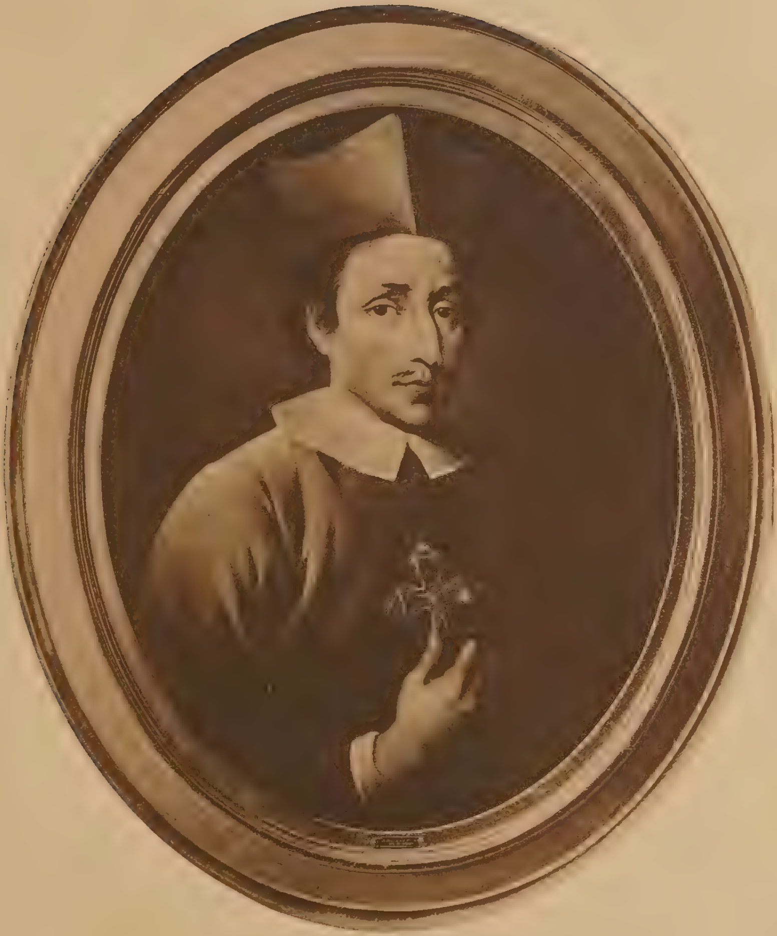
[1] Plenkers, *Niels Stensen* (pp. 122, 123), quotes, among other letters, an interesting appeal to Spinoza. The latter did not reply.

[2] An old bishopric in Isauria.

[3] *Vita del letteratissimo Mons. N. Stenone* (Florence, 1775), p. 229; quoted by Plenkers, *Niels Stensen*, p. 131.

[4] This statement is inexact; Steno did not go to Denmark in 1670, and in 1674 his objective was Florence, not Rome.

Plate VI.



Portrait of Steno as Vicar of Schwerin.

trouble he began to long for the peace and friendships of Italy,[1] and was preparing to return when the missionary post at Schwerin was offered to him. He accepted it in 1685 as a call to further service. But the change meant only increased fasting and abject poverty, to which he succumbed November 26, 1686. At the request of Cosimo III Steno's body was taken to Florence and laid in the famous San Lorenzo.

The physical change which Steno's self-denial entailed is strikingly shown in his portrait as Vicar of Schwerin. The original, by an unknown artist, is still in Schwerin. Until recently it was his only known portrait. An excellent copy, reproduced in our Plate VI, is in possession of the Anatomical Institute of the University of Copenhagen.

On the walls of the cloister of San Lorenzo there is to-day a medallion portrait of Steno, surrounded by a marble wreath, with the following inscription, in black letters, beneath it:

> NICOLAI · STENONIS · IMAGINEM · VIDES · HOSPES
> QVAM · AERE · COLLATO · DOCTI · AMPLIVS · MILLE
> EX · UNIVERSO · TERRARUM · ORBE · INSCULPENDAM
> CURARUNT · IN · MEMORIAM · EJUS · DIEI · IV · CAL · OC-
> TOBR · AN · M · D · CCC · LXXXI · QUO · GEOLOGI · POST · CON
> VENTUM · BONONIAE · HABITUM · PRAESIDE · JOANNE
> CAPELLINIO · EQUITE · HUC · PEREGRINATI · SUNT · AT-
> QUE · ADSTANTIBUS · LEGATIS · FLOR · MUNICIPII · ET
> R · INSTITUTI · ALTIORUM · DOCTRINARUM · CINERES
> VIRI · INTER · GEOLOGOS · ET · ANATOMICOS · PRAE-
> STANTISSIMI · IN · HUJUS · TEMPLI · HYPOGEO · LAUREA
> CORONA · HONORIS · GRATIQUE · ANIMI · ERGO · HONE-
> STAVERUNT

The medallion portrait is by Vincenzo Consani. Plenkers (*Niels Stensen*, p. 88) quotes the inscription, but does not divide it properly into lines. My own transcript was made in Florence June 20, 1911. I add a translation:

---

[1] Indicated in the letters to Madame Arnolfini (Plenkers, *Niels Stensen*, p. 178).

'Friend, you behold the likeness of Nicolaus Steno. To it more than a thousand men of learning, from all parts of the world, contributed. They made provision for the carving of it in memory of this day, the twenty-eighth of September, in the year 1881, when the Geologists, after the Congress at Bologna, under the Presidency of Cavaliere Giovanni Capellini, journeyed hither, and in the presence of delegates representing the City of Florence and the Royal Institute of Higher Studies, in the cloister of this church, as a testimonial of respect and gratitude honored with a laurel crown a man of surpassing distinction among Geologists and Anatomists.'

The official account of the events recorded in the inscription is in itself a commentary of sufficient interest to warrant reprinting here, particularly because of its estimate of the value of Steno's contribution to geology:[1]

Ils allèrent en suite rendre hommage aux restes de Sténon qui reposent dans une tombe de plus modestes, dans la crypte souterraine de la chapelle des Médicis, à San Lorenzo. Les chanoines de la Basilique se tenaient, pour les recevoir, au pied de l'escalier qui descend dans la crypte.

Là, M. le président Capellini invita à prendre la parole l'éminent représentant des études d'archéologie préhistorique, M. Waldmar Schmidt, de Copenhague. Notre savant confrère s'exprima en ces termes:

"Messieurs, Au moment où les membres du second Congrès géologique international sont réunis dans la célèbre église de San Lorenzo, devant la tombe de Nicolas Sténon, vous permettrez, je l'espère, au seul représentant du pays où est né Sténon, d'exprimer au noms de ses compatriotes les plus chaleureux remercîments à la ville de Florence pour l'hommage qu'elle a rendu à leur concitoyen.

"Comme vous le savez, à une époque où les sciences naturelles n'étaient pas encore sorties de leur première enfance, Nicolas Sténon a jeté les fondements de la géologie; et par ses études, par ses observations, par son génie perspicace, il est arrivé à énoncer, sur divers points de la science, des vues dont les géologues de notre siècle, après tant de nouvelles recherches, ont reconnu l'exactitude.

"Sténon était né en Danemark et c'est là qu'il fit ses premières

---

[1] *Congrès Géologique International . . . Compte Rendu de la 2me Session, Bologne*, 1881, pp. 249–251. See also the brief account in *Bolletino del R. Comitato Geologico d' Italia*, vol. 12 (1881), pp. 379, 380.

études. Mais c'est en Italie qu'il a accompli ses merveilleuses découvertes et posé les bases de la géologie. Il y fut reçu avec cette splendide hospitalité qui nous a nous mêmes accueillis partout d'abord à Bologne, aujourd'hui à Florence.

" L'Italie fut sa seconde patrie, et ses restes mortels reposent dans ce temple magnifique, dans lequel on admire les œuvres des plus grands artistes du monde.

" Quand Sténon abandonna le Danemark pour venir se fixer dans ce beau pays, la science géologique ne déserta pas avec lui la patrie scandinave.

" Ne dois-je pas, Messieurs, vous rappeler à cette occasion que si le Danemark a eu Sténon, un autre pays scandinave, la Suède, a eu Linné. Comme l'un avait établi les fondaments de la stratigraphie, l'autre posa les bases de la géologie physique. . . . Vous me permettrez donc, Messieurs, de joindre à mes remercîments pour la ville de Florence et son syndic qui nous ont fait un si magnifique accueil, l'expression de ma reconnaissance pour celui qui par un beau travail a fait connaître, je ne dirai pas le nom de Sténon qui était déjà assez connu, mais sa vie, son origine et son pays natal: toute ma gratitude à M. Capellini, auteur de la Vie de Sténon et président du deuxième Congrès international de géologie à Bologne."

Il (Capellini) ajoute que son but était de faire mieux connaître ce grand homme dont le souvenir doit être sacré pour tous les géologues, et sur la tombe duquel il est heureux de tendre la main à M. W. Schmidt, afin de reserrer entre l'Italie et le Danemark les liens d'affection que rappelle cette illustrée mémoire. . . .

Le soir, le cercle philologique, le cercle des ingénieurs et le club alpin ouvrirent gracieusement leur salles aux congressistes. Mais avant de se rendre à ces amiables invitations, ils furent conviés à un dîner à l'hôtel Minerva par le président Capellini, et, après, une souscription fut ouverte par ses soins pour placer sur le tombe de Sténon une pierre dont l'inscription rappellerait à la fois les glorieux titres scientifiques du célèbre Danois, et la visite faite à sa tombe par les membres du Congrès géologique international. . . .

II. THE WRITINGS OF STENO

Steno's published works may be grouped under three heads: Anatomy, Geology, and Theology. The scientific treatises have all been reprinted by Maar, *Opera Philosophica* (2 vols., Copenhagen, 1910); no complete edition of his many interesting letters, and of his theological writings, has yet appeared. The following list, compiled from Maar and Plenkers, gives the full title of each published work, its date, and place of original publication. In the case of the scientific treatises references are given also to Maar's edition.

I. ANATOMY

1. *a. Disputatio Anatomica de Glandulis Oris et Nuper Observatis inde Prodeuntibus Vasis Prima.* Leyden, 1661 (July 6).

b. Disputatio . . . Secunda. Leyden, 1661 (July 9).

These two articles appeared together in:

De Glandulis Oris et Novis earundem Vasis Observationes Anatomicae. Leyden, 1661. Printed by Maar, Vol. I, pp. 9–51 (Number II).

2. *Observationes Anatomicae, Quibus Varia Oris, Oculorum, et Narium Vasa Describuntur, Novique Salivae, Lacrymarum et Muci Fontes Deteguntur, et Novum Nobilissimi Bilsii de Lymphae Motu et Usu Commentum Examinatur et Rejicitur.* Leyden, 1662.

This volume includes four treatises:

a. De Glandulis Oris, etc., pp. 1–54; a reprint of the two Disputationes. Maar, Vol. I, pp. 9–51 (No. II).

b. Responsio ad Vindicias Hepatis Redivivi, Qua Tela, Quae in Praesidem Celeberr. Dn. Johannem van Horne direxerat Clar. Antonius Deusingius, a Thesium Authore Excipiuntur, et Evanida Ostenduntur. Pp. 55–78.

This treatise bears the date 28 November, 1661. Maar, Vol. I, pp. 59–73 (No. IV).

c. De Glandulis Oculorum Novisque earundem Vasis Observationes Anatomicae, Quibus Veri Lacrymarum Fontes Deteguntur. Pp. 79–100. Maar, Vol. I, pp. 75–90 (No. V).

d. Appendix de Narium Vasis. Pp. 101–108. Maar, Vol. I, pp. 91–97 (No. VI).

3. *Apologiae Prodromus, Quo Demonstratur, Judicem Blasianum et Rei Anatomicae Imperitum Esse, et Affectuum Suorum Servum.* Leyden, 1663. Maar, Vol. I, pp. 143–154 (No. XIII).

4. *De Musculis et Glandulis Observationum Specimen Cum Epistolis Duabus Anatomicis.* Copenhagen, 1664. The *De Musculis*, etc., is printed by Maar, Vol. I, pp. 161–192 (No. XV).

The first of the two letters was written to Willem Piso, and is entitled *De Anatome Rajae Epistola.* Dated April 24, 1664, Copenhagen. Pp. 48–70. Maar, Vol. I, pp. 193–207 (No. XVI). The second was written to Paul Barbette, and is entitled *De Vitelli in Intestina Pulli Transitu Epistola.* Dated June 12, 1664, Copenhagen. Pp. 71–84. Maar, Vol. I, pp. 209–218 (No. XVII).

5. *De Prima Ductus Salivalis Exterioris Inventione, et Bilsianis Experimentis.* A letter to Thomas Bartholin, dated April 22, 1661, Leyden. First printed in Bartholin's *Epistolae Medicinae*, Cent. III, 1667, No. XXIV. Maar, Vol. I, pp. 1–7 (No. I).

6. *Variae in Oculis et Naso Observationes Novae.* A letter to Bartholin, dated September 12, 1661, Leyden. First printed in *Epist. Med.*, Cent. III, 1667, No. LVII. Maar, Vol. I, pp. 53–58 (No. III).

7. *Sudorum Origo ex Glandulis. De Insertione et Valvula Lactei Thoracici et Lymphaticorum.* A letter to Bartholin, dated January 9, 1662, Leyden. First printed in *Epist. Med.*, Cent. III, 1667, No. LXV. Maar, Vol. I, pp. 99–103 (No. VII).

8. *Cur Nicotinae Pulvis Oculos Clariores Reddat. De Lactea Gelatina Observatio.* A letter to Bartholin, dated May 21, 1662, Leyden. First printed in *Epist. Med.*, Cent. IV, 1667, No. I. Maar, Vol. I, pp. 105–111 (No. VIII).

9. *Observationes Anatomicae in Avibus et Cuniculis.* A letter to Bartholin, dated August 26, 1662, Leyden. First printed in *Epist.*

Med., Cent. IV, 1667, No. XXVI. Maar, Vol. I, pp. 113–120 (No. IX).

10. *De Vesiculis in Pulmone. Anatome Cuniculi Praegnantis. In Pulmonibus Experimenta. De Lacteis Mammarum. In Cygno Observationes.* A letter to Bartholin, dated March 5, 1663, Leyden. First printed in *Epist. Med.*, Cent. IV, 1667, No. LV. Maar, Vol. I, pp. 129–136 (No. XI).

11. *Nova Musculorum et Cordis Fabrica.* A letter to Bartholin, dated April, 1663, Leyden. First printed in *Epist. Med.*, Cent. IV, 1667, No. LXX. Maar, Vol. I, pp. 155–160 (No. XIV).

12. *Elementorum Myologiae Specimen, seu Musculi Descriptio Geometrica. Cui Accedunt Canis Carchariae Dissectum Caput, et Dissectus Piscis ex Canum Genere.* Florence, 1667.
The *Elementorum Myologiae Specimen* is printed by Maar, Vol. II, pp. 61–111 (No. XXII). The second treatise, entitled *Canis Carchariae*, etc., is printed by Maar, Vol. II, pp. 113–145 (No. XXIII). The third, *Dissectus Piscis*, etc., is printed by Maar, Vol. II, pp. 147–155 (No. XXIV).

13. *Discours sur l'Anatomie du Cerveau.* Paris, 1669. Maar, Vol. II, pp. 1–35 (No. XVIII).

14. *Figurae Explicatio. Receptaculi Sanguinis Circulus per Ventriculorum Cordis Separationem ab Invicem Manifestior Redditus.*
First printed in Bartholin's *Anatome, Quartum Renovata*, Leyden, 1673, pp. 805–807. Maar, Vol. II, pp. 279–282 (No. XXXIII).

15. *Embryo Monstro Affinis Parisiis Dissectus.* First printed in *Acta Hafniensia*, Vol. I, 1673, pp. 200–203. Maar, Vol. II, pp. 49–53 (No. XX).

16. *Uterus Leporis Proprium Foetum Resolventis.* In *Acta Hafn.*, Vol. I, 1673, pp. 203–207. Maar, Vol. II, pp. 55–60 (No. XXI).

17. *De Vitulo Hydrocephalo.* In *Acta Hafn.*, Vol. I, 1673, pp. 249–262. Maar, Vol. II, pp. 229–239 (No. XXVIII).

18. *In Ovo et Pullo Observationes.* In *Acta Hafn.*, Vol. II, 1675, pp. 81–92. Maar, Vol. II, pp. 37–47 (No. XIX).

19. *Ex Variorum Animalium Sectionibus hinc inde factis Excerptae Observationes circa Motum Cordis, Auricularumque et Venae Cavae.* In *Acta Hafn.*, Vol. II, 1675, pp. 141–147. Maar, Vol. I, pp. 121–127 (No. X).

20. *Observationes Anatomicae Spectantes Ova Viviparorum.* In *Acta Hafn.*, Vol. II, 1675, pp. 210–218. Maar, Vol. II, pp. 157–166 (No. XXV).

21. *Ova Viviparorum Spectantes Observationes factae Jussu Serenissimi Magni Ducis Hetruriae.* In *Acta Hafn.*, Vol. II, 1675, pp. 219–232. Maar, Vol. II, pp. 167–179 (No. XXVI).

22. *Lymphaticorum Varietas.* In *Acta Hafn.*, Vol. II, 1675, pp. 240–241. Maar, Vol. I, pp. 137–142 (No. XII).

23. *Historia Musculorum Aquilae.* In *Acta Hafn.*, Vol. II, 1675, pp. 320–345. Maar, Vol. II, pp. 257–277 (No. XXXII).

24. *Prooemium Demonstrationum Anatomicarum in Theatro Hafniensi Anni 1673.* In *Acta Hafn.*, Vol. II, 1675, pp. 359–366. Maar, Vol. II, pp. 249–256 (No. XXXI).

In addition to the foregoing treatises, Maar prints extracts from various sources, *op. cit.*, Vol. II, pp. 283–310 (Appendix, Nos. XXXIV, XXXV, XXXVI).

II. GEOLOGY

1. *De Solido intra Solidum Naturaliter Contento Dissertationis Prodromus.* Florence, 1669. Maar, Vol. II, pp. 181–227 (No. XXVII).

2. Letter to Cosimo III, in Italian, *On the Grotto above Gresta.* First printed by Fabroni, *Lettere Inedite di Uomini Illustri* (Florence, 1773–1775), Vol. II, no. 141, pp. 318–321. Maar, Vol. II, pp. 239–242 (No. XXIX).

3. Letter to Cosimo III, in Italian, *On the Grotto of Moncodine.* Dated August 19, 1671. First printed by Fabroni, *op. cit.*, no. 142, pp. 321–327. Maar, Vol. II, 243–248 (No. XXX).

III. THEOLOGY

1. *Ad Virum Eruditum cum Quo in Unitate S. R. E. desiderat Aeternam Amicitiam inire, Epistola detegens Illorum Artes Qui Suum de Interprete S. Scripturae Errorem S. Patrum Testimonio confirmare nituntur.* Florence, 1675.

2. *Epistola exponens Methodum Convincendi Acatholicum juxta D. Chrysostomum ex ejusdem Homilia XXXIII in Act. Apostolorum.* Florence, 1675.

3. *Epistola ad Novae Philosophiae Reformatorem de Vera Philosophia.* Florence, 1675.

4. *Epistola de Propria Conversione.* Florence, 1677.

5. *Scrutinium Reformatorum ad Demonstrandum Reformatores Morum fuisse a Deo, Reformatores autem Fidei et Doctrinae non fuisse.* Florence, 1677.

6. *Epistola de Philosophia Cartesiana.* Florence, 1677.

7. *Scrutinium Reformatorum d. i. Kurtzer Beweis dass Diejenigen Lehrer, so die Sitten der Menschen zu Verbessern Getrachtet, von Gott Gewesen, mit Nichten aber die Andern, so die Glaubenslehre zu Verbessern Gesuchet.* Hannover, 1678.

8. *Occasio Sermonum de Religione cum Jo. Sylvio.* Hannover, 1678.

9. *Examen Objectionis circa Diversas Scripturas Sacras et Earum Interpretationes Tamquam Divinas a Diversis Ecclesiis Propositas, D. Jo. Sylvio per Litteras a. 1670 Transmissum, modo Distinctius et Auctius in Lucem Editum, Ubi Omnes, Qui Reformatos Se Credunt, Nobis Nulla Unquam Fidei Reformatione Indigis Objiciunt, Se Solos Certos esse, Quod Deo Credant, Nostram autem Fidem Non Divina, Sed Humana Auctoritate niti.* Hannover, 1678.

10. *Tractatio de Purgatorio Cum Discursu utrum Pontificii an Protestantes in Religionis Negotio Conscientiae Suae Rectius Consulant.* Hannover, 1678.

11. *Katholische Glaubenslehre vom Fegfeur, mit Klaren Zeugnüssen aus dem H. Augustino Bewehret; nebenst Entdeckung Vier Grober Irrthümer des Dorschäi, indem Er Vorgibt dass Bellarminus das Fegfeur aus den H. H. Vättern Nicht Habe Erweisen Können*, etc. Hannover, 1678.

12. *Defensio et Plenior Elucidatio Scrutinii Reformatorum.* Hannover, 1679.

13. *Defensio et Plenior Elucidatio Epistolae de Propria Conversione.* Hannover, 1680.

14. *Parochorum Hoc Age, seu Evidens Demonstratio Quod Parochus Tenetur Omnes Alias Occupationes dimittere et Suae attendere Perfectioni ut Commissas Sibi Oves ad Statum Salutis Aeternae Ipsis a Christo Praeparatum Perducat.* Florence, 1683.

III. BIBLIOGRAPHY OF THE PRODROMUS

I. THE ORIGINAL EDITION

Nicolai Stenonis De Solido Intra Solidum Naturaliter Contento — Dissertationis Prodromus. Ad Serenissimum Ferdinandum II Magnum Etruriae Ducem. Florentiae. Ex Typographia sub signo Stellae MDCLXIX. Superiorum Permissu.[1]

The volume is a small quarto, the type page measuring seven and one sixteenth by four and three sixteenths inches, with wide margins. Including the margins, the page measures nine and one half by six and five sixteenths inches. Maar observes (*Opera Philosophica*, Vol. II, p. 355) that copies are extant without the wide margins, although he does not state where either these or the former may be found. The title-page is in two colors, as may be seen from our reproduction of it (Plate VII). We present also a reproduction of the first page of the treatise, with its tastefully designed headpiece and initial letter (Plate VIII). The volume is a creditable example of seventeenth century Italian printing.

The text fills 76 pages, and ends with a quaint tailpiece, which we have reproduced at the end of the translation (p. 270). The authorizations of publication fill the greater part of the next two pages. The figures follow, brought together in a single large folding plate. In this volume for the convenience of readers we have distributed the figures among three full-page plates (numbered IX,

---

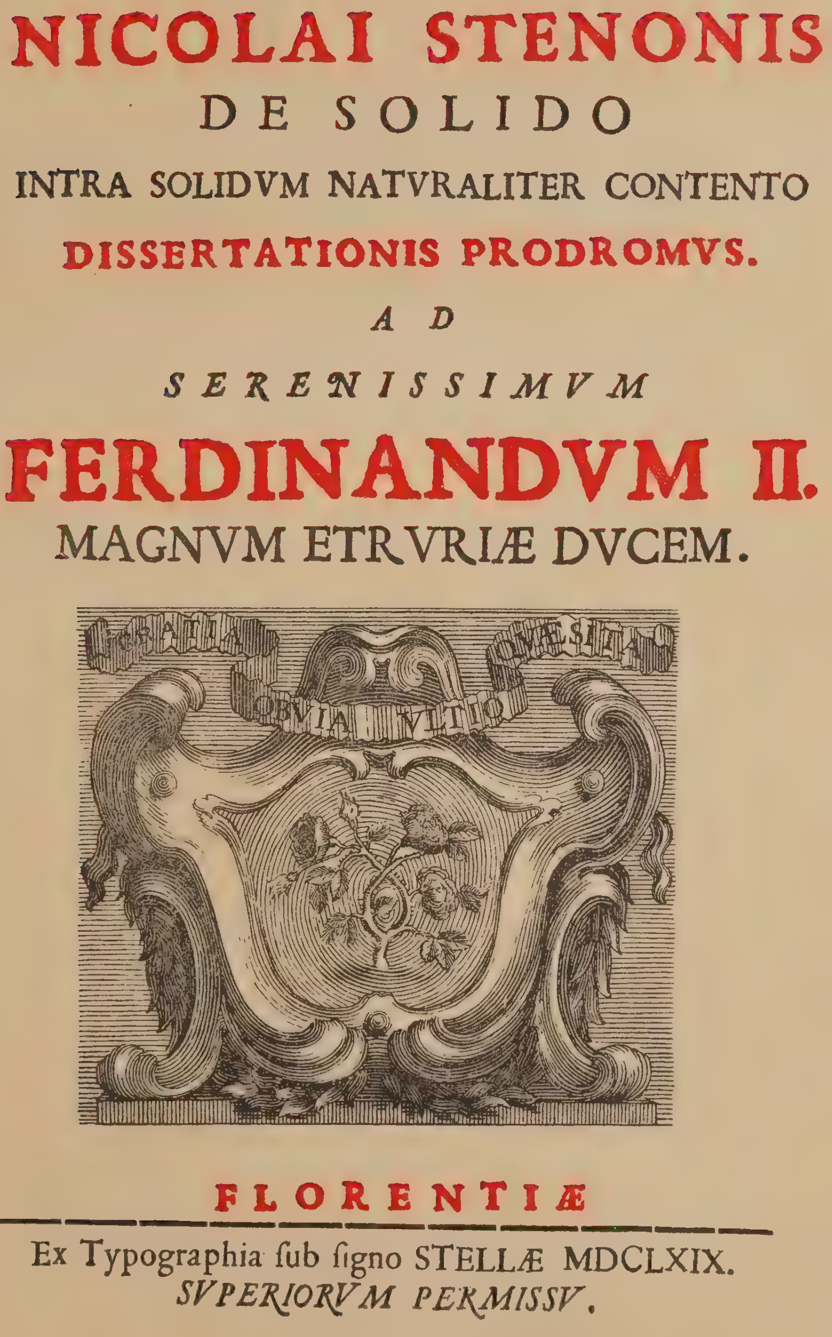
[1] Translation:
The Prodromus of Nicolaus Steno's
Dissertation
Concerning a solid naturally contained
Within a solid

To
The Most Serene
Ferdinand II
Grand Duke of Tuscany

Florence

From the press under the sign of the star, MDCLXIX
By order of the superiors.

[Plate VII.]



NICOLAI STENONIS
DE SOLIDO
INTRA SOLIDVM NATVRALITER CONTENTO
DISSERTATIONIS PRODROMVS.
AD
SERENISSIMVM
FERDINANDVM II.
MAGNVM ETRVRIÆ DVCEM.

GRATIA
QVÆSITA
OBVIA VLTIO

FLORENTIÆ

Ex Typographia ſub ſigno STELLÆ MDCLXIX.
SVPERIORVM PERMISSV.

X, XI), reproducing them, however, in the original size,[1] and preserving the numerical order. The 'explanation of the figures' in the original edition is printed on an accompanying double-page folding inset. Page 79, not numbered, contains a list of corrections, which is far from complete.

There are two copies of the original edition in the British Museum. The copy which I examined (press mark 537. b. 1) has a plate preceding the text and a duplicate of it following the *explicatio figurarum*. A number of marginal corrections of the text have been made by some scholarly reader, for they are essential in every case. There is also a copy in the Library of the Royal Society, London; it is bound up with various tracts, and contains marginal corrections, but by a less careful hand. In this copy the plate and 'explanation of figures' are inserted in the front of the treatise.

The only copy which I have found in the United States is in the New York Public Library, and carries the stamp of the Astor Library on its title-page. It is in perfect condition and without marginal corrections.

II. REPRINTS OF THE ORIGINAL EDITION

1. *Nicolai Stenonis De Solido Intra Solidum Naturaliter Contento — Dissertationis Prodromus. Ad Serenissimum Ferdinandum II Magnum Etruriae Ducem.* Lugd. Batav. Apud Jacobum Moukee, 1679.

The copy of the Leyden reprint in the British Museum is a neatly printed duodecimo of 115 pages, followed by the *explicatio figurarum* and plate. The lower half of the plate has been torn off, and the upper part is bound upside down. Several other treatises are bound up with the *Prodromus.*

2. *Viri Celeberrimi Nicolai Stenonis Dani De Solido Intra Solidum Naturaliter Contento — Dissertationis Prodromus. Ad Serenissimum Ferdinandum II Magnum Etruriae Ducem Editio Secunda Etrusca.* Pistorii A. S. MDCCLXIII. Ex Typographio Publici. Praesidibus Permittentibus. Prostant etiam Florentiae apud Vicentium Landi Bibliopilam prope Monasterium Monachorum Cassinensium.

According to Maar,[2] the Bibliotheca Nazionale Braidense, Milan,

---

[1] Figure 17 alone is slightly reduced.

[2] *Opera Philosophica*, Vol. II, p. 355.

contains 28 pages and a reduced plate. The text is accompanied by brief notes in Latin. This edition, like that of Beaumont, is very incomplete.

IV. TRANSLATIONS

1. *The Prodromus to a Dissertation Concerning Solids Naturally Contained within Solids. Laying a Foundation for the Rendering a Rational Accompt both of the Frame and the several Changes of the Masse of the EARTH, as also of the various Productions in the same. By Nicolaus Steno.* English'd by H. O. London. Printed by J. Winter, and are to be Sold by Moses Pitt at the White-Hart in Little Brittain, 1671.

This translation is a bibliographical rarity, as are all editions of Steno. H. O. is undoubtedly Henry Oldenburg, who was elected Secretary of the Royal Society, April 22, 1663, and continued to act in that capacity until November 30, 1677, at a salary of forty pounds a year.[1]

Maar[2] mentions only the copies in the Royal Library of Copenhagen and the British Museum. There is also a copy in the Library of the Geological Society, London, and another in the Harvard University Library. It is odd that the Royal Society should not have been presented with a copy by the author; possibly one may have been presented, and lost. The British Museum copy is imperfect, the plate being lacking. The page facing the preface bears the name *Jos. Banks*. This can be no other than Sir Joseph Banks, the celebrated English naturalist, who became President of the

---

[1] See *Record of Royal Society*, 3d ed., 1912, p. 207; and Birch, *History of the Royal Society*, Vol. II (1756), p. 376. Oldenburg was succeeded by Robert Hooke. Under the date of June 25, 1667, Samuel Pepys remarks: "I was told, yesterday, that Mr. Oldenburg, our Secretary at Gresham College, is put into the Tower, for writing news to a virtuoso in France, with whom he constantly corresponds in philosophical matters; which makes it very unsafe at this time to write, or almost do any thing." And again, under date of April 30, 1669: "This morning I did visit Mr. Oldenburgh, and did see the instrument for perspective made by Dr. Wren, of which I have one making by Browne; and the sight of this do please me mightily."

For the records of Oldenburg's arrest on June 20, 1667, and his release on August 26 of the same year by orders of Charles II, see C. R. Weld, *History of the Royal Society*, London, 1848, vol. I, pp. 201–204. An account of Oldenburg's life may be found *ibid.*, pp. 259–261.

A brief notice of the H. O. edition was contributed, probably by Oldenburg himself, to the Royal Society. See *Philosophical Transactions*, vol. VI (1671), p. 2179 ff.; *Abridged Edition of Philosophical Transactions*, vol. I (1665–1672), London, 1809, pp. 605, 606.

[2] *Opera Philosophica*, Vol. II, p. 356.

Royal Society in 1778 and who is known to have bequeathed his collection of books and botanical specimens to the British Museum. The copy in the Library of the Geological Society is inscribed with the name *Rob. Dav* on the upper right-hand corner of the fly-leaf, where the last part of the name, Davis, has been worn away.

The Harvard copy is thus described by its donor, Professor J. B. Woodworth, in *Science*, Vol. 25 (1907), pp. 738, 739: "There are sixteen pages of preface with the title-page, and 112 pages of text and one plate; the size of the printed part of the page measures 2.75 inches wide by 5.5 inches high. . . . The copy in the writer's possession is bound up as a separately paged tract at the end of a small volume of the celebrated Robert Boyle's 'Essays of Effluvium, etc.,' containing also his 'Essay about the Origine and Virtue of Gems' of 1672.[1] A general title-page gives reference to Steno's work. This title-page is dated 1673.[2] All of the contained tracts appear to have been separately printed at different dates between 1671 and 1673, at which last date they were brought out in the form above described."[3]

---

[1] In this connection the following item from the *History of the Royal Society* (Vol. III, p. 55) is of interest: "Mr. O. presented from Mr. Boyle his *Essay about the Origin and Virtues of Gems*, printed at London, 1672, in 8vo."

[2] Woodworth's title-page, as reproduced in *Science*, vol. 25, p. 738, ascribes the publication of the treatise to *F. Winter*. The letter is not *F* but a quaintly formed *J*, as is clear from the reappearance of the same letter in the spelling of the word *juyces* in the "Interpreter to the Reader." Maar, *op. cit.*, Vol. II, p. 336 also printed *F*, but in a letter in reply to my contention writes: "Of course you are quite right. It is a *J* and not an *F* on the title-page of Steno's treatise."

[3] Dr. Maar is the possessor of a similar copy which is not described in his *Opera Philosophica*. The title-page reads:

ESSAYS
Of the
Strange Subtility
Determinate Nature } of EFFLUVIUMS
Great Efficacy
To which are annext
New Experiments to make FIRE
and FLAME Ponderable.
Together with
A Discovery of the Perviousness of *Glass*.
ALSO
An ESSAY, about the Origine
and Virtue of GEMS.
By the Honorable Robert Boyle,
Fellow of the Royal Society.
To which is added
The PRODROMUS to a Dissertation
concerning *Solids* naturally contained with-

The translation is preceded by an address or preface bearing the title: "The Interpreter to the Reader." Since this is of unusual interest because of the writer's testimony regarding the results of the independent investigation of the nature of gems by Robert Boyle, we print it here in full:

"Reader,

"This Ingenious Piece, lately publish't in Italy, (where 'twas Printed in Latin), and thence come to the hands of the Interpreter, was thought fit to be English'd, chiefly upon this occasion, That the Stationer, that hath Printed it, did, upon Information given Him of the Valuable Contents thereof, earnestly sollicite, that it might forth-with be put into this Language; he not only conceiving, that there being now very little or no commerce between the English Book-sellers, and those of Italy, the conveyance of this Book, (as it doth of others there Printed) into England would prove very tardy; but also considering, that though within a reasonable time some Copies of it should come over, yet there would not be enough of them, to serve all sorts of curious English-Men, nor even that number of English Readers versed in the Latin Tongue, which this Considerable Discourse is like to meet with, forasmuch as it giveth very fair hopes, That by a due weighing of the particulars, therein laid down, the sagacious Inquirers into Nature may be much assisted to penetrate into the true knowledge of one of the Great Masses of the World, the EARTH, and therein to find out not only the Constitution of the Whole, but also the several Changes, and the various Productions made in the Parts thereof; as the Excellent Robert Boyle[1] hath of late Years, with great Acuteness as well as

---

in *Solids* Giving an Account of the *Earth*
and its Productions.
By *Nicholas Steno*. Englished by H. O.
London, Printed by *W. G.* for *M. Pitt*, at
the *Angel* near the little North Door
of St *Paul's* Church. 1673.

A copy of this edition, consigned to the translator, was lost in the sinking of the "Hesperian," September 6, 1915. Another copy of the H. O. translation, now in the translator's possession, is an independent volume in its original calf binding. To the bibliographical data presented above, it may be interesting to add that the first page of the translation is adorned with the conventional emblems of the rose, fleur-de-lis, and thistle, each surmounted by the crown of Charles II as Sovereign of the United Kingdom. The book is very rare.

[1] The reference is to *Memoirs for a General History of the Air* (which, however, was not published until 1692), by Robert Boyle (1627–1691), in the edition of P. Shaw, Vol. III, Lon-

unwearied Industry, led us on a great way in the knowledge of another of the great Masses, the AIR, though the same also hath not been unmindful of considering this very subject, here treated of; forasmuch as He, before he would see or hear any thing of this Prodromus, did upon occasion candidly declare to the Author of this Version, (who bona fide here publickly attests it,)

" First, That he doth, upon several inducements, suppose, the generality of Transparent Gems or Precious Stones to have been once Liquid substances, and many of them, whilst they were either fluid, or at least soft, to have been imbraced with Mineral Tinctures, that con-coagulated with them; whence he conceiveth, that divers of the real qualities and vertues of Gems (for he doubts, most ascribed to them are fabulous) may be probably derived. And as for Opacous Gems and other Medical Stones, as Bloud-Stones, Jaspers, Magnets, Emery, etc. He esteems them to have, for the most part, been Earth (perhaps in some Cases very much diluted and soft,) impregnated with the more copious proportion of fine Metallin or other Mineral Juyces or Particles; all which were afterwards reduced into the forme of Stone by the supervenience (or the exalted action) of some already in-existent petrescent liquor or petrifick Spirit, which he supposeth may sometimes ascend in the forme of Steams; from whence may be probably deduced not only divers of the Medical Vertues of such stones, but some of their other Qualities, as Colour, Weight, etc. and also explained, How it may happen what He hath (and, he doubts not, others may have also) observed of Stones of another kind, or Marcasites, or even Vegetable and perhaps Animal substances, that have been found inclosed in solid Stones; For, these Substances may easily be conceived to have been lodged in the Earth, whilst it was but Mineral Earth or Mud; and afterwards to have been, as 'twere cased up by the supervenient Petrifick Agents that pervaded it.[1]

" Nor are these Petrescent liquors the only ones, to which he supposeth that many Fossils may owe their Origin, since he thinks,

---

don, 1725, pp. 15–98. The edition of Boyle's works by Thomas Birch (first edition, 1744, in five volumes, the second, 1772, in six) has not been accessible to me.

[1] " Of these Pretious Stones this Noble Philosopher was pleased to leave with the Publisher a Manuscript of his composure, now ready to be Printed, which he assur'd him it had been several Years ago."

H. O. refers to Boyle's *The Origin and Virtues of Gems*, published in 1672. Cf. Shaw, Vol. III, pp. 99–143, and above, p. 198.

there may be, (if one may so speak) both Metallescent and Mineralescent Juyces in the bowels of the Earth, and that sometimes they may there exist and operate under the forme of Spirits or Steams.[1]

"Beside this, we cannot but take notice here of what was intimated a good while ago in Numb. 32. of the Phil. Transactions, p. 628, viz. That Mr. Robert Hook had at that time ready some Discourses upon this very Argument, which, by reason of the many avocations he hath met with in the rebuilding of the City of London, and his attendance on the R. Society, he hath not yet been able quite to finish for the Press.

"Now this being so, that several judicious Persons do employ themselves in the inquiry after the Observables in the greater Parts of the World, there is no question but many remarkable things will be detected therein; and, (to speak more generally on this occasion,) since 'tis apparent, that the Ingenious and Diligent almost everywhere are entring more and more into Philosophical Leagues, for the discovery of the works of God and the Operations of Nature, we cannot but entertain pregnant hopes, that notwithstanding all the oppositions of Lazy and Envious Men, a good harvest of considerable and useful knowledge will be reaped in time, and thence good store of fruitful seed be ministred for large successive crops of the same kind, for the magnifying of our great Creatour, and the Enobling and benefiting of Man-kind."

2. *Extrait De La Dissertation de Nicolas Sténon sur les corps solides qui se trouvent contenus naturellement dans d'autres corps solides.* In *Collection Académique de Dijon, Partie Etrangère*, IV, 1757, pp. 377–414.

3. *Prodromus d'une dissertation sur le solide contenu naturellement dans un autre solide; extrait et traduit par M. Elie de Beaumont.* Paris, 1832.

This translation appeared in *Annales des Sciences Naturelles*, Paris, 1832, Vol. 25, pp. 337–377. The article is entitled *Fragmens géologiques tirés de Stenon, de Kazwini, de Strabon et du Boun-Dehesch.*

Copies of Beaumont's translation may be found in the libraries of the Royal Society and the Geological Society, London. The title-page of the edition in the Geological Society shows that Beaumont based his work on the Leyden edition of 1679. It is, however, a summary rather than a translation, emphasizing chiefly the section dealing with the origin of mountains.

---

[1] "About which he also was willing not only to shew to the Publisher several Observations and Collections of his in the forme of Discourses, but also to put them into his hands to peruse the same."

4. *Nicolaus Steno Foreløbig Meddelelse Til en Afhandling Om Faste Legemer, Der Findes Naturlig Indlejrede I Andre Faste Legemer I Oversættelse Ved August Krogh Og Vilhelm Maar Med Indledning Og Noter*, København, MCMII. Gyldendalske Boghandels Forlag Langkjærs Bogtrykkeri.

This is a quarto volume containing a portrait of Steno, an Introduction, pp. i–xii, the Translation, pp. 3–89, the Explanation of Figures, pp. 91–93, and a half-sized reproduction of the original plate. The attestations are given on page 97, not numbered, and page 98. Pages 101–106 contain notes. The edition is limited to 700 numbered copies.

V. SELECTED REFERENCES

Angelis, de, Article *Sténon* in *Biographie Universelle (Michaud), Ancienne et Moderne*, Nouvelle Edition, Tome Quarantième, Paris, pp. 209–211.

Capellini, G., *Di Nicola Stenone e dei suoi studi geologici in Italia*, University of Bologna, 1870.

Chéreau, Article *Sténon* in *Dictionnaire Encyclopédique des Sciences Médicales*, Troisième Série, Tome Onzième, Paris, 1883, pp. 689–691.

Eloy, N., *Dictionnaire Historique de la Médicine*, Tome Second, Liège, 1755, pp. 391–393.

Fabronius, A., *Vitæ Italorum Doctrina Excellentium Qui Saeculis XVII et XVIII Floruerunt* (Pisis, 1778–1805), Vol. III (1779).

Geikie, A., *The Founders of Geology* (2d ed., London, 1905), pp. 53–60.

Gosch, C. C. A., *Udsigt over den danske Zoologiske Literatur*, 2 Afdeling, 1 Hefte, København, 1872.

Hughes, T. M., *Steno*, in *Nature*, Vol. 25 (1882), pp. 484–486.

Huxley, T. H., *The Rise and Progress of Palæontology, Discourse at York* (Meeting of the British Association), in *Nature*, Vol. 24 (1881), pp. 452–455.

Jorgensen, A. D., *Niels Stensen*, København, 1884.

Köcher, A., *Herzog Johann Friedrich, Bischof Steno, u. Pastor Petersen*, in *Zeitschrift des historischen Vereins für Niedersachsen*, 1889, pp. 204–212.

Lorenzen, A., *Niels Stensen, Der Vater der Geologie*, in *Die Natur*, Vol. 3 (1854), p. 220 ff.

Lyell, C., *Principles of Geology* (9th ed., New York, 1853), pp. 21–24.

Manni, D. M., *Vita del Litteratissimo Mgr. Niccolo Stenone*, Firenze, 1775.

Plenkers, W., *Der Däne Niels Stensen, Ein Lebensbild nach den Zeugnissen der Mit- und Nachwelt entworfen*, Freiburg im Br., 1884.

Spencer, L. J., Article *Crystallography*, in *Encyclopædia Britannica*, 11th ed., Vol. VII, pp. 569, 570; cf. also Article *Steno*, *ibid.*, Vol. XXV, p. 879.

Sollas, W. J., *The Influence of Oxford on the History of Geology*, in *Science Progress*, Vol. 7 (1898), pp. 25–29.

Wichfeld, J., *Erindringer om den Danske Videnskabsmand Niels Stensen*, in *Historisk Tidsskrift*, 3 Række, 4 Bind (Kjøbenhavn, 1865), pp. 1–109.

Woodworth, J. B., *Steno*, in *Science*, Vol. 25 (1907), pp. 738, 739.

Von Zittel, K. A., *Geschichte der Geologie und Paläontologie bis Ende des 19 Jahrhunderts* (München u. Leipzig, 1899), pp. 32–36.

Steno ist der erste Forscher welcher geologische Probleme auf inductivem Wege zu lösen versuchte und zugleich eine klare Vorstellung davon hatte, dass die Geschichte der Erde aus ihrer Zusammensetzung und ihrem Aufbau ermittelt werden könne. Für die Entwickelung der Geologie blieben leider die Schriften dieses Scharfsinnigen Forschers ohne jegliche Bedeutung; sie wurden von den Zeitgenossen kaum beachtet, geriethen in Vergessenheit und fanden erst in diesem Jahrhundert durch Elie de Beaumont und Alexander von Humboldt die verdiente Anerkennung.

— *Von Zittel, Geschichte der Geologie und Paläontologie*, pp. 35, 36.

THE PRODROMUS

Most Serene Grand Duke:[1]

Travellers into unknown realms frequently find, as they hasten on over rough mountain paths toward a summit city, that it seems very near to them when first they descry it, whereas manifold turnings may wear even their hope to weariness. For they behold only the nearest peaks, while the things which are hidden from them by the interposition of those same peaks, whether heights of hills, or depths of valleys, or levels of plains, far and away surpass their guesses; since by flattering themselves they measure the intervening distances by their desire.

P. 2.[2] So, and not otherwise, is it with those who proceed to true knowledge by way of experiments; for as soon as certain tokens of the unknown truth have become clear to them, they are of a mind that the entire matter shall be straightway disclosed. And they will never be able to form in advance a due estimate of the time which is necessary for loosing that knotted chain of difficulties which, by coming forth one by one, and from concealment, as it were, delay, by the constant interposition of obstacles, them that are hastening toward the end. The beginning of the task merely reveals certain common, and commonly known, difficulties, whereas the matters which are comprised in these difficulties—now untruths which must be overthrown, now truths which must be established; sometimes dark places which must be illumined, and again, unknown facts which must be revealed—shall rarely be disclosed by any one before the clew of his search shall lead him thither. Democritus,[3] not unwisely, was wont to use the illustration of a well, wherein one could scarcely estimate aright the task and time of draining it dry, except by draining it dry, since both the number and the volume of the hidden springs leave the amount of the intake a matter of doubt.

---

[1] Ferdinand II, Grand Duke of Tuscany; see p. 179.

[2] The pagination is that of the original publication, which is reproduced in the Berlin Facsimile (p. 196).

[3] Steno doubtless had in mind the proverb recorded by Diogenes Laertius (IX. 72); ἐτεῇ δὲ οὐδὲν ἴδμεν· ἐν βυθῷ γὰρ ἡ ἀλήθεια, 'In reality we know naught, for truth lies in a well.' βυθός, strictly speaking, denotes the depth of the sea (cf. Æschylus, *Prometheus*, 432). It is in this sense that Cicero (*Academica Prior.*, ii. 10, 32) repeats the proverb: *naturam accusa, quae in profundo veritatem, ut ait Democritus, penitus abstruserit*, 'Accuse nature, which has

Do not be surprised, therefore, Most Serene Prince, if, for a whole year's time, and, what is more, almost daily, I have said that the investigation for which the teeth of the shark[1] had furnished an opportunity, was very near an end. For having once or twice seen regions where shells and other similar deposits of the sea are dug up, when I observed that those lands were sediments of the turbid sea and that an estimate could be formed of how often the sea had been turbid in each place, I not only
P. 3. over-hastily fancied, but also dauntlessly informed others, that a complete investigation on the spot was the work of a very short time. But thereafter, while I was examining more carefully the details of both places and bodies, these day by day presented points of doubt to me as they followed one another in indissoluble connection, so that I saw myself again and again brought back to the starting-place, as it were, when I thought I was nearest the goal. I might compare those doubts to the heads of the Lernean Hydra, since when one of them had been got rid of, numberless others were born; at any rate, I saw that I was wandering about in a sort of labyrinth, where the nearer one approaches the exit, the wider circuits does one tread.[2]

But I shall not tarry to excuse this tardiness of mine, since it is abundantly evident to you, from long experience, how per-

---

hidden the truth completely, as Democritus says, in the depth.' Cf. also *Acad. Post.*, i. 12, 44: *Democritus (dixit) in profundo veritatem esse demersam.*

Steno's use of the word *puteus* ('well') accords with the expression of the proverb in his time. Rabelais (*Pantagruel*, iii. 36) incorrectly ascribes the saying to Heraclitus: *Je suis descendu au puiz tenebreux, auquel disoit Heraclitus estre vérité cachée.* A curious addition to the original was made by Francis Bacon (edition of Spedding, Ellis, Heath, Vol. XIII, p. 383) when he wrote, "Democritus said 'that truth did lie in profound pits, and when it was got, it needed much refining.'"

The proverb has given rise to several allegorical paintings of Truth in which the well figures prominently. Among these are Paul Baudry's "Truth," J. J. Lefebure's "Truth," both in the Luxembourg; and Titian's so-called "Sacred and Profane Love," in the Borghese Gallery.

[1] *Canis Carcharia.* Steno's treatise *Canis Carchariae Dissectum Caput* is dated 1667 and is reprinted by Maar, *N. Stenonis Opera Philosophica*, Vol. II, pp. 113–145. Cf. p. 125, especially.

[2] The language is reminiscent of Seneca, *Epistles*, 44. 7.

plexing is a matter which is involved in a series of experiments. But the fact that, after a large part of the task assayed had been completed, I should drop everything and ask your leave to return to my native land to pursue anatomical investigation — this indeed would demand an excuse did I not know that this obedience on the part of a subject of another prince would not be displeasing to you, which in a similar circumstance would please you on the part of your own subjects. And this hope of mine concerning your kindness is made surer by that exceptional goodwill, whereby, through devoting generous assistance[1] to the advancement of my studies, you wished that unrestricted opportunity for learning should be left to me whenever occasion might arise. Therefore, since I no longer dare to hope for the time neces-
P. 4. sary for finishing the tasks which I have begun, I shall do in the payment of my promises what has been conceded by common custom to debtors; when they have not the means to pay in full, they pay what they have, in order that they may not be forced to withdraw from business. Since, then, I am unable to complete all the things which were to be shown to you, I shall offer the chief of what I have found, in order that I may not appear to have deceived you.

I should gladly have postponed everything until it had been possible for me, on my return to my native land, to perfect the details, were I not awaiting the same fortune there which I have hitherto experienced everywhere, in that new tasks have constantly stood in the way of finishing those first undertaken. While I was intent upon counting the glands of the entire body,[2] the wonderful structure of the heart[3] carried me away into an examination of it; and the deaths of my kin[4] interrupted the studies I had begun on the heart. Your seas furnished us a shark[5] of marvellous size to keep me from applying myself to the detailed description of the muscles; and now, when I am wholly devoted to my present experiments, he whose command the law of nature bids me heed, and whose great kindness toward me and mine constrains me, calls me to other things.[1]

---

[1] See p. 179.

[2] For Steno's work on the glands, see pp. 176, 188 f.

[3] Steno first mentions his study of the heart in a letter to Thomas Bartholin dated "the last of April," 1663, Leyden. See Maar, *Opera Philosophica*, Vol. I, p. 155. In 1667 Steno published his *Elementorum Myologiae Specimen, Seu Musculi Descriptio Geometrica.* Cf. p. 190.

[4] While studying in Leyden, 1664, Steno learned of the death of his step-father, Johannes Stichman. The death of Steno's mother occurred soon after his arrival in Copenhagen. See p. 178.

[5] For the treatise *Canis Carchariae Caput*, see p. 206, note 1. Compare also *Historia Dissecti Piscis Ex Canum Genere*, Maar, *op. cit.*, Vol. II, pp. 147–155.

To what end all these matters may come, I do not care to inquire anxiously, lest it be, peradventure, to accredit to myself things which are due to a higher cause. If long contemplation had added something of my own, as it were, to discoveries not my own, certainly if I had tarried longer in working out one discovery, I should myself have shut the door to the finding of the rest. And so, not knowing what other experiments and P. 5. studies may await me elsewhere, I thought it best to set forth here these matters *concerning a solid naturally contained within a solid*, which shall be a pledge to you of gratitude for the favors I have received and shall afford to others, who are enjoying their desired leisure, an opportunity of pursuing their studies of physics and geography with greater profit.

As regards the production of a solid naturally contained within a solid, I shall first sketch briefly the method of my Dissertation, then explain concisely the more noteworthy matters which appear in it.

The Dissertation itself I had divided into four parts, of which the first, taking the place of an introduction, shows that the inquiry concerning sea objects found at a distance from the sea, is old, delightful, and useful; but that its true solution, less doubtful in the earliest times, in the ages immediately following was rendered exceedingly uncertain. Then after setting forth the reasons why later thinkers abandoned the belief of the ancients, and why, although one may read a great many excellent works by many authors, the question at issue has hitherto been settled by no one anew,[2] I show, returning at length to you, that besides very many other things which under your auspices have in part been discovered, and in part freed from old doubts, to you is due our trust that the finishing touch shall soon be put upon this investigation also.

---

[1] Steno refers to the invitation of Frederik III; cf. p. 181. He got no farther than Amsterdam, however. See p. 182.

[2] There is an interesting discussion on the nature and origin of fossil shells in Bernard Palissy's *Des Pierres* (*Discours Admirables*, etc., 8vo, Paris, 1580). Referring to Palissy, Fontenelle (*Histoire de l'Académie des Sciences*, Année 1720, p. 5) remarks: "Un potier de terre, qui ne savait ni latin ni grec, fut le premier qui, vers la fin du XVI^e siècle, osa dire dans Paris, et à la face de tous les docteurs, que les coquilles fossiles étaient de véritable coquilles déposées autrefois par la mer dans les lieux où elles se trouvaient alors, que des animaux, et surtout des poissons, avaient donné aux pierres figurées toutes leurs différentes figures; et il

In the second part is solved a universal problem upon which depends the unravelling of every difficulty, and it is this: *given a substance possessed of a certain figure, and produced according* P. 6. *to the laws of nature, to find in the substance itself evidences disclosing the place and manner of its production.* In this connection, before I proceed to unfold the solution of the problem, I shall strive to expound all its terms, with the view of leaving no school of philosophers in doubt, and in dispute, as to their significance.

The third part I have reserved for the investigation of different solids contained within a solid, in accordance with the laws discovered in the solution of the problem.

The fourth part describes various conditions in Tuscany not treated by historians and writers upon natural subjects, and sets forth a process of the universal deluge which is not at variance with the laws governing movements of nature.

I had indeed begun to set forth these things in Italian, both because I knew this would please you, and in order that it might appear to the illustrious Academy[1] which has enrolled

---

défia hardiment toute l'école d'Aristote d'attaquer ses preuves." The quotation is taken from Flourens, *De la Longévité humaine et de la Quantité de Vie sur le Globe*, Paris, 1855, pp. 200, 201.

The dialogue in Palissy's *Discours* is between *Theorique* and *Practique*, whose contention may be illustrated by the following quotation:

"Et par ce qu'il se trouue aussi des pierres remplies de coquilles, iusques au sommet des plus hautes montagnes, il ne faut que tu penses que lesdites coquilles soyent formees, comme aucuns disent que nature se iouë à faire quelque chose de nouveau. Quand i'ay eu de bien pres regardé aux formes des pierres, i'ay trouué que nulle d'icelles ne peut prendre forme de coquille ny d'autre animal, si l'animal mesme n'a basti sa forme: parquoy te faut croire qu'il y a eu iusques au plus haut des montaignes des poissons armez et autres, qui se sont engendrez dedans certains cassars ou receptacles d'eau, laquelle eau meslee de terre e d'un sel congelatif et generatif, le tout s'est reduit en pierre auec l'armure du poisson, laquelle est demeuree en sa forme. . . . Il faut donc conclure que auparauant que cesdites coquilles fussent petrifiées, les poissons qui les ont formées estoyent viuans dedans l'eau qui reposoit dans les receptacles desdites montagnes, et que depuis l'eau et les poissons se sont petrifiez en un mesme temps, et de ce ne faut douter." *Œuvres Complètes de Bernard Palissy*, by Paul-Antoine Cap, Paris, 1844, pp. 277, 279.

[1] The Accademia del Cimento; see p. 180. This Academy came to an end in 1667 when its founder, Leopold de' Medici, became a Cardinal.

me among its members, that as I am least worthy of so great an honor, so am I most desirous of proving the attempts whereby I am striving to attain some knowledge of the Tuscan tongue. But I am not grieved that the necessity has been placed upon me of postponing that writing; for as my present journey promises me a fuller knowledge of matters serving to elucidate my investigation, so the delay assures me of a happier advancement in my study of the language.

It would be a long task to write out in detail all my observations, together with the conclusions drawn from them, developed in accordance with the method suggested; wherefore, I shall report sometimes conclusions, and again observations, as may seem best, in order to explain the chief points briefly, and
P. 7. as clearly as possible.

The reason why, in the solution of natural questions, not only do many doubts remain undecided but, for the most part, such doubts multiply with the number of writers, seems to me to depend chiefly upon two causes.

The first cause is that few take it for granted that all those difficulties, without whose solution the settlement of the question itself is left marred and incomplete, must be removed. The present inquiry illustrates this point. Only a single difficulty vexed the ancients, that is in what way marine bodies had been left in places far from the sea; and the question was never raised whether similar substances had been produced elsewhere than in the sea.[1]

---

[1] The presence of fossil shells in places remote from the sea is discussed in the *Geography* of Strabo (*circa* 67 B.C.–19 A.D.), C. 49, 50 (I. 3, 4):

'He (Eratosthenes) says that a particularly interesting subject of inquiry is afforded by the fact that an abundance of cockle, oyster, and scallop shells, and salt-water lakes are frequently seen far inland, two or three thousand stadia from the sea, as in the case of the temple of Ammon and the road leading up to it for a distance of three thousand stadia. For a profusion of oyster shells, salt beds, and salt springs can still be found there at the present time. In addition to this, wrecks of sea-going vessels are pointed out which were said to have been cast up through some chasm. . . .

'In this he agrees with the opinion of Strato, the physicist, and of Xanthus, the Lydian. Xanthus asserts that in the reign of Artaxerxes there was so great a drought that the rivers, lakes, and wells dried up; and that he had frequently found, far from the sea, fossil shells, some like cockles and others like scallops, as well as salt lakes in Armenia, Matiana, and Lower Phrygia. On this account he was convinced that what are now plains had once been sea. Strato, who searched more deeply for the causes of these phenomena, believed that the Euxine formerly had no outlet at Byzantium, but the rivers which emptied into the Euxine had forced an opening, and that the water thereupon fell into the Propontis and the Hellespont.

In more recent periods the difficulty of the ancients was pressed with less insistence, since almost all were concerned with tracing out the origin of the bodies mentioned. They who ascribed them to the sea accomplished this result: they proved that bodies of this character could not have been produced by any other agency. They who attributed these bodies to the land, denied that the sea could have covered the places where they were found, and were wholly engaged in praising the forces of a Nature of which they had little knowledge — forces fitted to produce anything whatsoever. Perhaps a third opinion may properly be admitted, in accordance with which a part of the bodies under consideration is regarded as attributable to the land, and a part to the sea. Nevertheless there is deep silence almost everywhere concerning the doubt of the ancients, except that some make mention of floods, and a sort of immemorial succession of years, but merely in passing and, as it were, in P. 8. treating another subject. In order, therefore, to satisfy, to the best of my ability, the laws of analysis, I wove and unwove the web of this research, and scrutinized its various details again and again, until I saw no difficulty left any longer in the reading of authors, or in the objections of friends, or in the examination of places, which I had not either solved, or at least determined, from facts hitherto known to me, to what extent a solution was possible.

The first question was, whether *Glossopetrae Melitenses*[1] were

---

'The same thing happened in the case of the Mediterranean. For the sea, after having been filled by the rivers emptying into it, had broken a passage through at the Pillars (Gibraltar), and the places formerly covered with shoal water, were left dry by this eruption. He (Strato) finds the cause for this, first, in the fact that the bottoms of the Atlantic and of the Mediterranean are not on the same level, and, secondly, in the fact that even now a sand-bank runs beneath the water from Europe to Libya, bearing witness to the time when the Mediterranean and the Atlantic were not united. Strato also said that the waters around Pontus are very shallow, whereas off Crete, Sicily, and Sardinia they are very deep. . . .

'Egypt, too, he said, was formerly covered by the sea as far as the marshes near Pelusium (Tineh) and Mount Casius (El-Kas) and Lake Sirbonis (Lake Sebaket-Bardoil). Even now, when salt is dug in Egypt, the beds are found beneath layers of sand and mixed with fossil shells, as if the country had formerly been under the sea, and all the region around Casium and Gerra (Maseli) had a shoal extending to the Arabian Gulf.'

[1] The literal translation of *Glossopetrae Melitenses* is 'tongue-stones from Malta.' In the treatise *Canis Carchariae Dissectum Caput* (1667), Steno was not free from doubt as to the origin of the 'stones,' as shown by the following passage (Maar, *op. cit.*, Vol. II, pp. 127, 128):

No decision has yet been reached regarding the larger *glossopetrae*, as to whether they are shark's teeth or stones formed in the earth. Some have maintained that substances found in once the teeth of sharks; this, it was at once apparent, is identical with the general question whether bodies which are similar to marine bodies, and which are found far from the sea, were once produced in the sea. But since there are found also on land other bodies resembling those which grow in fresh waters, in the air, and in other fluids, if we grant to the earth the power of producing these bodies, we cannot deny to it the possibility of bringing forth the rest. It was necessary, therefore, to extend the investigation to all those bodies which, dug from the earth, are observed to be like those bodies which we elsewhere see growing in a fluid. But many other bodies, also, are found among the rocks, possessed of a certain form; and if one should say that they were produced by the force of the

---

the earth, resembling parts of animals, are the remains of animals which once lived on the spot; while others believe that such substances were formed in the earth without reference to animals. I have not sufficient knowledge in these matters to venture an opinion at this time. Although my travels [Steno accompanied Ferdinand II in his travels through Tuscany; cf. Plenkers, *Niels Stensen*, pp. 31, 58] have led me through various regions of this sort, I would not presume to assert that the places which I shall see in the rest of my journey will correspond to those I have already examined; especially since I have not yet seen the regions which my distinguished teacher Bartholin has examined in his journey in Malta. Just as in court, therefore, one man takes the rôle of defendant, another of plaintiff, while both submit to the decision of the judge, so I shall present, as a result of my observations, the reasons for ascribing such substances to animals. At another time I may set forth the reasons for the opposite belief, but I shall always await a true decision from those who are better informed.' For Steno's illustrations of *glossopetrae* see Maar, *op. cit.*, Vol. II, Tab. III.

Pliny (*Natural History*, XXXVII. 164) states that the *glossopetra*, resembling the human tongue, 'is not produced (*nasci*) in the earth, as tradition relates, but falls from heaven at the time of the waning moon.' Compare, further, O. Abel, article *Paläontologie und Paläozoologie*, in *Die Kultur der Gegenwart*, *Dritter Teil*, *Vierte Abteilung*, *Vierte Band* (Leipzig, 1914), pp. 313, 314:

"Albertus Magnus hatte noch die Möglichkeit zugegeben, dass die Versteinerungen nicht ausschliesslich Produkte der Virtus formativa seien, sondern dass auch die Leichenreste fossiler Tiere und Pflanzen dort zu Stein werden könnten, wo eine steinmachende Kraft ihren Einfluss ausüben könne. Ungefähr in denselben Bahnen bewegen sich die Vorstellungen von Georg Bauer, genannt Agricola (1494–1555); Haifischzähne, die er nach dem Vorbilde des älteren Plinius 'Glossopetren' nennt (eine Bezeichnung, die noch G. W. Leibniz für fossile Pottwalzähne 1749 gebrauchte), sind nach Agricola 'verhärtete Wassergemenge.'"

Ibid., p. 344: "Die Glossopetren des Plinius sind fossile Haifischzähne, und da solche in tertiären Bildungen zu den häufigeren Wirbeltierresten gehören, so erregten sie schon frühzeitig die Aufmerksamkeit. Es berührt eigentümlich, noch heute einen einfachen Taglöhner, der gewiss nicht die mindeste Ahnung von der Literatur der Scholastenzeit besitzt, einen fossilen Haifischzahn, den er in seinem Bruche fand, als 'Vogelzunge' bezeichnen zu hören. Bis zur Zeit Knorrs und Walchs gingen Haifischzähne vorwiegend unter der Bezeichnung Zungensteine, Vogelzungen, Schwalbenzungen oder Schwalbensteine, Lamiodonten, Schlangenzungen usw. durch die Literatur, und noch Leibniz hielt an der Bezeichnung 'Glossopetra' des älteren Plinius fest."

place, one must confess that all the rest were produced by the same force. And so I saw the matter finally brought to the point that any given solid naturally contained within a solid must be examined in order to ascertain whether it was produced in the same place in which it is found; that is, the character **P. 9.** not only of the place where it is found, but also of the place where it was produced, must be investigated. But no one, in truth, will easily determine the place of production who does not know the manner of production, and all discussion concerning the manner of production is idle unless we gain some certain knowledge concerning the nature of matter. From this it is clear how many questions must be solved in order that a single question may be set at rest.

The second cause, the nurse of doubts, seems to me to be the fact that in the consideration of questions relating to nature those points which cannot be definitely determined, are not distinguished from those which can be settled with certainty. And the result is that the leading schools of philosophers are reduced to two classes. Some religiously refrain from putting credence even in demonstrations, out of fear that there be lurking in them the error which they have often found in other asserted truths. Others, on the contrary, would by no means allow themselves to be bound to consider as certain only those matters to which no one of sound mind and sound senses could deny credence, but believe that all things are true which have seemed to themselves fine and clever. Nay, the very advocates of experiment have rarely had sufficient self-control to refrain either from casting aside even most certain fundamental facts of nature, or from considering the fundamental facts discovered by themselves as proved. In order that I might, therefore, avoid [1] this rock also, I decided that in the sciences of nature we must enforce the principle which Seneca [2]

---

[1] In the Florentine edition of 1669, *evitare* is an obvious misprint for *evitarem*.

[2] Seneca nowhere, so far as I know, gives expression to the first part of Steno's sentence. But the language of the last part is identical with that of *Epistles*, 29. 11: *ex omni domo conclamabunt, Peripatetici, Academici, Stoici, Cynici*. The only point of difference is that Seneca has the future tense, *conclamabunt*, where Steno has the present, *conclamant*. The first part of Seneca's sentence does indeed mention 'the people,' but scarcely in a manner apropos of Steno's argument. Epicurus is represented as saying: 'I never wished to please the people. For the people does not approve what I know, and what the people approves, I do not know.' Steno may have been thinking of *de Beneficiis*, I. 11, 1, where benefits are classified as neces-

again and again inculcates concerning maxims of morality; he says that those are the best maxims of morality which are
P. 10. common, which are of the people, which all of every school proclaim, Peripatetics, Academics, Stoics, and Cynics. And certainly those statements of the fundamental facts of nature cannot fail to be best which are common, which are accepted of the people, which all of every school are held to acknowledge, both those who in everything are desirous of novelty, and those who are devoted to ancient doctrines.

I do not determine, therefore, whether the particles of a natural body, can or cannot undergo change, as its form does; whether there are or are not minute interstices; whether in those particles there is present, besides extension and hardness, something else unknown to us; for these expressions are not of common acceptance, and it is a weak argument to deny that there is anything else in a certain object because I do not discern anything else in it.

I do assert, however, without hesitation:

1. That a natural body is an aggregate of imperceptible particles which is subject to the operation of forces proceeding from the magnet, fire, and sometimes light also; in whatever way, indeed, passages may be found, whether between the particles, or in the particles themselves, or in both.

2. That a solid differs from a fluid in that in a fluid the imperceptible particles are in constant motion, and mutually withdraw from one another; while in a solid, although the imperceptible particles may sometimes be in motion, they hardly ever withdraw from one another so long as that solid remains a solid and intact.

P. 11. 3. That while a solid body is being produced, its particles are in motion from place to place.

4. That as yet we know of nothing in the nature of matter by the aid of which the principle of motion, and the perception of motion, can be explained; but that the determination of natural motions can be altered by three causes:

(1) By the motion of a fluid permeating all bodies; and we

---

sary, useful, and pleasant, and of *Ep.*, 12. 11, where 'the best things' are said to be 'common': *quae optima sunt, esse communia.*

say that those things which are produced in this way are produced naturally.

(2) By the motion of living beings; and many of those things which in this way are produced by man, are said to be artificial.

(3) By the first and unknown cause of motion; and even the pagans believed that there was something divine in motions which originate in this way. Surely to deny to this cause the power of accomplishing results contrary to the usual course of nature, is the same as to deny to man the power of changing the courses of rivers; or of battling with sails against the winds; or of kindling fire in places where without man fire would never be kindled; or of quenching a light which would not wane except with the ceasing of its supply; or of ingrafting the shoot of one plant upon the branch of another; or of producing summer fruits in mid-winter months; or of making ice in the very heat of summer; or of a thousand other things of the kind which are in conflict with the usual laws of Nature. For if we ourselves, who know not the structure of our own and other bodies, change the determination of natural motions every day, why cannot He change their
P. 12. determination who not only knows our structure and that of all things, but also made them? To be ready, again, to marvel at the cleverness of man acting with free will in the case of things done by his skill, and to deny a free agent to the products of Nature, would indeed seem to me to betoken great lack of penetration; since when man has performed the most ingenious things, he cannot, save through a cloud, discern what he has done, or what instrument he has used, or what that cause is by which the instrument is moved.

These details, proved both by experiments and by arguments, I shall set forth at greater length in the Dissertation itself, in order that it may be clear that there is no philosopher who does not say the same thing, although not always in the same words; or, if he has said otherwise, who does not, nevertheless, agree to the principles from which these details necessarily follow. For the statements I have affirmed concerning matter hold true in all cases, whether one considers matter as atoms, or particles changeable in a thousand ways, or the four elements, or whatsoever chemical elements may be assumed to suit the differences of opinion among chemists. And further, the statements which I have made concerning the determination of motion, are consistent with every agent, whether you call the agent the form, or the qualities proceeding from the form, or the idea, or the tenuous common substance, or the tenuous particular substance, or the individual soul, or the world soul, or the immediate act of God.

After these things I shall explain the various modes of speaking admitted by common usage, whereby we explain in **P. 13.** different ways the different production of different, and sometimes the same, substances; for whatever contributes anything to the production of any substance, does this either as place, or as matter, or as the agent. Hence when like produces like, it contributes to that object the place, the matter, and the motion of production, just as the small plant enclosed within the seed of some plant receives from that parent plant the matter in which it has been produced, the matter from which it has been produced, and the motion of the particles by which it has received its form; this same thing is true of animals enclosed within the egg of similar animals.

While the particular form or soul is producing something, the motion of particles in the production of that body is determined by some particular agent, whether this be the agent of another similar body or something else similar to this agent.

The things which are said to be produced by the sun receive the motion of their particles from the sun's rays, just as those which are attributed to the influences of the stars may receive the motion of their particles from the stars; for since it is certain that our eyes are affected by the light of the heavenly bodies, it will also be beyond cavil that the rest of matter may be affected by them in the same way.

The things which the earth bears receive from the earth nothing except the place in which they are produced and the matter supplied to them through the pores of the place.

The things which are produced by Nature receive the motion P. 14. of their particles from the motion of a penetrating fluid, whether this fluid come from the sun, or from the fire contained within the matter of the earth, or from some other cause unknown to us, as the agency of the soul, and so on.

He, therefore, who attributes to Nature the production of any thing, names the universal agent which appears in the production of all things; he who calls the sun to share, limits that agent a little more; he who names the soul or the particular form, mentions a more limited cause than the rest: but one who nevertheless duly weighs the answers of all, finds nothing known, seeing that Nature, the sun's rays, the soul, and the particular form, are things known only by name. But since, besides the agent, matter and place ought to be taken account of in the production of substances, it is clear that the answer ("produced by Nature") is not only more unknown than the very thing under investigation, but altogether incomplete; as, for example, mollusks found on land are said to have been produced by Nature, while those that grow in the sea are also Nature's work. Nature indeed produces all things, seeing that the penetrating fluid has a place in the production of all things; but one may also say with truth that Nature produces nothing, since that fluid by itself accomplishes nothing; its determination depends upon the place and the matter to be moved. We find an illustration in man: he can produce anything if all the necessary things are at hand, but if they are wanting, can produce nothing.

P. 15. He who attributes the production of anything to the earth, names the place indeed, but since the earth affords place, in part at least, to all the things of earth, the place alone does not account for the production of the body. The same thing can be said about the earth as about Nature; that is, the things which are formed in the earth are all produced by the earth, and of those things which are formed in the earth none is produced by the earth.

The few points set forth above suffice for the solution of all the doubtful issues in our inquiry, and I have desired to sum them up now in the three following propositions:

I

If a solid body is enclosed on all sides by another solid body, of the two bodies that one first became hard which, in the mutual contact, expresses on its own surface the properties of the other surface. Hence it follows:

1. That in the case of those solids, whether of earth, or rock, which enclose on all sides and contain crystals, selenites, marcasites,[1] plants and their parts, bones and the shells of animals, and other bodies of this kind which are possessed of a smooth surface, these same bodies had already become hard at the time when the matter of the earth and rock containing them was still fluid. And not only did the earth and rock not produce the bodies contained in them, but they did not even exist as **P. 16.** such when those bodies were produced in them.

2. That if a crystal is enclosed in part by a crystal, a selenite by a selenite, a marcasite by a marcasite, those contained bodies had already become hard when a part of the containing bodies was still fluid.

3. That in the earth and rock in which crystalline and petrified shells, veins of marble, of lapis lazuli, silver, mercury, antimony, cinnabar, copper, and other minerals of this kind are contained, the containing bodies had already become hard at the time when the matter of the contained bodies was still fluid; and that, consequently, the marcasites were produced first, then the stones in which the marcasites are enclosed, and, finally, the veins of minerals which fill the fissures of the rock.

II

If a solid substance is in every way like another solid substance, not only as regards the conditions of surface, but also as regards the inner arrangement of parts and particles, it will also be like it as regards the manner and place of production, if you except those conditions of place which are found time and again in some place to furnish neither any advantage nor disadvantage to the production of the body. Whence it follows:

---

[1] By the term "crystals," Steno means mineral quartz (cf. p. 237); *selenites* refers to crystals of gypsum, and *marcasites* to pyrites (cf. p. 225, n. 1).

P. 17. 1. That the strata of the earth, as regards the place and manner of production, agree with those strata which turbid water deposits.

2. That the crystals of mountains, as regards the manner and place of production, agree with the crystals of niter,[1] although it is not therefore essential that the fluid in which they were produced should have been aqueous.

3. That those bodies which are dug from the earth and which are in every way like the parts of plants and animals, were produced in precisely the same manner and place as the parts of the plants and the animals were themselves produced. But in order that no uncertain interpretation of the term place may beget new doubts, I shall forestall that difficulty here.

By the term place I mean that matter which with its own surface is in immediate contact with the surface of the body which is said to be in that place. But that matter allows sundry differences, for:

(1) It is either wholly solid, or wholly fluid, or partly solid and partly fluid.

(2) It is either wholly perceptible by itself, or partly perceptible by itself and partly perceptible through tests.

(3) It is either wholly contiguous to the body which it contains within itself, or even partly continuous with the same body.

(4) It is either always the same or undergoes change gradually. Thus the place in which a plant is produced is the matter, like that of the plant, within which the minute plant receives its form. Thus the place in which the plant grows is all that matter which, with its own surface, is in immediate contact with the entire surface of the plant, consisting sometimes of P. 18. earth and air, sometimes of earth and water, sometimes of earth, water, and air; sometimes of only stone and air, as in underground places I have time

---

[1] Steno was, of course, ignorant of the chemical difference between quartz and niter.

and again seen the roots of small plants, without any covering of earth at all, clinging to the surface of the tuff. Thus the place where the orange grows, after the blossom has fallen, is partly the peduncle continuous with it, and partly the air contiguous to it. Thus the place where the first growth occurs in animals, is partly the amniotic fluid contiguous to it, and partly the umbilical vessels scattered through the chorion, continuous with it.

III

If a solid body has been produced according to the laws of nature, it has been produced from a fluid.

In the production of a solid body, both its first outlines and its growth should be taken into account; but as I freely confess that the outline of most of the bodies is not only doubtful to me, but wholly unknown, so do I believe, without any hesitation, that nearly all the following statements concerning their growth are true.

A body grows while new particles, secreted from an external fluid, are being added to its particles. This addition, moreover, takes place either immediately from an external fluid, or through one or more mediating internal fluids.

The additions which are made directly to a solid from an **P. 19.** external fluid sometimes fall to the bottom from their own weight, as sediments do; sometimes the additions are made to a solid on all sides from a fluid bearing matter toward the solid, as in the case of incrustations; or the additions are made only to certain places of the solid surface, as in the case of those bodies which present fibres, branches, and angular bodies.[1] Here it must be noted in passing that the processes mentioned sometimes continue until an entire cavity is filled with such additions, and hence replacements occur which are sometimes simple, are sometimes formed from incrustations, sometimes from sediments, sometimes from angular bodies, and sometimes from various bodies variously intermingled.

The particles which are added to a solid by a mediating internal fluid either take on the form of fibres (since they are partly added through open pores along the length of the extended fibril, and are partly disposed in the interstices of the fibrils into the form of a new fibril by the permeating fluid), or form simple replacements: and in these two ways plants and animals are formed. Since I am less familiar with the anatomy of plants, I do not decide whether there are present several internal fluids; but it is certain that in animals different internal fluids are to be found, and I shall try to reduce these to a definite classification.

---

[1] *Angulata corpora* is the phrase used by Steno to denote crystals in general; *crystallus* is confined to quartz. See p. 218, n. 1.

Besides the attenuated fluid permeating all things, we note in the case of animals at least three kinds of fluids, of which the first is external; the second is internal and common, the third P. 20. an internal fluid peculiar to each part. By the term external fluid I mean that fluid in animals which not only surrounds the surface exposed to our eyes, as the atmosphere, but also that which is in contact with all the remaining surfaces of the body which are continuous with it through the larger foramina of the surface, such as the entire surface of the trachea, with which the air inhaled in breathing comes in contact; the entire surface of the alimentary canal, by which I mean the mouth, the œsophagus, the stomach, and the intestines; the entire surface of the bladder, and of the urethra; the entire surface which communicates with the uterus, especially in the years of puberty; the entire surface of all the excretory organs from the capillaries even to the orifices which discharge their contents into the ears, eyelids, nose, eyes, alimentary canal, bladder, urethra, uterus, and skin — a separate enumeration of which would show that many are truly external which are commonly considered internal, nay, even internal in the highest degree, and hence it follows:

1. That worms and calculi are generated within our body and that most are formed in the external fluid.
2. That many parts are essential to certain animals because they have them, not because the animal cannot exist without them.

A fluid which is in contact with these surfaces I call external because it communicates with the surrounding fluid by means of canals without intermediate capillary veins, that is, without P. 21. cribration; and the result is that although the cavities containing the fluids mentioned may be closed at times, still whenever they are opened, they discharge all their retained fluid without a dividing membrane.

I call that fluid internal which does not communicate with the external fluid except through the intermediate strainers of the capillary veins, and therefore never discharges all its contents naturally into the external fluid without a dividing membrane.

The internal common fluid is that which is contained in the veins, arteries, and lymphatic ducts, at least in those ducts which connect the conglobate glands[1] and the veins. I call this fluid common because it is distributed over all parts of the body. Concerning another common fluid which resides in the nerve substance, I have nothing to say, because it is unknown.

The internal peculiar fluid is that fluid which is sent about in the capillary veins of the common fluid, and which varies with the diversity of places; for it is one thing in bloody parenchymata,[2] another in bloodless parenchymata, another around the muscle fibres,[3] another in the capsule of the ovum, another in the substance of the uterus, and still another in other places. For that belief accords with neither reason nor experience, which supposes that the ends of the veins and arteries terminate in the smallest possible particle of the body for the distribution of warmth and nourishment at that point.[4] But there are cavities everywhere, and the elements which have been secreted into these cavities from the blood are mingled with the fluid of that

---

[1] The lymphatic glands, as shown by Steno's treatise *De Glandulis Oris et Novis inde Prodeuntibus Salivae Vasis*, printed by Maar (Vol. I, p. 20, and note, p. 227).

[2] The name *parenchyma* was given by Erasistratus to the peculiar substance of the lungs, liver, kidneys, and spleen, on the theory that this substance, as distinguished from the flesh of the muscles, was formed from the blood which flowed from the veins and coagulated in the organs mentioned. See Galen, Περὶ τῆς τῶν Φαρμάκων Κράσεως καὶ Δυνάμεως, book XI, prooemium; and εἰς τὸ περὶ φύσεως ἀνθρώπου Βιβλίον Ἱπποκράτους Ὑπόμνημα πρῶτον, 4 (edition of Mewaldt, Helmreich, and Westenberger, Leipzig, 1914, p. 6).

[3] Steno's phrase is *circa fibras motrices*. This is defined in *Elementorum Myologiae Specimen*, Maar, *op. cit.*, Vol. II, p. 69: 'The *fibra motrix* is a certain bundle of very minute fibrillae closely joined longitudinally. . . . I call such a fibre *motrix* because it seems to me to be the true organ of motion in an animal. For the muscle is nothing except a collection of such fibres.'

[4] Steno refers to the theory commonly accepted before Harvey's demonstration of the circulation of the blood. This is the view expounded in Plato's *Timaeus* (79, 80), a work which, in a crude Latin translation, profoundly influenced the science of the Middle Ages.

P. 22. particular place, thereupon to be added to the solid parts, just as the particles worn from the solid parts flow back into the same cavities, to be restored to the blood again in order that with its help they may be carried back to the external fluid. The doctrine concerning the fluid of these cavities agrees in many ways with the teaching of the great Hippocrates[1] concerning air.

Although I may not be able to determine why different fluids are secreted in different places from the same blood, I hope, nevertheless, that a few things hold true for determining that question; since it is certain that such secretion does not depend upon the blood, but upon the places themselves. An examination of this matter is comprised under these three heads:

1. The consideration of the capillary veins of the internal common fluid; with which alone they concern themselves who attribute all things to cribration through the different pores — with whom I, too, was for a long time numbered.

2. The consideration of the internal peculiar fluid, with which alone they busy themselves who ascribe a special ferment to each part. Their belief may be partially true, although the name ferment rests upon a comparison taken[2] from too specific a process.

3. The consideration of the particular parts of a solid; and upon this they especially lay emphasis who acknowledge, by ascribing to each part its own form, that they recognize in it something peculiar to that part, but which is unknown to us, and which, according to the knowledge of matter which we have so far gained, can be nothing else than the porous surface of P. 23. that solid, and the attenuated fluid permeating the pores.

I should wander too far afield if I were to apply the foregoing statements to the explanation of those things which take place in our body every day, and still they cannot otherwise be explained. It will suffice to have hinted here that the particles which separate from the external fluid in various ways are carried into the internal common fluid through the intervening cribration, and that after having been secreted from it likewise in various ways, and having been transmitted into the internal peculiar fluid by fresh cribration, they are added to the solid parts in the form of either fibres or parenchymata, according as they have been directed by the property of a given part, unknown to us, included in the consideration of the three foregoing statements.

---

[1] The Περὶ Φυσῶν of Hippocrates assigns air in the body as the cause of all diseases, and different diseases as merely due to different organs thus affected; cf. the edition of Hippocrates by Littré, Vol. VI, pp. 92, 104–106, and Maar, *op. cit.*, Vol. II, p. 335.

[2] For *desumpta* in the Florentine edition of 1669 read *desumptae*.

If, now, we wish to reduce to definite classes the solids naturally contained within solids in the fashion indicated, we shall find some of them produced by accretion from the external fluid, which are due either to deposits, as the strata of the earth; or to incrustations, as the agate, onyx, chalcedony, eaglestone,[1] bezoar,[2] and so on; or to filaments, as the amianthus, feathery alum, different kinds of veins which I have observed in the fissures of rocks; or to dendrites, as those forms of plants which are seen in the chinks of stones, except that certain ramifications in an agate which I have seen, whose trunks rested on the surface of the outer lamella but whose branches spread throughout the substance of the inner lamella, are merely superficial; or to angular bodies, as the crystals of P. 24. mountains, the angular bodies of iron and copper, cubes[1] of marcasites, diamonds, amethysts, and the like; or to replacements, as variegated marbles of every kind, granites, dendrites, petrified mollusks, crystalline substances, metallic plants, and many similar bodies filling the places of bodies which have been destroyed.

---

[1] The Latin word is *aëtites*. The 'eaglestone' is defined thus by the *New Oxford Dictionary:* "A hollow nodule or pebble of argillaceous oxide of iron, having a loose nucleus, which derived its name from being fabled to be found in the eagle's nest, and to which medicinal and magical properties were ascribed."

Some of these properties are mentioned by Damigeron, *de Lapidibus, Lapis Aëtites:*

'The aëtites is a very great safeguard of nature; God gave this stone to men as a protection to health. The eagle carries the stone to its nest from the uttermost parts of the earth for the sake of guarding its eggs. . . . The aëtites has a purple color and a very rough appearance, and has another stone within it, as if it were pregnant. It is useful to pregnant women, for when bound upon the left arm it prevents abortion. It is also very useful for accelerating parturition. For if taken from the woman's arm, and ground and placed upon her back, it will bring her immediate release. Furthermore, it will preserve the one who wears it, for it will make him sober and superior to all things; it will increase his wealth and spread about his good repute, and he will be most agreeable. . . . It is a remedy for insanity and unspeakable terrors, preserving the sufferers from dreaming and frequently falling. If you suspect that there is a poisoner in your home, put the stone in a relish and invite the suspect to dinner. If he is a poisoner, he will eat nothing, and if he ventures to swallow, he will not be able to do so. Such power has this stone. But if you remove the stone from the relish, the criminal will begin to eat and make merry. The wearing of the stone also greatly lessens anger incurred from powerful men. This stone is a sort of safeguard; the eagle uses it as a preventive against harm. For the eagle carries it from a never-failing river and puts it against the young in the nest to keep them from being harmed by another bird.' Abel, *Orphei Lithica, accedit Damigeron de Lapidibus* (Berlin, 1881), pp. 163, 164.

Similar statements are found in Pliny, *N. H.*, X. 12 (3); XXX. 130 (14). Val. Rose, in his study of the sources of Damigeron (*Hermes*, Vol. IX, 1875, pp. 471–491), cites an interesting parallel from Demosthenes, *Concerning Stones*, pp. 481, 482.

Palissy (cf. p. 208) ventures an independent opinion as to the origin of the aëtites: "Il y a beaucoup d'autres pierres qui sont formées selon le suiet qu'ils ont pris, comme quelques autres pierres que i'ay veuës que l'on nomme Pierre d'Aigle. Quelque chose que l'on en die, ie croy que ce n'est autre chose qu'un fruit lapifié, et ce qui iouë dedans est le noyau, qui estant amoindry quand on secouë ladite pierre, ledit noyau frappe des deux costez d'icelle." *Œuvres Complètes de Bernard Palissy*, by Paul-Antoine Cap, Paris, 1844, p. 284.

[2] An account of the medical history of the *Bezoar* is given by A. Laboulbène in *Diction-*

Other solids are produced by accretion from the internal fluid; and these are due either to simple replacements, as fat, the callus uniting broken bones, the cartilaginous substance joining severed tendons, the tissues which chiefly form the substance of the viscera, the medulla in both plants and animals; or to fibrous growths, as the fibrous parts of plants, the nerve fibres and muscle fibres in animals, also, which are all solid bodies and are naturally enclosed, for the most part, within solids.

If, therefore, every solid has had its accretions, at any rate, from a fluid, if bodies similar to one another in all respects were also produced in a similar way, and if of two contiguous solids that one first became hard which exhibits on its own surface the

---

naire Encyclopédique des Sciences Médicales, Tome Neuvième (Paris, 1868), pp. 221–225. Steno refers, of course, to the fossil, which is briefly alluded to in the work mentioned, p. 225: "Le bézoard fossile était composé de masses globuleuses de carbonate de chaux, réunies en couches concentriques."

A fuller description of the stone is quoted by Maar (*op. cit.*, Vol. II, p. 336) from the *Dictionnaire Raisonné Universel d'Histoire Naturelle*, by Valmont-Bomare, 3d ed., Lyon, 1791, Vol. II, p. 230:

"Une pierre arrondie, de couleur cendrée, composée de couches concentriques, friables, depuis la grosseur d'une aveline jusq'à celle d'un œuf d'oie. Au centre de cette pierre est quelquefois un grain de sable, une petite coquille, ou un morceau de charbon de terre. Une de ces matieres a servi de noyau, de point d'appui, et venant à rouler sur des terres molles, à demi-trempées, elle s'est ainsi accrue par couches roulées comme une pelotte de rubans."

[1] For *ubi* of the Florentine edition read *cubi*, with Maar, *op. cit.*, Vol. II, p. 195. By "cubes of marcasite" Steno, in common with the older scientists, means pyrites. Marcasite has the same chemical composition as pyrites, FeS_2, being iron disulphide. But marcasite crystallizes in orthorhombic form, whereas pyrites crystallizes in the cubic system. See H. N. Stokes, *On Pyrite and Marcasite*, in *Bulletin of United States Geological Survey*, No. 186 (1901).

characteristics of the other's surface, it will be easy, granted a solid and the place in which it is, to affirm something definite about the place of its production. And this, indeed, is the general question *of a solid contained within a solid.*

I pass to the more particular investigation of those solids dug from the earth which have given rise to many disputes; espe- P. 25. cially incrustations, deposits, angular bodies, the shells of marine animals, of mollusks, and the forms of plants. Under incrustations belong rocks of every kind consisting of layers, whose two surfaces are indeed parallel but not extended in the same plane. The place where incrustations are formed is the entire common boundary of fluid and solid; and the result is that the form of the layers or crusts corresponds to the form of the place, and it is easy to determine which of them hardened first, which last. For if the place was concave, the outer layers were formed first; if convex, the inner;[1] if the place was uneven because of various larger projections, the new layers were produced in the larger spaces when the narrower spaces had been filled with the formation of the first layers.

From this fact it is easy to account for all the differences of form which are seen in sections of similar rocks, whether they show the round veins of a tree cut transversely, or resemble the sinuous folds of serpents, or run along, curved in any other way, without law. Nor is it surprising that agates and other kinds of incrustations seem, so far as regards their outer surface, rough like ordinary stones,[2] since the outer surface of the outer layer imitates the roughness of the place. In torrents, however, incrustations of this kind are more frequently found outside of the place of their production, because the matter of the place has been scattered by a breaking up of the strata.

Concerning the manner in which particles of the layers P. 26. which are to be added to a solid are separated from the fluid, the following at least is certain:

---

[1] The reference is doubtless to the formation of secretions in the first instance, and concretions in the second.

[2] The Florentine edition reads *saxis ignobilis instar asperos;* this is partially corrected in the Leyden edition of 1679 to *saxi,* etc. The correct reading *saxi ignobilis instar asperas* is given by Maar, *op. cit.*, Vol. II, p. 196.

1. That there is in it no place for buoyancy or gravity.

2. That the particles are added to surfaces of every kind, since surfaces smooth, rough, plane, curved, and consisting of several planes at different angles of inclination, are found overspread by the layers.

3. That movement of the fluid causes them no hindrance.

Whether the substance under consideration which flows from a solid, be different from that substance which moves the parts of the fluid, or whether something else is to be sought, I leave undecided.

Different kinds of layers in the same place can be caused either by a difference of the particles which withdraw from the fluid one after the other, as this same fluid is gradually disintegrated more and more, or from different fluids carried thither at different times. From this fact it follows that the same arrangement of layers sometimes recurs in the same place, and often evident traces revealing the entrance of new matter remain. But all the matter of the layers seems to be a finer substance emanating from the stones, as will further appear in the following.

THE STRATA OF THE EARTH

The strata of the earth are due to the deposits of a fluid:

1. Because the comminuted matter of the strata could not have been reduced to that form unless, having been mixed with P. 27. some fluid and then falling from its own weight, it had been spread out by the movement of the same superincumbent fluid.

2. Because the larger bodies contained in these same strata obey, for the most part, the laws of gravity, not only with respect to the position of any substance by itself, but also with respect to the relative position of different bodies to each other.

3. Because the comminuted matter of the strata has so adjusted itself to the bodies contained in it that it has not only filled all the smallest cavities of the contained body, but has also expressed the smoothness and lustre of the body in that part of its own surface where it is in contact with the body, although the roughness of the comminuted matter by no means admits of similar smoothness and lustre.

Sediments, moreover, are formed so long as the contents in a fluid fall to the bottom of their own weight, whether the said contents have been carried thither from some other where, or have been secreted gradually from the particles of the fluid, that too, either in the upper surface, or equally from all the particles of the fluid. Although a close relationship exists between crusts and sediments, they can nevertheless be distinguished easily because the upper surface of crusts is parallel to the lower surface, however rough[1] this may be from various larger projections, while the upper surface of sediments is parallel to the horizon, or deviates but slightly therefrom. So in rivers, the mineral layers, now green, now yellow, now reddish, do not remove the unevenness of a stony bottom, while a sediment of sand or clay makes all level; and it is due to this fact **P. 28.** that in the formation of the different composite strata of the earth I have easily distinguished crusts from sediments.

Concerning the matter of the strata the following can be affirmed:

1. If all the particles in a stony stratum are seen to be of the same character, and fine, it can in no wise be denied that this stratum was produced at the time of the creation from a fluid which at that time covered all things; and Descartes[2] also accounts for the origin of the earth's strata in this way.

2. If in a certain stratum the fragments of another stratum, or the parts of animals and plants are found, it is certain that the said stratum must not be reckoned among the strata which settled down from the first fluid at the time of the creation.

3. If in a certain stratum we discover traces of salt of the sea, the remains of marine animals, the timbers of ships, and a substance similar to the bottom of the sea, it is certain that the sea was at one time in that place, whatever be the way it came there, whether by an overflow of its own or by the upheaval of mountains.

4. If in a certain stratum we find a great abundance of rush, grass, pine cones, trunks and branches of trees, and similar ob-

---

[1] *aspera*, Florentine edition, is an error for *asperae*.

[2] The reference is to Descartes, *Principia Philosophiae* (first edition Amsterdam, 1644), *Pars Quarta*, XXXII ff. See *Œuvres de Descartes, Publiées par Charles Adam et Paul Tannery*, Paris, Vol. VIII (1905), p. 220 ff.

jects, we rightly surmise that this matter was swept thither by the flooding of a river, or the inflowing of a torrent.

5. If in a certain stratum pieces of charcoal, ashes, pumice-stone, bitumen,[1] and calcined matter appear, it is certain that a P. 29. fire occurred in the neighborhood of the fluid; the more so if the entire stratum is composed throughout of ash and charcoal, such as I have seen outside the city of Rome, where the material for burnt bricks is dug.

6. If the matter of all the strata in the same place be the same, it is certain that that fluid did not take in fluids of a different character flowing in from different places at different times.

7. If in the same place the matter of the strata be different, either fluids of a different kind streamed in thither from different places at different times (whether a change of winds or an unusually violent downpour of rains in certain localities be the cause) or the matter in the same sediment was of varying gravity, so that first the heavier particles, then the lighter, sought the bottom. And a succession of storms might have given rise to this diversity, especially in places where a like diversity of soils is seen.

8. If within certain earthy strata stony beds are found, it is certain either that a spring of petrifying waters existed in the neighborhood of that place, or that occasionally eruptions of subterranean vapors occurred, or that the fluid, leaving the sediment which had been deposited, again returned when the upper crust had become hardened by the sun's heat.

Concerning the position of strata, the following can be considered as certain:

1. At the time when a given stratum was being formed, there was beneath it another substance which prevented the further P. 30. descent of the comminuted matter; and so at the time when the lowest stratum was being formed either another solid substance was beneath it, or if some fluid existed there, then it was not only of a different character from the upper fluid, but also heavier than the solid sediment[2] of the upper fluid.

---

[1] The inclusion of bitumen in the list indicates that Steno was ignorant of its true nature as an organic compound.

[2] *sedimenta*, Florentine edition, is an error for *sedimento*.

2. At the time when one of the upper strata was being formed, the lower stratum had already gained the consistency of a solid.

3. At the time when any given stratum was being formed it was either encompassed on its sides by another solid substance, or it covered the entire spherical surface of the earth. Hence it follows that in whatever place the bared sides of the strata are seen, either a continuation of the same strata must be sought, or another solid substance must be found which kept the matter of the strata from dispersion.

4. At the time when any given stratum was being formed, all the matter resting upon it was fluid, and, therefore, at the time when the lowest stratum was being formed, none of the upper strata existed.

As regards form, it is certain that at the time when any given stratum was being produced its lower surface, as also its lateral surfaces, corresponded to the surfaces of the lower substance and lateral substances, but that the upper surface was parallel to the horizon, so far as possible; and that all strata, therefore, except the lowest, were bounded by two planes parallel to the P. 31. horizon. Hence it follows that strata either perpendicular to the horizon or inclined toward it, were at one time parallel to the horizon.

Moreover, the changed position of strata and their exposed sides, such as are seen to-day in many places, do not contradict my statements; since in the neighborhood of those places evident traces of fires and waters are to be found. For just as water disintegrating earthy material carries it down sloping places not only on the surface of the earth but also in the earth's cavities; so fire, breaking up whatever solids oppose it, not only drives out their lighter particles but also sometimes hurls forth their heaviest weights; and the result is that on the surface of the earth are formed steeps, channels, and hollows, while in the bowels of the earth subterranean passages and caverns are produced.

By reason of these causes the earth's strata can change position in two ways:

The first process is the violent thrusting up of the strata, whether this be due to a sudden burning of subterranean gases, or be brought about through the violent explosion of air due to other great downfalls near by. This thrusting up of the strata is followed by a scattering of the earthy matter as dust and the breaking up of rocky matter into lapilli and rough fragments.

The second process is the spontaneous slipping or downfall of the upper strata after they have begun to form cracks, in consequence of the withdrawal of the underlying substance, or foundation. Hence by reason of the diversity of the cavities P. 32. and cracks the broken strata assume different positions; while some remain parallel to the horizon, others become perpendicular to it, many form oblique angles with it, and not a few are twisted into curves because their substance is tenacious. This change can take place either in all the strata overlying a cavity, or in certain lower strata only, the upper strata being left unbroken.

The altered position of the strata affords an easy explanation of a variety of matters otherwise obscure. Herein may be found a reason for that unevenness in the surface of the earth which furnishes occasion for so many controversies; an unevenness manifest in mountains, valleys, elevated bodies of water, elevated plains, and low plains. But passing over the rest, I shall now treat briefly certain points concerning mountains.

THE ORIGIN OF MOUNTAINS

That alteration in the position of strata is the chief cause of mountain formation is clear from the fact that in any given range of mountains there may be seen:

1. Large level spaces on the summits of some mountains.

2. Many strata parallel to the horizon.

3. Various strata on the sides of the mountains inclined at different angles to the horizon.

4. Broken strata on the opposite sides of hills, showing absolute agreement in form and material.

5. Exposed edges of strata.

6. Fragments of broken strata at the foot of the same range, P. 33. partly piled into hills, and partly scattered over the adjoining country.

7. Either in the rock of the mountains themselves, or in their neighborhood, very clear traces of subterranean fire; just as many springs are found around hills which are made up of strata of earth. And here it must be observed in passing that the hills which are formed of earthy strata, for the most part, have as their foundation larger fragments of stony strata; these in many places keep the earthy strata placed upon them from being swept away by the current of neighboring rivers and torrents. Further, they often protect entire regions against the violence of the ocean, as the extended reefs of Brazil[1] and exposed craggy shores everywhere bear witness.

Mountains can also be formed in other ways, as by the eruption of fires which belch forth ashes and stones together with sulphur and bitumen; and also by the violence of rains and torrents, whereby the stony strata, which have already become rent apart by the alternations of heat and cold, are tumbled headlong, while the earthy strata, forming cracks under great blasts of heat, are broken up into various parts. And from this it is clear that the chief classes of mountains and hills are two: first, of those which consist of strata; of these there are two kinds, since in some, strata of rock prevail, in others, strata of earth. The second class is composed of mountains which rise without order or arrangement from fragments of strata and from parts, further, which have been worn away.

Hence it could be easily shown:

1. That all present mountains did not exist from the beginning of things.

P. 34. 2. That there is no growing[2] of mountains.

---

[1] Steno's information regarding Brazil was probably gained from a book called *Historia Naturalis Brasiliae*, Amsterdam, 1648. The volume contains Piso's *De Medicina Brasiliensi Libri Quatuor*, and George Musgrave's *Historiae Rerum Naturalium Brasiliae Libri Octo.* No doubt Casper Barlaeus's *Rerum per Octennium in Brasilia sub Praefectura Mauritii Nasovii Historia* (Amsterdam, 1647) was also known to him.

Willem Piso (1611–1678) was a member of the Brazilian expedition of Count Jan Maurits from 1636 to 1644. Steno had known Piso in Leyden and in 1664 addressed to him the letter on the Anatomy of the Ray (*De Anatome Rajae Epistola*) printed by Maar, *Opera Philosophica*, Vol. I, pp. 193–207. Robert Boyle refers frequently to Piso's *History of Brazil.*

[2] Steno's word is *vegetatio*, which suggests the growth of an organism; but he does not hesitate to use *crescere* of inorganic accretions. The passage quoted by Maar, *op. cit.*, Vol. II, p. 338, from Fabronius (*Vitae Italorum* (p. 202), Vol. III, p. 72), is singularly apposite. In

3. That the rocks or mountains have nothing in common with the bones of animals except a certain resemblance in hardness, since they agree in neither matter nor manner of production, nor in composition, nor in function, if one may be

1657 Montanari and Boni, master of the mint in Vienna, journeyed to Stiermark, Bohemia, and Hungary to examine the mines. I translate:

'They also investigated whether metals grow in the same manner as plants do, that is by means of a circulating sap of the earth. They thought they knew, from the surest proofs, that metals do indeed grow (*crescere*) — iron rather rapidly and gold more slowly. But how this took place they were unable to decide, although Montanari inclined to believe that the growth (*maturitatem*) was caused by accretion (*fermentationi*). . . . And he made fun of the levity and weakness of those who believed the testimony of George Agricola that gnomes flit and wander about the mines, by whom the workmen are often disturbed.'

Agricola (1494–1555), scientist though he was, fully believed in gnomes. Compare *De Re Metallica*, Book VI, p. 217, edition of Hoover (London, 1912):

"In some of our mines, however, though in very few, there are other pernicious pests. These are demons of ferocious aspect, about which I have spoken in my book *De Animantibus Subterraneis* (the last paragraph). Demons of this kind are expelled and put to flight by prayer and fasting."

Agricola's credulity, however, did not extend to a belief in the "growth" of mountains. On the contrary, he was perhaps the first to recognize clearly the fundamental agencies of mountain sculpture, as appears from *De Ortu et Causis Subterraneorum*, Book II (*De Re Metallica*, edition of Hoover, pp. 595, 596):

"Hills and mountains are produced by two forces, one of which is the power of water, and the other the strength of the wind. There are three forces which loosen and demolish the mountains, for in this case, to the power of the water and the strength of the wind we must add the fire in the interior of the earth. Now we can plainly see that a great abundance of water produces mountains, for the torrents first of all wash out the soft earth, next carry away the harder earth, and then roll down the rocks, and thus in a few years they excavate the plains or slopes to a considerable depth; this may be noticed in mountainous regions even by unskilled observers. By such excavation to a great depth through many ages, there rises an immense eminence on each side. When an eminence has thus arisen, the earth rolls down loosened by constant rain and split away by frost, and the rocks, unless they are exceedingly firm, since their seams are similarly softened by the damp, roll down into the excavations below. This continues until the steep eminence is changed into a slope. Each side of the excavation is said to be a mountain, just as the bottom is called a valley.

"Streams, moreover, and to a far greater extent rivers, effect the same results by their rushing and washing; for this reason they are frequently seen flowing either way between very high mountains which they have created, or close by the shore which borders them. . . . Nor did the hollow places which now contain the seas all formerly exist, nor yet the mountains which check and break their advance, but in many parts there was a level plain, until the force of winds let loose upon it a tumultuous sea and a scathing tide. By a similar process the impact of water entirely overthrows and flattens out hills and mountains. But these changes of local conditions, numerous and important as they are, are not noticed by the common people to be taking place at the very moment when they are happening, because, through their antiquity, the time, place, and manner in which they began is far prior to human memory.

"The wind produces hills and mountains in two ways: either when set loose and free from bonds, it violently moves and agitates the sand; or else when, after having been driven into the hidden recesses of the earth by cold, as into a prison, it struggles with a great effort to burst out. For hills and mountains are created in hot countries, whether they are situated by permitted to affirm aught about a subject otherwise so little known as are the functions of things.

4. That the extension of crests of mountains, or chains, as some prefer to call them, along the lines of certain definite zones of the earth, accords with neither reason nor experience.[1]

5. That mountains can be overthrown, and fields carried over from one side of a high road across to the other; that peaks of mountains can be raised and lowered, that the earth can be opened and closed again, and that other things of this kind occur which those who in their reading of history wish to escape the name of credulous, consider myths.[2]

PASSAGE-WAYS FOR THINGS ISSUING FROM THE EARTH

The same alteration in the position of strata affords a passage-way for things issuing from the earth, such as:

1. Waters, which are shut up in mountain caves away from the air, gushing forth on the mountains, whether those waters come from subterranean reservoirs, or have been condensed in a place away from the upper air and then ejected. And this I believe to be very common, since in many caverns I have observed that everything both above and below was solid though the water nevertheless trickled there abundantly.

2. Winds breaking forth from mountains, whether those

---

the sea coasts or in districts remote from the sea, by the force of winds; these no longer held in check by the valleys, but set free, heap up the sand and dust, which they gather from all sides, to one spot, and a mass arises and grows together. If time and space allow, it grows together and hardens, but if it be not allowed (and in truth this is more often the case), the same force again scatters the sand far and wide. . . .

"Then, on the other hand, an earthquake either rends and tears away part of a mountain, or engulfs and devours the whole mountain in some fearful chasm. In this way it is recorded the Cybotus was destroyed, and it is believed that within the memory of man an island under the rule of Denmark disappeared. Historians tell us that Taygetus suffered a loss in this way, and that Therasia was swallowed up with the island of Thera. Thus it is clear that water and the powerful winds produce mountains, and also scatter and destroy them. Fire only consumes them, and does not produce at all, for part of the mountains — usually the inner part — takes fire."

[1] Steno is not referring to mountain-chains in the modern sense of the term; he is rejecting Kircher's theory of chains running from north to south and east to west over the entire surface of the earth. This is set forth in *Mundus Subterraneus*, Amstelodami (1665), Vol. I, c. ix, p. 68 ff. Cf. Maar, *op. cit.*, Vol. II, p. 337.

[2] For a modern exposition of Tuscan earth features, see Murchison, *Geological Structure of the Alps*, in *Quarterly Journal of the Geological Society*, vol. 5 (1849), pp. 157–312, especially pp. 263–308.

winds be air expanded by heat or whether different fluids of
P. 35. the air made violent by collision produced them.

3. Ill-smelling exhalations, fiery or frigid ebullitions, and so on. And there is no longer any doubt of the fact that cold and dry places boil up without any trace of heat whenever water is poured upon them; that a hot spring issues by the side of a very cold spring; that in consequence of an earthquake a hot spring may be turned into a cold spring, and rivers change their course; that valleys shut in on all sides discharge their gathered rain water into lower places; that rivers gliding underneath the earth's surface are in places returned to the light of day; that in laying foundations architects sometimes lose all their labor when they encounter a quicksand, as it is called;[1] that in certain places on digging wells water is at first found near the surface of the ground, then after digging to the depth of several yards new waters are discovered which at first, on the opening of a passage-way, leap forth beyond the height of the water already found; that whole fields with trees and buildings sink gradually, or are engulfed suddenly, and hence vast lakes now exist where once stood cities; that a plain is a source of danger to its inhabitants from catastrophes of this kind unless they have made themselves sure about its foundation of rock; that abysses emitting a deadly gas are sometimes found which are again stopped up when a number of bodies have been cast into them.

THE ORIGIN OF VARIEGATED STONES AND THE REPOSITORIES OF MINERALS

The same alteration in the position of strata has given rise to variegated stones of every kind, and at the same time
P. 36. afforded a repository for most minerals, whether the deposition took place in the cracks of the strata, or in those fissures which were, in respect to matter, dry but not yet hard, either between the layers or in their clefts; or in the interstices between the upper and the lower strata after the falling of lower strata; or in the places left empty by the decomposition of bodies therein contained. Whence it can be shown:

1. That on the very slightest foundation, nay, apparently on

---

[1] *arena viva*, 'living sand,' is Steno's phrase.

no foundation, have been based those minute and all but inconceivable subdivisions of veins made use of by diggers of minerals; and that divination for the abundance of metal by means of roots and branches is, in consequence, as doubtful as is the ridiculous belief of certain Chinese concerning the head and tail of the dragon which they employ in finding a favorable place of burial in the mountains.[1]

2. That most of the minerals for which man's labor is spent did not exist at the beginning of things.

3. That in the investigation of rocks many things can be disclosed which are attempted in vain in the study of the minerals themselves, seeing that it is more than probable that all those minerals which fill either the clefts or expanded spaces of rocks had as their matter the vapor forced from the rocks themselves, whether the deposition took place before the strata changed their position, as I believe happened in the mountains of Peru,[2] or when the strata had already changed their position; and that a new metal can therefore form in the place of an P. 37. exhausted one, as is believed rather than known concerning the mining of iron among the people of Elba, for the miner's tools and the idols which have been found there were surrounded not with iron but with earth.[3]

And these things concerning the strata of the earth I thought ought to be investigated the more carefully, not only because the strata themselves are solids naturally enclosed within solids but also because in them are contained almost all those bodies which gave rise to the question propounded.

---

[1] This practice is mentioned by Kircher, *China Illustrata* (Amstelodami, 1667), p. 135, who quotes Trigautius, *De Christiana Expeditione apud Sinas Suscepta* (Augustae Vind., 1615), *lib.* 1, ix, p. 95. 'One characteristic of the Chinese can be mentioned. In seeking a spot for building private and public structures, or for burying their dead, they examine the spot with the head, tail, and feet of various dragons which are supposed to live beneath it; and they believe that all their adversity and prosperity depend upon the dragons.' Maar, *op. cit.*, Vol. II, p. 337.

What is perhaps the first published description of the divining rod and its use in finding minerals or water, is given by Agricola, *De Re Metallica*, II, edition of Hoover, pp. 38–42. See also Robert Boyle, edition of Shaw, Vol. I, pp. 172, 173.

[2] The mineral deposits in the Peruvian mountains were familiar to Steno from de Acosta's *Historia Natural y Moral de Las Indias* (Seuilla, 1590), iv, iv–v, and from de Rosnel's *Le Mercure Indien, ou Le Tresor des Indes* (Paris, 1667), Première Partie, Livre Premier, I–III. Maar, *op. cit.*, Vol. II, p. 337.

[3] The belief that iron would "grow" or replace itself in process of time, probably arose from finding limonite upon the tools mentioned in the text. See above, p. 232, note 2.

CONCERNING THE CRYSTAL

As regards the formation of crystal, I would not venture to declare in what manner its first shape is produced; this at any rate is beyond dispute, that the things which it has been my lot to read in other writers concerning this subject are not to the point; for neither irradiations, nor a shape of the particles resembling the shape of the whole, nor the perfection of the hexagonal form[1] and the assembling of the parts about a common centre, nor other things of this kind, accord with fact; as will be clear from various propositions which I shall bring forward, proved elsewhere by conclusive experiments. But that no room may be left for doubt, it is well to explain beforehand the terms which I employ in naming the parts of a crystal.

A crystal consists of two hexagonal pyramids and an intermediate prism likewise hexagonal. I call those angles the *terminal solid angles* which form the apexes of the pyramids, but those angles the *intermediate solid angles* which are formed by the union of the pyramids with the prism. In the same way P. 38. I call the planes of the pyramids *terminal planes*, and the planes of the prism the *intermediate planes*. The *plane of the base* is the section perpendicular to all the intermediate planes; a *plane of the axis* is a section in which lies the axis of the crystal, which consists of the axes of the pyramids and the axis of the prism.

The place where the first hardening of a crystal begins, whether it be between a fluid and a fluid, or between a fluid and a solid, or even in a fluid itself, may remain in doubt; but the place in which the crystal grows after it has already begun to form, is a solid in that part where the crystal is supported on it, whether the place be a stone or another crystal already formed. The remaining portion is fluid, if you except the obstructions which can present themselves to it from the unevenness of the rock or even from other crystals already formed. I would not venture to affirm whether the surrounding fluid is aqueous; and it matters not what is said about the water enclosed within crystals, since it is certain that air together with water is contained therein, and that many crystals are found which enclose air alone. But if the crystal had indeed grown in an aqueous fluid, all the spaces enclosed on every side would be filled with water,[1] since it is an undisputed fact that water kept in that way never vanishes in any number of centuries.

---

[1] By crystal Steno meant rock crystal, which is the mineral quartz and has a hexagonal form.

The cavities of the rocks, formed in different ways, as has been said above, afford this place for crystals, and the fact that entire hills consist of earthy substance packed with crystals, is no disproof, seeing that in the vicinity of the same hills are found mountains of rock suited to the formation of crystals. P. 39. And in those hills of earthy matter, large unburied rocks are pulled out which have been rent from neighboring mountains, and some of these show fissures filled with the material of marble, precisely as the fissures of strata in mountains of rock are filled. The same cause, moreover, which hurled upon the hills the fragments of strata rent from the neighboring mountains, can likewise have sown over them broadcast the crystals which had been shaken out from cavities of the same strata.

The following propositions will show what can be determined concerning the place of the crystal to which new crystalline matter is being added:

I

A crystal grows while new crystalline matter is being added to the external planes of the crystal already formed. No room at all is here left for the belief of those who affirm that crystals grow, plantlike, by nourishment, and that they draw their nourishment on the side where they are attached to the matrix, and that the particles thus received from the fluid of the rock, and transmitted into the fluid of the crystal, are inwardly added to the particles of the crystal.

II

This new crystalline matter is not added to all the planes but, for the most part, to the planes of the apex only, or to the terminal planes, with the result:

---

[1] This would not necessarily follow.

1. That the intermediate planes, or the quadrilateral planes, are formed by the bases of the terminal planes, and hence the intermediate planes are larger in some crystals, smaller in others, and wholly wanting in still others.

2. That the intermediate planes are almost always striated, while the terminal planes retain traces of the matter added to them.

III

P. 40. The crystalline matter is not added to all the terminal planes at the same time, nor in the same amount. Hence it comes to pass:

1. That the axis of the pyramids does not always continue the same straight line with the axis of the prism.

2. That the terminal faces are rarely of a size, whence follows an inequality of the intermediate planes.

3. That the terminal faces are not always triangular, just as all the intermediate planes are not always quadrilateral.

4. That the terminal solid angle is broken up into several solid angles, this being the case frequently also with the solid intermediate angles.

IV

An entire plane is not always covered by crystalline matter, but exposed places are left sometimes toward the angles, sometimes toward the sides, and sometimes in the centre of the plane. Hence it happens:

1. That the same plane, commonly so-called, does not have all its parts located in the same plane, but in different planes extending above it in different ways.

2. That a plane, commonly so-called, in many places is seen to be not a plane but a protuberance.

3. That in the intermediate planes inequalities rise like the steps of stairs.

The crystalline matter added to planes upon the same planes is spread out by the enveloping fluid, and gradually hardens, with the result:

1. That the surface of the crystal comes forth the smoother the more slowly the added matter has hardened, and is left

wholly rough if the matter has hardened before it has spread sufficiently.

P. 41. 2. That the manner in which the crystalline matter is added to the crystal can be distinguished, since where it has hardened suddenly, it reveals a surface full of small elevations like variolar postules, as it were, just as small drops of oily fluid are wont to float upon an aqueous fluid; sometimes it shows also trilateral and depressed pyramids, if it has hardened somewhat more slowly. The tortuous fringes of the descending matter show now the place to which the fluid matter was being added, now the place toward which it was being advanced, now the arrangement of the matter added, that is, which came first, and which last. And in this way certain roughnesses always appear in the crystals of mountains, nor have I ever seen a crystal whose still unbroken surfaces possess the lustre which the rent sides of the same crystal show after it has been broken, however prolix writers on subjects relating to nature become in extolling the lustre of the crystal which is extracted from the mountains.

3. That certain intruding solid bodies are enclosed within the crystal itself, as if they had been coated with a sort of glue, in case the crystals did not yet present a hardened surface.

4. That the crystalline matter sometimes seems to flow down over neighboring planes.

5. That when certain small areas on those planes have been left without added crystalline matter, new crystalline matter approaches, and, spreading over the areas, forms cavities there, sometimes producing several layers; sometimes, again, enclosing a part of the external fluid, which in some instances is nothing but air, in others water and air.

P. 42. The external fluid receives crystalline matter from the substance of the harder stratum, with the result:

1. That rocks of a different kind, emitting a different fluid, give rise to crystals of a different hue.

2. That in the same place sometimes the first, sometimes the last, crystals are the darker: but in the same crystal the parts first hardened are sometimes darker than the parts last hardened.

3. That when oysters, mussels, and other bodies have decomposed within the earth, the cavities are filled with crystals.

The movement of the crystalline matter toward a point where the planes of the crystal already formed are fixed, does not arise from some common cause of motion in the surrounding fluid, but varies in any given crystal; so that in reality it depends upon the movement of the tenuous fluid flowing from the crystal already formed, and the result is:

1. That in the same place crystalline matter is added to planes which face the horizon from different angles.

2. That in the same fluid crystals of different shapes are produced. Whether the fluid is that by the aid of which refraction is caused, or there is still some fluid different from it, I leave to wiser minds to study.

That the efficiency of a penetrating fluid is certainly great, is illustrated by the row of iron filings which rise about the poles[1] of a magnet, not only when the filings are in direct contact with the magnet but also when they are separated from the magnet by an intervening sheet of paper. When, for example, the magnet is moved in various ways below the paper, P. 43. while one end remains at rest, filings of this kind describe on the paper all the arcs which can be drawn within a hemisphere. Now all advance from place to place like armed soldiers; now, deflected by the approach of another magnet, they form an arch just as if the individual parts of the filings had been glued together and had united into a solid body.[2]

---

[1] *Poros* in the original edition is an obvious error for *polos*.

[2] This experiment with the magnet was, no doubt, a scholastic commonplace. A curious analogue to Steno's illustration may be found in Robert Boyle's *The Effects of Languid Motion Consider'd* (edition of P. Shaw, London, 1725), Vol. I, pp. 477, 478:

"The load-stone is acknowledged to act by the emission of insensible particles. For tho' iron and steel be solid bodies, and magnetic effluvia corpuscles so very minute, as readily to get in at the pores even of glass itself; yet entring the steel in swarms, they may operate so violently on it, as to attract above fifty times the weight of the magnet. For to these I rather ascribe magnetical attraction and suspension, than to the pressure of the ambient air; because I have found on trial, that such a pressure is not absolutely necessary to magnetical operations.

"And farther, as to the power of magnetical effluvia upon iron, I took filings of iron fresh made, that the virtue might not be diminished by rust, and having laid them in a little heap upon paper held level, I applied to the lower side of it, just beneath the heap, the pole of a vigorous load-stone, whose emissions diffusing themselves thro' the metal, manifestly alter'd

In a similar way I should suppose that by the help of a permeating fluid those minute drops mutually cohere which have formed in a receiver from the material forced out of a retort. At first they cling together on the inside of the upper part of the receiver, but later, when a number of drops have come together in the upper part of the receiver, they fall down and form globular masses which sometimes cling, with their extremities, to the sides of the receiver, and sometimes join other filaments. Filaments of this kind, which I have sometimes noticed in the humor of the eye, I should believe to consist of globular masses and to have been formed in a similar way, and so, too, should I believe filaments and branches to have been produced in the fluid by accretion from without.

But however the case may stand concerning these things, in the growth of a crystal, we must take into account two movements: one, the movement whereby it is brought to pass that crystalline matter is added to certain places of the crystal and not to others — a movement which I fancy must be attributed to the attenuated permeating fluid, and is to be illustrated by the example of the magnet which I have given; the other the movement whereby the new crystalline matter added to the crystal is spread forth over the plane — and this movement must be derived from the surrounding fluid, just as, when the iron filings have risen up above the magnet, through the movement of the air, whatever is struck off from one filing is added to another. To this movement of the surrounding fluid I should attribute the fact that not only in a crystal, but also in many other angular bodies, any given opposite planes are parallel.

P. 44.

From the arguments presented it might be possible to prove that the efficient cause of the crystal is not extreme cold; that it is not ashes only, burned out by the force of fire, which turn into glass; that the force of fire alone is not the producer of glass; that not all crystals were produced in the beginning of things, but that they are even now being produced from day to day; that it is not a task beyond man's power to disclose the

their appearance, and produced many erect aggregates of filings, placed one above another, like little needles: and as these needles stood erected upon the flat paper, so they would run to and fro, according as the load-stone, which was held underneath, moved one way or the other; and as soon as that was taken away, all this little stand of pikes would fall again into a confused heap."

formation of glass without the agency of fire, provided one undertake a careful analysis of the rocks in whose cavities the best crystals are formed. For it is certain that, just as a crystal has formed from a fluid, so that same crystal can be dissolved into a fluid, provided one know how to imitate the real menstruum[1] of Nature. And it is no disproof that certain solid bodies, when once the dissolving fluid, or their menstruum, has been taken away, can be no further disintegrated by the same or a similar solvent; for this occurs in bodies from which the entire menstruum is freed by the force of fire. But the crystal, and all angular bodies which form in the midst of a solvent fluid, or in the midst of a menstruum, can never come out so pure but that some particles of the menstruum are left within the particles of the angular body. And upon this fact depends the main cause of variation whereby crystal differs from glass not
P 45. only in refraction but also in other properties, since in glass no parts of the dissolving fluid are present, inasmuch as they have been driven forth by the violence of fire. For the fluid, in which the crystal is formed, bears the same relation to the crystal that ordinary water bears to salts; this could easily be proved by setting forth the characteristics which the formation of salts holds in common with the formation of crystal.

But since I should be wandering too far from my subject if I should allude to all these things here, I shall mention but one example, which seemed exceedingly beautiful to me. In various places within the same stone the receding layers were filled with crystals, of which some were watery, others very clear, some white, many amethystine, mingled together without any blending of hues; exactly as experiments with salts made here show that vitriol and alum, dissolved in the same water, after a part of the water has been taken up, have each formed by themselves without any mixing of parts.

---

[1] The term *menstruum*, used by Steno, was commonly employed by the alchemists and physicists to denote a solvent fluid. Compare Littré, *Dictionaire de la Langue Française*, *s.v. menstrue*: "Terme de chimie. Liqueur propre à dissoudre les corps solides. L'eau régale est le menstrue de l'or (*aqua regia*). On dit aujourd'hui de préférence dissolvant."

Excellent examples of this usage can be found in Robert Boyle's works; compare, *e.g.*, *Experiments and Observations upon Colours* (edition of Shaw, Vol. II, p. 96): "That gold, dissolv'd in *Aqua regia*, communicates its own colour to the menstruum, is a common observation; but the solutions of mercury, in *Aqua fortis*, are not generally observ'd to give any notable tincture to the menstruum. See also *New Oxford Dictionary*, *s.v. menstruum*.

ANGULAR BODIES OF IRON

The angular bodies of iron which it has hitherto been my fortune to see[1] reduce to three classes. Of these the first is plane and, being thicker in the middle, gradually grows thinner towards the margins, where it terminates in an edge sharp on every side; the second is bounded by twelve planes, the third by twenty-four planes. Sometimes an angular body of the second class is bounded by six planes, resembling two trilateral P. 46. pyramids so joined along the bases that the angles of one base bisect the sides of the other.

The second and third classes of angular bodies of iron agree with crystals in the following particulars:

1. In the place of production; since the place where iron is formed is partly solid, partly fluid, and is a cavity in the rock.

2. As regards the place to which matter is added; since in iron also it is added not to all the planes, but to some only, and not always to the whole of these, nor always at the same time, but now to one, now to another; now towards the margins, and now towards the middle.

3. As regards the place from which the iron matter comes, since this matter, also, seems to have flowed forth from the pores of a more solid body.

4. As regards the manner in which the same matter is directed toward the solid by the help of the permeating fluid, and is spread forth and smoothed out upon the plane by the movement of the surrounding fluid.

Iron and crystal differ in matter and form, because the matter of the crystal is translucent, while the matter of iron is opaque. The form of the crystal is bounded by eighteen planes, of which the twelve terminal are brilliant, while the six intermediate are striated. In the second class of iron, however, twelve planes may be counted, of which six are terminal and striated, the other six intermediate and brilliant; and in the third class of P. 47. iron twenty-four planes may be counted, of which the six terminal are striated, and the intermediate eighteen brilliant. Between the terminal striated planes there sometimes lie six other glistening planes resembling the truncated sides of triangular pyramids.

---

[1] Steno refers to crystals of hematite from the mines on the island of Elba.

It seemed to me worthy of notice that by truncating a cube at the very extremity the entire number of planes in the third kind of angular bodies of iron can be shown; for it has six pentagonal planes which coincide with the planes of the cube, and which, at the four angles,[1] bisect the individual sides of the cube's planes. All the remaining planes are found at the cube's angles when they are truncated in a certain way.

In the angular bodies of iron there is also another thing equally surprising. In the second class of angular bodies of iron the terminal planes, which are striated and five-sided, are in process of time changed to three sides, while intermediate planes, which are three-sided and brilliant, pass into five-sided with two right angles adjacent to each other.[2] Between two five-sided planes, however, where their right angles are adjacent, a pair of triangles, or two three-sided planes, are formed, likewise brilliant, whose bases coincide with the perpendicular side of the five-sided planes; so that the second class of iron is thus changed into the third.

That in this same way a body of twenty-four planes is formed from a body of twelve, I am convinced for the following **P. 48.** reasons: (1) Because in the same mass of iron bodies almost all the thinner bodies have only twelve planes, while the thicker ones have twenty-four. (2) Because in certain bodies of twelve planes are seen the beginnings of triangular planes which are accessory and which, if continued, form a body of twenty-four planes.

In triangular planes I have sometimes noticed a smoothness so perfect that not the slightest unevenness was apparent to the eye, — something which it has never yet been my lot to see in any crystal.[3] In other instances I have seen smaller curved planes piled above larger, of which the higher were, for the most part, nearest the triangular apex, so that one may therefore question whether the five-sided planes are not formed by the bases of triangular planes, since traces of striæ appear in them parallel to the bases.

---

[1] The polyhedral angles. Steno is apparently referring to the relation of the rhombohedron to the cube.

[2] Steno evidently thought that the various modifications of hematite resulted from an evolution in time of new crystal forms.

[3] Quartz or rock crystal.

That in the case of copper, angular bodies are formed in the same way that has been mentioned in the case of the crystal and iron, is inferred from those bits of copper which you[1] preserve among the curiosities of nature,[2] but since the abundance of the matter has filled all the interstices of the bodies, it is difficult to ascertain the original form of the bodies. And precisely the same is true of the angular bodies of silver sent to you from Germany.

CONCERNING THE DIAMOND

Concerning the diamond, the same thing is inferred touching the place and manner of production which is inferred from the crystal, namely:

1. That diamonds have been produced in a fluid enclosed in P. 49. the cavities of rocks, although a distinguished writer on India attempts to prove that diamonds are again produced in a certain period of years, in the earth from which they have once been dug.[3]

2. That they have been produced from a fluid by the accretion of diamond matter.

3. That in their production the workings of both the attenuated permeating fluid and the surrounding fluid must be taken into account.

For the rest, as regards the form of diamonds, it varies greatly, since some are bounded by eight planes, others by nine,[4] others by eighteen, others by twenty-four planes; and among these most of the planes were striated, while some were also smooth. Although some diamonds might be angular, they nevertheless could have some surfaces curved rather than plane.

---

[1] Ferdinand II. See p. 205.

[2] Steno refers to the collection in the Pitti Palace. See p. 182.

[3] Maar (*op. cit.*, Vol. II, p. 338) observes that Steno may have had in mind P. de Rosnel's *Le Mercure Indien* (Paris, 1667), *Seconde parte, livre premier*, Chapter II, p. 12: "Monardes en son livre . . . remarque que les grands diamants prennent d'ordinaire leur naissance de la partie inferieure de la mine, et que les petits prennent la leur de la partie superieure."

[4] Not a regular form of diamond but doubtless due to the disappearance of certain faces.

CONCERNING MARCASITES

The substance of marcasites assumes divers forms, for sometimes it incrusts the surface of a place, sometimes it is condensed into bodies of many planes, sometimes it forms rectangular parallelopipeds[1] which, after the usual mode of speech, we shall call cubes, although regularity of all the planes is found in but a few.

Since I have had the opportunity to note various matters concerning the cubes of marcasites, both the cubes themselves and the place where they are found, I shall speak concerning those matters only; but their formation, nevertheless, differs from the formation of a crystal:

1. In time; since the cubes of marcasites were formed before
P. 50. the formation of the strata in which they are contained, whereas crystals hardened after the formation of the strata.

2. In place of production; for a crystal, at least while it was forming, was resting upon a solid body and so was contained partly in a solid place, partly in a fluid, while the cubes of the marcasites seem to have formed between two fluids, since there are no traces, even in the larger cubes, of cohesion with another body; although small cubes are frequently found which, while growing, adhere to one another in the surface of the fluid. Moreover we are taught by the weighty proofs of the great Galileo[2] that heavier substances of this kind can cling together on the surface of a fluid while one of their surfaces is in immediate contact with an overlying and lighter fluid of another kind. That one of the fluids referred to was aqueous, is shown by the matter of the stratum which results from the same fluid.

3. In the manner and place of accretion; for the matter of the marcasite is added to all the planes of the cubes in a manner different from that which we have indicated in the case of crystals. This fact is clearly shown by the uniformity of all the surfaces of the cubes which I have myself cut from rocks; all the planes of these had striæ parallel to two sides, in such

---

[1] For Steno's use of the word *marcasites*, see p. 225, note 1.

[2] The treatise of Galileo (1564–1642) to which Steno refers is entitled *Discorso al Serenissimo Don Cosimo II, Gran Duca di Toscana, Intorno alle Cose che Stanno in Su L'Acqua O Che in Quella Si Muovono.* Cf. *Le Opere di Galileo Galilei, Edizione Nationale,* Firenze, Vol. IV, 1894, pp. 63–141.

a way indeed that the striæ in opposite planes ran along in the same direction, while planes adjacent to each other showed a different direction of the striæ. From the direction of the striæ it follows that the surrounding fluid was directed about every cube by a threefold movement.[1] Of these movements P. 51. one was perpendicular to the horizon; the remaining two, parallel to the horizon, were perpendicular in relation to each other. And it is not difficult to account for this threefold movement; for while the fluid is trying to withdraw from the earth's centre, that direct movement is checked by the base of the cube, with the result that the fluid is deflected toward the narrower sides, inasmuch as the force of the ascending fluid is stronger along the wider sides and so allows no approach in that quarter; and in this way two pairs of planes are marked out by the traces of the striæ. The third pair of planes receives its striæ from that part of the fluid which passes between the cube and the fluid rebounding from the base of the cube.

4. In perfection of form; for in crystals scarcely a single one is found in whose form something is not lacking. Cubes of marcasites, however, rarely have a missing part; and the explanation is not difficult. For, since all the solid angles in the crystal, except the terminal angles, are obtuse, and the crystalline matter is added little by little to their separate planes, any given plane remains imperfect, if the adjacent planes change their shape, in just the degree that more substance is added to that one alone. Since in cubes of marcasites, however, all the solid angles are right angles, even if new matter be added to one plane only, that same plane always retains the same dimension, provided the adjacent planes do not change their form.

Various other things may be noted in the cubes of marcasites, such as cubes enclosed in cubes; the transparent matter[2] P. 52. enveloped in the substance of the marcasite which encloses another marcasite; and other matters of this kind, which I keep for the Dissertation itself.

There are also angular bodies which are broken up into layers, just as rhomboidal selenites are rhomboid bodies which are broken up into other rhomboidal bodies.[1] And there are various other bodies which, although differing from the crystal in many respects, still all agree in this, that they were formed in a fluid and from a fluid. This is true also of talc, the most famous among chemical substances; so that they are by no means mistaken who believe that the solid body of talc can be resolved into a fluid body, seeing it is beyond cavil that talc was formed from a fluid. But there is no doubt that they are as far as possible astray from the truth who strive to wrench this token from it by means of fire's violence; for talc, accustomed to kindlier treatment at Nature's hands, scorning so great barbarity in lovers of beauty, by way of revenge yields to the fire-god that function of self-destruction which it keeps closed within itself.[2]

---

[1] Steno was probably the first to observe that the cube surfaces of pyrite are commonly striated parallel to three intersecting edges.

[2] Not clear, because pyrite is opaque.

If a careful investigation of angular bodies should be begun, touching not only their composition but also their decomposition, we should soon gain a sure knowledge concerning the diversity of the motion by which the particles of both the attenuated fluid and the surrounding fluid are driven on; and this branch of physics is as essential to all for a true understanding of the workings of Nature as few they be that pursue it.

SHELLS OF MOLLUSKS

P. 53. Among solids naturally enclosed in a solid none occurs more commonly, or occasions greater doubt, than the shells of mollusks. Concerning these, therefore, I shall speak at somewhat greater length, considering first shells taken from the sea, and then those which are dug from mountains.

---

[1] The reference appears to be to the cleavage of selenite.

[2] Talc was "famous" in alchemy. Compare White, *The Hermetic and Alchemical Writings of Paracelsus*, Vol. II, London, 1894, p. 383 (*A Short Lexicon of Alchemy*):

"The older alchemists have often made reference to what they term an Oil of Talc, to which they have attributed so many virtues that subsequently chemists have exerted all their power to compose it. They have calcined, purified, and sublimed the matter in question, but have met with no success. The reason is that the term was used allegorically, and that the reference was to the Oil of the Philosophers, the elixir at the white."

See also Robert Boyle, *The Usefulness of Philosophy* (edition of P. Shaw, London, 1725, Vol. I, p. 67):

"But a credible person, disciple to Cornelius Drebell, cou'd do more than this. He assur'd me, he had a way of building furnaces, wherein he, by the single force of fire, made *Venetian* talc flow; which I confess myself unable to do by the fire of a glass-house." "Talc, usually employ'd in cosmetics, is of so very difficult calcination, that eminent chymists have look'd on all calces of talcs as counterfeit." *Ibid.*, p. 158.

Shells of every kind which at one time had a living creature enclosed in them, reveal to our perceptions the following characteristics:

1. The entire shells are themselves resolved into subdivisions, the subdivisions, again, are divided into filaments, and these filaments are reduced to two kinds differing from each other in color, composition, and place.

2. In the subdivisions of the shell the upper and lower surfaces are nothing but the ends of filaments, while the surface of the edge is the sides of those same filaments located in the edge of the subdivision.

3. The inner surface of the shell itself is identical with the inner surface of the inmost or largest subdivision, while the outer surface is composed of the outer surface of the smallest subdivision, and of the surface of all the edges of the intermediate subdivisions.

Regarding the manner in which shells on animals are formed, the following points can be clearly shown:

1. That the substance of the filaments is like the perspiration of animals, in that it is the fluid exuded through the outer surface of the animal.

2. That the form of the filaments can be produced in two P. 54. ways: either in the animal's very pores, through which they are exuded, or the surface of the growing animal, having become larger than the surface of the subdivisions already hardened, separates from it, and so partly draws the viscous fluid contained between the two surfaces into filaments (a process which is common to viscous fluids), and partly adds to it by the exudation of fresh fluid, because no other substance can enter between the two surfaces.

3. That the difference of the filaments depends upon a difference of the pores by which the surface of the animal is perforated, and upon a difference of the substance which is exuded through the pores; for animals of this kind possess a twofold substance in their surface, of which the one is harder, the other

softer, and both fibrous; a careful examination of these is as illuminating as an investigation of bones.

4. That all the subdivisions, if you exclude the outermost or smallest, were produced between the outer shell and the body of the animal itself, and so have received their forms, not from themselves, but from their place; the result of this is that in the case of oysters the motion of the animal, and the amount of substance, often give rise to some diversity of form. With regard to the outermost shell there can be a doubt whether the surrounding fluid has touched the outer surface or whether it has been protected by a membrane. I should, however, believe that the last view alone is correct: (1) Because the filaments of all the rest of the subdivisions were untouched by the surrounding fluid at the time when they formed. (2) Because in **P. 55.** prickly cockles we see that something like a membrane or skin covers the outside of the shells. But the inquiry concerns something almost outside the realm of vision, and it can be said that the filaments of the first subdivision had already hardened within the egg, since experimental knowledge proves that oysters and other testacea spring from eggs, not from decaying matter.[1]

From what has been said it is easy to explain:

1. All the diversity of hues and of spines which arouse the wonderment of many in the case of shells not only from our own land, but also from other lands; for it has no other origin than the edge of the animal enclosed in the shell. This edge, gradually growing and expanding from something exceedingly small, leaves its impress upon each margin of the subdivisions, since these margins either form from the fluid which is exuded from the outer edge of the animal, or are themselves the creature's outer edges which, like the teeth of the shark, grow

---

[1] Theories of spontaneous generation were common among the Greek philosophers; as, *e.g.* Anaximander, in Diels, *Fragmente der Vorsokratiker* (Zweite Auflage, Berlin, 1906), p. 17, and especially Aristotle, *de Animalium Historia*, V. 1, 3, and *de Generatione Animalium*, I. 23; III. 9, 10, and 11. Steno's friends were the first to combat them scientifically; so Harvey, *Exercitationes de Generatione Animalium* (London, 1651); Francesco Redi, *Esperienze intorno alla Generazione degl' Insetti* (Florence, 1668); Swammerdam, *Historia Insectorum Generalis* (Utrecht, 1669). See also Huxley, *Address before the British Association*, 1870, in *Lay Sermons, Addresses, and Reviews* (New York, 1877), pp. 345-378.

up anew, perhaps, in the place of the earlier edge and, like those same teeth, are gradually thrust outward.

2. The formation of pearls, not only of those which, clinging to the shells, have a form not quite round, but also of those which, after the pores in the surface of the animal have closed, acquire a round form within the pores themselves. For between the integuments of pearls and the subdivisions of shell of pearl-bearing mollusks there is merely this difference, that the filaments of the shells are located in the same plane, as it were, while the integuments of pearls have their filaments
P. 56. distributed over a spherical surface.

A happy instance of this was furnished by a pearl which, with others, I broke at your command. This pearl, although white without, enclosed within it a black body resembling a grain of pepper in both color and size; in that black body the position of the filaments tending toward the centre was very clear, and the arrangements or spheres of the same filaments could be distinguished. At the same time I saw:

1. That the excrescences on various pearls are nothing else than very small pearls enclosed by the same common crusts.

2. That many pearls of yellowish hue are imbued with a yellow color not only in the outermost surface of the sphere, but in all the inner spheres; so that it is thus no longer possible to doubt that the yellow color must be attributed to the changing fluids of the animal, and that he who seeks to wash it clear, washes an Ethiopian;[1] unless the color has either been acquired, as, for instance, the tint gained from being worn at the throat, or else was yellow in the outermost sphere only, as might be the case if, for instance, the fluids of the animal had changed when the inner spheres were being formed.

In view of these facts the mistake is apparent of those who, without a knowledge of Nature, cleverly attempt the imitation of pearls; for hardly any one could assay that feat successfully unless another Lucullus should fill his aquaria[1] with pearl-bearing mollusks, and either seek in the animals themselves the methods of increasing them, or learn thence the difficulty of imitating the works of Nature. I would not deny that one can P. 57 form, artificially, globular masses consisting of various integuments, but to arrange these integuments from a succession of filaments and unite them according to a system, upon which the natural lustre of pearls depends, this I should consider indeed most difficult.

---

[1] The allusion is to a fable of Æsop, who flourished about the middle of the sixth century, B.C. (Halm, *Fabulae Aesopicae*, Leipzig, 1875, XIII): 'A man bought an Æthiopian believing his color to be what it was through the neglect of his former owner. Upon taking him home, the new owner applied to him soaps of all kinds and tried to whiten him with baths of every description. But he was unable to change the color and was ready to fall sick from his toil. Characteristics remain what they were.'

The shells which lie buried in the earth are reduced to three classes.

The first class consists of those which are as like the shells just described as an egg is to an egg; since both the shells themselves are resolved into subdivisions, and the subdivisions into filaments; and there is the same difference and position of filaments. An examination of the shell itself proves that these shells were parts of animals at one time living in a fluid, even if marine testacea had never been seen, as will appear from the example of bivalve mussels.

At the time when bivalve mussels were formed, the substance contained within the mussel,

1. Had a smooth surface pierced with countless pores, and a twofold variation of pores.
2. Had a substance pliable and less hard than the shell itself.
3. Was in communication on the one side with the surrounding matter, on the other had no communication with it.
4. Gradually withdrew, from the side where communication with the outer matter was denied it, toward the side where it had free communication with that same matter.

---

[1] The fish-ponds of Lucullus were famous in antiquity, but our sources do not indicate that he stocked them with pearl-bearing oysters. Nor does Steno imply that he did so. The *locus classicus* is Pliny's *Natural History*, IX, 170 (54): 'Lucullus cut away a mountain near Naples at greater expense than it had cost him to build his villa. He let in the sea-water, and for this reason Pompeius Magnus used to call him the Roman Xerxes. After the death of Lucullus the fish were sold for four thousand sestertia' (more than $150,000). With this may be compared Pliny, *N. H.*, VIII, 211 (52) and Plutarch, *Lucullus*, 39.

P. 58. 5. Was able to open itself at times in proportion to the size of the angle which the hinges of the shells allow.
6. Grew from a small to a large size.
7. Transmitted through its own substance the matter of which the subdivisions of shell were made.

As regards the outer matter surrounding mussels: (1) If it was not wholly a fluid, at least its power of resistance was less than the power of expansion inherent in the substance within the shells. (2) It contained a fluid substance suited to the formation of the filaments of the subdivisions of shell.

All these conditions of both the inner and the outer place, which are proved by arguments and drawings in the Dissertation itself, fully show that there was an animal within the shells, and a fluid without the shells.

The second class consists of those shells which are in other respects like those just described, but differ from them in color and weight. While some are lighter, others are heavier, because the heavier shells have their pores filled with an extraneous fluid, while the pores of the former have been enlarged by the ejection of the lighter parts; I add nothing further in regard to them because they are nothing but the shells of animals, either petrified or calcined.

The third class consists of shells which in form only are similar to those just described, in other respects differing from them completely; since neither subdivisions nor filaments, much less differences of filaments, are found in them. Some of these are filled with air; others, either black or yellow in
P. 59. color, with stone; others with marble; others with crystal; and still others with other matter. The formation of all of these I account for in the following way.

When the penetrating force of fluids has dissolved the substance of a shell, the fluids have either been drained away by the earth, leaving empty spaces in the shells (which I call shells filled with air),[1] or have been changed by the addition of new matter and have filled the spaces in the shells with crystals or marble or stone, according to the diversity of the matter. And from this source that most beautiful kind of marble which is called Nephiri[1] has its origin, which is nothing else than a deposit of the sea filled with shells of every description, in which a stony substance takes the place of the decomposed substance of the shells.

---

[1] The Latin phrase is *testas aereas;* porous shells are meant.

The limitation of my plan does not allow me to give an exposition of all the things which I have remarked worthy of notice in the different kinds of shells dug from the earth; wherefore, passing by other matters, I shall mention here only the following:

1. A pearl-bearing mussel, found in Tuscany, in which the pearl was clinging to the shell itself.
2. A part of an unusually large pinna in which, after the decomposition of the byssus, the color of the byssus remained in the earthy matter which had filled the shell.
3. There are shells of oysters of marvellous size in which are found several oblong cavities eaten out by worms, quite

---

[1] The term *Nephiri* is unintelligible. Neither nepheline nor nephrite, to which the word bears closest resemblance, fulfils the requirements of a marine deposit. Maar's note (*Om Faste Legemer*, p. 105) leaves the difficulty unsolved: "Vi har intetsteds kunnet finde nogen Oplysning om *Nephiri*. Professor Heiberg, til hvem vi har henvendt os, antager det for en Trykfejl for *nephriti* og formoder, at marmor nephrites er det, der nu hedder Breccia (Marmarkonglomerat). Hertil maa dog bemaerkes, at nefritisk Marmor naeppe indeholder noget, der af Steno kunde antages for Muslingeforsteninger. Tozzetti (*Reisen* I, p. 127) omtaler nefritisk Marmor fra Bygninger i Pisa, og angiver, at det i Virkeligheden er en Slags Serpentine." See also *Opera Philosophica*, Vol. II, p. 340.

It is more probable that Steno wrote *Septarium*, which was converted to *Nephiri* by a printer's error. The change is not difficult to account for palæographically. The final *um* following a vowel was usually indicated by ~ ; *S* was taken for *N* ; *ph* was an error for *pt*, and *i* for *a*. Steno's chirography was none too clear, as may be seen from the facsimile letter inserted by Wichfeld, *Erindringer om den Danske Videnskabsmand Niels Stensen*, *Dansk historisk Tidsskrift*, 3 Raekke, 4, opposite p. 108. That the error was not corrected in the *legenda*, p. 79, of the original edition cannot be urged against this conjecture, inasmuch as many mistakes in the text escaped notice.

Septaria are thus described by Chamberlain and Salisbury (*A College Text-book of Geology*, New York, 1909, p. 48): "Concretions sometimes develop cracks within themselves, and these may then be filled with mineral matter differing in composition or color from that of the original concretions (Fig. 28). Concretions the cracks of which have been filled by deposition from solution, are called *septaria*. They are especially abundant in some of the Cretaceous shales and clays. In not a few cases the filling of the cracks appears to have wedged segments of the original concretion farther and farther apart, until the outer surface of the septarium is made up more largely of vein-matter than of the original concretion. The development of concretions in rock is not commonly looked upon as metamorphism, but it is really a metamorphic change in the broadest sense of that term."

like those which[1] a certain kind of mussel inhabits in the rocks of Ancona, Naples, and Sicily. These cavities in the **P. 60.** rocks, unless they were formed by insects building a nest out of mud (a thing which I can scarcely believe, since the substance of the middle of the rock, where no cavities are found, is identical with the substance of the rock containing the cavities, which are all confined to the surfaces), must have been eaten out by worms; and this view is not only commended by the surface of the cavity, but also proved by a body composed of rather thick filaments which is found in many cavities, and which answers to the cavity itself in size and shape. Surely, the cavities were made neither by mussels nor around mussels, since testacea of this kind lack the organs for gnawing, and no cavity corresponds to the shape of the shells.[2] Nor is it surprising that rocks exposed to the sea afford a resting place, in their cavities, for mussels' eggs which have been cast up by the sea, for I have not yet seen any of those cavities lacking an evident exit. But if one say that the cavities were made by a petrifying fluid which became hard around certain bodies, some cavities would have been found enveloped by that same substance on every side, and lacking an exit.

4. A shell partly destroyed on the inside, where a marble incrustation covered by various balanoids had supplied the loss of the substance eaten away; so that it is possible to infer with certainty that the shell had been left upon land by the sea, next carried down into the sea; again covered by a new deposit, and abandoned by the sea.

5. Very small eggs and helical shells hardly visible except with the aid of the microscope.

P. 61. 6. Pectens, helical shells, and bivalve mollusks not covered with crystal but crystalline in all their substance.

7. Various tubes of sea worms.

---

[1] The Florentine edition has *quos:* read *quas.*

The cavities, containing the thick filaments referred to by Steno, were probably made by the Lithophagus (Lithodomus). This Lamellibranch, of the family Mytilidae, perforates shells of the Lamellibranchs Melina, Ostrea, and Pectin, and produces a flask-shaped excavation. See von Zittel, *Grundzüge der Paläontologie, Dritte Auflage, 1 Abteilung* (München, 1910), p. 322 and fig. 632 *c.*

[2] The borer is a mussel ; cf. the preceding note.

OTHER PARTS OF ANIMALS

What has been said concerning shells must also be said concerning other parts of animals, and animals themselves buried in the earth. Here belong the teeth of sharks, the teeth of the eagle-fish,[1] the vertebræ of fishes, whole fish of every kind, the crania, horns, teeth, femurs, and other bones of land animals; since all these are either wholly like true parts of animals, or differ from them only in weight and color, or have nothing in common with them except the outer shape alone.

A great difficulty is caused by the countless number of teeth which every year are carried away from the island of Malta; for hardly a single ship touches there without bringing back with it some proofs of that marvel. But I find no other answer to this difficulty than:

1. That there are six hundred and more teeth to each shark, and all the while the sharks live new teeth seem to be growing.
2. That the sea, driven by winds, is wont to thrust the bodies in its path toward some one place and to heap them up there.
3. That sharks come in shoals and so the teeth of many sharks can be left in the same place.
4. That in lumps of earth brought here from Malta,[2] besides **P. 62.** different teeth of different sharks, various mollusks are also found, so that even if the number of teeth favors attributing their production to the earth, yet the structure of these same teeth, the abundance in each animal, the earth resembling the bottom of the sea, and the other sea objects found in the same place, all alike support the opposite view.

Others find great difficulty in the size of the femurs, crania, and teeth, and other bones, which are dug from the earth. But the objection, that an extraordinary size makes it necessary to conclude the size to be beyond the powers of Nature, is not of so great moment, seeing that:

---

[1] Steno's phrase is *piscis aquilae*, 'eagle-fish.' The reference is to a family of rays known scientifically as *Myliobatidae* and popularly as "eagle-rays," "devil-fishes," and "sea-devils." The teeth are flat molars, adapted for crushing hard substances.

[2] Cf. p. 211.

1. In our own time bodies of men of exceedingly tall stature have been seen.

2. It is certain that men of unnatural size existed at one time.[1]

3. The bones of other animals are often thought to be human bones.

4. To ascribe to Nature the production of truly fibrous bones is the same as saying that Nature can produce a man's hand without the rest of the man.

There are those to whom the great length of time seems to destroy the force of the remaining arguments, since the recollection of no age affirms that floods rose to the place where many marine objects are found to-day, if you exclude the universal deluge, four thousand years, more or less, before our time. Nor does it seem in accord with reason that a part of an animal's body could withstand the ravages of so many years, **P. 63.** since we see that the same bodies are often destroyed completely in the space of a few years. But this doubt is easily answered, since the result depends wholly upon the diversity of soil; for I have seen strata of a certain kind of clay which by the thinness of their fluid decomposed all the bodies enclosed within them. I have noticed many other sandy strata which preserved whole all that was entrusted to them. And by this test it might be possible to come to a knowledge of that fluid which disintegrates solid bodies. But that which is certain, that the formation of many mollusks which we find to-day must be referred to times coincident with the universal deluge, is sufficiently shown by the following argument.

It is certain that before the foundations of the city of Rome were laid, the city of Volterra was already powerful. But in the exceedingly large stones which are found in certain places (the remains of the oldest walls) at Volterra, shells of every kind are found,[2] and not so very long ago there was hewn from the midst of the forum a stone packed full of striated shells; hence it is certain that the shells found to-day in the stones had already been formed at the time when the walls of Volterra were being built.

---

[1] The belief in the existence of giants, based upon the finding of fossil bones of beasts, was widespread. See E. B. Tylor, *Researches into the Early History of Mankind* (London, 1865), pp. 314–317; *Primitive Culture*, 4th edition (London, 1903), Vol. I, p. 387.

[2] The courses of massive masonry within the impressive Porta all' Arco are of a yellow conchiliferous sandstone, called *panchina*. See Dennis, *Cities and Cemeteries of Etruria* (London, 1878), Vol. II, p. 144.

And in order that no one may say that the shells only have turned into stone, or that having been enclosed within the stone they have suffered no destruction from the tooth of time, we may remark that the whole hill upon which the most ancient of Etruscan cities is built, rises from the deposits of the sea, placed one above the other, and parallel to the horizon; and in these deposits many strata, not of stone, abound in
P. 64. mollusks that are real and have suffered no change at all; so it is possible to affirm that the unchanged shells which we dig from them to-day were formed three thousand and more years ago. From the founding of Rome to our own times, we reckon two thousand four hundred and twenty years and more; who will not grant that many ages elapsed from the time the first men transferred their homes to Volterra until it grew to the flourishing size it possessed at the time of the founding of Rome? And if to these centuries we add the time which intervened between the first sedimentary deposit of the hill of Volterra, and the time when that same hill was left by the sea and strangers flocked to it, we shall easily go back to the very times of the universal deluge.

The same authority of history forbids our doubting that those exceedingly large bones which are dug from the fields of Arezzo, have withstood the ravage of nineteen hundred years; for it is certain:

1. That the skulls of the pack-animals which are found there do not belong to animals of this climate, as neither do the huge femurs, and very long scapulæ, which are found in the same place.

2. It is certain that Hannibal crossed thither before he fought with the Romans at the Trasumene Lake.

3. It is certain that there were in his army African pack-animals and turret-bearing elephants of huge size.

4. It is certain that while he was coming down from the
P. 65. mountains of Fiesole a large part of the animals kept for carry-

ing packs perished in the marshy places from the excessive floods.[1]

5. It is certain that the place whence are dug the bones under discussion, was heaped up from various strata which are full of stones rolled down by the force of torrents from the surrounding mountains; so that the manifest agreement in all details can no longer remain hidden from one who compares the character of the place and of the bones with the historical record.

PLANTS

What has been said regarding animals and their parts holds equally true of plants and the parts of plants, whether they are dug from earthy strata or lie hidden within rocky substance; for they either completely resemble actual plants and parts of plants (this kind is found rather rarely), or they differ from actual plants only in color and in weight (this kind occurs more frequently, sometimes burnt in charcoal, sometimes impregnated in a petrifying fluid), or they correspond to actual plants in form only; of this last kind there is a great abundance in various places.

---

[1] Cuvier (*Recherches sur les Ossemens Fossiles*, 4th edition, Vol. II, Paris, 1834, p. 17) states that the skeleton of an elephant was found at Arezzo in 1663:

"C'est dans le val de Chiana, au territoire d'Arezzo, que le grand-duc Ferdinand II, ce généreux protecteur des sciences, fit déterrer un squelette entier en 1663, dont proviennent encore, selon Targioni (Tozzeti), une partie des os conservés à Florence, et dont paraissent avoir parlé Stenon et Boccone."

Steno's explanation is a naïve attempt to account for the presence, in very great numbers, of the fossil remains of elephants belonging to the Pleistocene period. Says Cuvier (*op. cit.*, p. 16): "Quand on passe de l'Etat de l'Eglise en Toscane, en suivant le Tibre, le Clanis ou Chiana et l'Arno, les os d'éléphans deviennent de plus en plus nombreux. Le val de Chiana, le val d'Arno et les vallées particulières qui y aboutissent, en contiennent d'étonnantes quantités."

Further references are: Forsyth Major, *Considerazioni sulla Fauna dei Mammiferi pliocenici e post-pliocenici della Toscana* in *Atti di Società Toscana di Scienze Naturale in Pisa*, Vol. I (1875), pp. 7–40, 223–245; III (1877), pp. 207–227; *Mammalian Fauna of the Val d'Arno* in *The Quarterly Journal of the Geological Society of London*, Vol. XLI (1885), pp. 1–8; Depéret, *Evolution of Tertiary Mammals, and the Importance of Their Migrations* in *The American Naturalist*, Vol. XLII (1908), pp. 109–114, 166–170, 303–307.

It may be interesting to note, apropos of Steno's "historical record," that both Livy and Polybius agree that Hannibal had only one elephant by the time he reached Arezzo. According to Eutropius (III. 8) and Polybius (III. 42), Hannibal entered Italy with thirty-seven elephants. Polybius states (III. 74) that all except one perished from the extreme cold immediately after the battle of the Trebia (218 B.C.), and Livy (XXI. 56) remarks that almost all (*prope omnis*) were overcome. In the attempt to cross the Apennines, a detail of the campaign which is not mentioned by Polybius and is a source of confusion in Livy, seven of these succumbed (Livy, XXI. 58). And when Hannibal reached Arezzo in the early spring of 217 B.C. (XXII. 2), Livy represents him as riding the sole survivor.

Regarding the first two classes there cannot be the least doubt that they were at one time actual plants, since the structure of their very bodies compels this view, and the character of the place whence they are dug does not oppose it. They who hold, in opposition, that the earth which had been carried over into houses in process of time changed into wood, cannot affirm this except of the surface of the earth enclosing the wood, where the earth, having become dry in time, and turned to dust, has brought to light the wood enclosed within it. Neither do the metallic filaments found in the pores of the same wood militate against our view, since I myself have drawn from the earth a trunk attesting its plant nature by the knots of its branches and by its bark, whose fissures had been filled with
P. 66. mineral matter. Hence, furthermore, it might throw no little light upon the lore of minerals if an investigation were made of the wood, and the place of the wood to determine what they could have contributed to the formation of minerals. Under the name of bitumen come many things which the channels of the fibres and the ashes of the burnt portions prove to be nothing but charcoal.

A greater difficulty is occasioned by the third class of plants, or the forms of plants marked upon stones, since we find forms of this kind in hoar-frost, in the mercury tree,[1] in various volatile salts, in a white substance[2] soluble in water, which not only forms in glass vessels on their inner surfaces but sometimes rises from the middle of the vessel into the air. But to one who duly weighs all considerations nothing will be found to be inconsistent with the views expressed; for the forms of plants inscribed on stones are reducible to two classes. Some of these forms are imprinted only on the surface of the clefts, which I would readily acknowledge to have been produced without an actual plant, although not without a fluid; others appear not only on the surface of the clefts but spread their branches everywhither throughout the substance of the stone. Hence it follows that at the time when a plant of the second type was

---

[1] The metallic crystals produced by mercury in a solution containing silver.

[2] Ammonium chloride, sal ammoniac.

" being produced, whether this took place after the manner of other plants, or in the fashion of the mercury tree, the substance of the stone had not yet laid aside the character of a fluid; this fact, again, is further assured not only by the softer consistency of the stone, but also by the angular bodies common in the den-
P. 67. drite of Elba, such as form only in a free fluid. But what need is there of other proofs, when experience itself speaks? I have seen various moist places, not only those exposed to the sun, but also underground, where, on account of water flowing by, a rock growing to moss and other plants was being covered with new moss of a different kind.

Hitherto I have reviewed the principal bodies whose place of finding has for many afforded no clue to the place of their production; and at the same time I have hinted how, from that which is perceived, a definite conclusion is formed in regard to that which cannot be perceived.

THE DIFFERENT CHANGES WHICH HAVE TAKEN PLACE IN TUSCANY

In what way the present condition of any thing discloses the past condition of the same thing, is above all other places clearly manifest in Tuscany; inequalities of surface observed in its appearance to-day contain within themselves plain tokens of different changes, and these I shall review in inverse order, proceeding from the last to the first.

1. At one time the inclined plane *A* [Pl. XI, fig. 20] was in the same plane with the higher, horizontal plane *B*, and the end of the same plane *A* thus raised, as also the end of the higher, horizontal plane *C*, were continuous, whether the lower, horizontal plane *D* was in the same plane with the higher horizontal planes *B*, *C*, or another solid body existed there, supporting the exposed sides of the higher planes. Or, what is the same thing, in the place where to-day rivers, swamps, sunken plains, steeps, and planes inclined between sand hills are seen,
P. 68. all was once level, and at that time all the waters, both of rains and of springs, were flooding that plain, or had opened for themselves underground channels beneath it; at any rate, there were cavities under the upper strata.

2. At the time when the plane *B*, *A*, *C* [Pl. XI, fig. 21]

was being formed, and other planes under it, the entire plane *B*, *A*, *C*, was covered with water; or, what is the same thing, the sea was at one time raised above sand hills, however high.

3. Before the plane *B*, *A*, *C* [Pl. XI, fig. 22] was formed, the planes *F*, *G*, *I* [Pl. XI, fig. 23] had the same position which they now hold; or, what is the same thing, before the strata of the sand hills were[1] formed, deep valleys existed in the same places.

4. At one time the inclined plane *I* [Pl. XI, fig. 23] appeared in the same plane with the horizontal planes *F* and *G*, and either the exposed sides of the planes *I* and *G* were continuous, or another solid existed there, supporting the exposed sides when the planes were being formed; or, what is the same thing, where valleys are seen to-day between the plane summits of the highest mountains, there was at one time a single continuous plane under which huge caverns had been formed before the downfall of the upper strata.

5. When the plane *F*, *G* [Pl. XI, figures 24 and 25] was being formed, a watery fluid lay upon it; or, what is the same thing, the plane summits of the highest mountains were at one time covered with water.

Six distinct aspects of Tuscany[2] we therefore recognize, P. 69. two when it was fluid, two when level and dry, two when it was broken; and as I prove this fact concerning Tuscany by inference from many places examined by me, so do I affirm it with reference to the entire earth, from the descriptions of different places contributed by different writers. But in order that no one may be alarmed by the novelty of my view, in a few words I shall set forth the agreement of Nature with Scripture by reviewing the chief difficulties which can be urged regarding the different aspects of the earth.

In regard to the first aspect of the earth Scripture and Nature agree in this, that all things were covered with water[3]; how and when this aspect began, and how long it lasted, Nature says not, Scripture relates. That there was a watery

---

[1] The alluvial deposits of the valley.

[2] This summary takes up the figures in inverse order, figures 25, 24, etc.

[3] See *Genesis*, I. 1–7.

fluid, however, at a time when animals and plants were not yet to be found, and that the fluid covered all things, is proved by the strata of the higher mountains, free from all heterogeneous material. And the form of these strata bears witness to the presence of a fluid, while the substance bears witness to the absence of heterogeneous bodies. But the similarity of matter and form in the strata of mountains which are different and distant from each other, proves that the fluid was universal. But if one say that the solids of a different kind contained in those strata were destroyed in course of time, he will by no means be able to deny that in that case a marked difference must have been noticed between the matter of the stratum and the matter which percolated through the pores of the stratum, filling up the spaces of the bodies which had been destroyed. If, however, other strata which are filled with different
P. 70. bodies are, in certain places, found above the strata of the first fluid, from this fact nothing would follow excepting that above the strata of the first fluid new strata were deposited by another fluid, whose matter could likewise have refilled the wastes of the strata left by the first fluid. Thus we must always come back to the fact that at the time when those strata of matter unmixed, and evident in all mountains, were being formed, the rest of the strata did not yet exist, but that all things were covered by a fluid free from plants and animals and other solids. Now since no one can deny that these strata are of a kind which could have been produced directly by the First Cause, we recognize in them the evident agreement of Scripture with Nature.

Concerning the time and manner of the second aspect of the earth, which was a plane and dry, Nature is likewise silent, Scripture speaks. As for the rest Nature, asserting that such an aspect did at one time exist, is confirmed by Scripture, which teaches us that the waters welling from a single source overflowed the whole earth.[1]

When the third aspect of the earth, which is determined to have been rough, began, neither Scripture nor Nature makes plain. Nature proves that the unevenness was great, while Scripture makes mention of mountains[2] at the time of the

---

[1] See *Genesis*, 2. 10–14.

[2] See *Genesis*, 7. 19–20.

flood. But when those mountains, of which Scripture in this connection makes mention, were formed, whether they were identical with mountains of the present day, whether at the beginning of the deluge there was the same depth of valleys
P. 71. as there is to-day, or whether new breaks in the strata opened new chasms to lower the surface of the rising waters, neither Scripture nor Nature declares.

The fourth aspect, when all things were sea, seems to cause more difficulty, although in truth nothing difficult is here presented. The formation of hills from the deposit of the sea bears witness to the fact that the sea was higher than it is now, that too not only in Tuscany but in very many places distant enough from the sea, from which the waters flow toward the Mediterranean; nay, even in those places from which the waters flow down into the ocean. Nature does not oppose Scripture in determining how great that height of the sea was, seeing that:

1. Definite traces of the sea remain in places raised several hundreds of feet above the level of the sea.

2. It cannot be denied that as all the solids of the earth were once, in the beginning of things, covered by a watery fluid, so they could have been covered by a watery fluid a second time, since the changing of the things of Nature is indeed constant, but in Nature there is no reduction of anything to nothing. But who has searched into the formation of the innermost parts of the earth, so that he dare deny that huge caverns may exist there, filled sometimes with a watery fluid, sometimes with a fluid akin to air?

3. It is wholly uncertain what the depth of valleys at the beginning of the deluge was; reason, however, may urge that in the first ages of the world smaller cavities had been eaten out by water and fire, and that in consequence not so deep breaks of strata followed from this cause; while the highest
P. 72. mountains of which Scripture speaks were the highest of those mountains which were in existence at that time, not of those which we see to-day.

4. If the movement of a living being can bring it to pass that places which have been overwhelmed with waters are arbitrarily made dry, and are again overwhelmed with waters,

why should we not voluntarily grant the same freedom and the same powers to the First Cause of all things?

In regard to the time of the universal deluge, secular history is not at variance with sacred history, which relates all things in detail. The ancient cities of Tuscany, of which some were built on hills formed by the sea, put back their birthdays beyond three thousand years; in Lydia, moreover, we come nearer to four thousand years: so that it is possible thence to infer that the time at which the earth was left by the sea agrees with the time of which Scripture speaks.[1]

As regards the manner of the rising waters, we could bring forward various agreements with the laws of Nature. But if some one say that in the earth the centre of gravity does not always coincide with the centre of the figure, but recedes now on one side, and now on the other, in proportion as subterranean cavities have formed in different places, it is possible to assign a simple reason why the fluid, which in the beginning covered all things, left certain places dry, and returned again to occupy them.

The universal deluge may be explained with the same ease if a sphere of water, or at least huge reservoirs, be conceived around a fire in the middle of the earth; thence, without the movement of the centre, the pouring forth of the pent-up water
P. 73. could be derived. But the following method also seems to me to be very simple, whereby both a lesser depth of the valleys and a sufficient amount of water are obtained without taking into account the centre, or figure, or gravity. For if we shall have conceded (1) That by the slipping of fragments of certain strata, the passages were stopped through which the sea penetrating into hollow places of the earth sends forth the water to bubbling springs; (2) That the water undoubtedly enclosed in the bowels of the earth, was, by the force of the known subterranean fire in part driven toward springs, and in part forced up into the air through the pores of the ground which had not

---

[1] Steno accepted the chronology of Archbishop Usher, which assigned the creation of the world to the year 4004 B.C. In this connection, see also J. Woodward, *An Essay Toward a Natural History of the Earth and Terrestrial Bodies, Especially Minerals*, etc., London, first edition, 1695; J. Arbuthnot, *An Examination of Dr. Woodward's Account of the Deluge. . . . With a Comparison between Steno's Philosophy and the Doctor's in the Case of Marine Bodies Dug out of the Earth*, London, 1697.

yet been covered with water; that, moreover, the water which not only is always present in the air but also was mixed with it in the manner previously described, fell in the form of rain; (3) That the bottom of the sea was raised through the enlarging of subterranean caverns; (4) That the cavities remaining on the surface of the earth were filled with earthy matter washed from the higher places by the constant falling of rains; (5) That the very surface of the earth was less uneven, because nearer to its beginnings — if we shall have granted all this, we shall have admitted nothing opposed to Scripture, or reason, or daily experience.

What happened on the surface of the earth while it was covered with water, neither Scripture nor Nature makes clear; this only can we assert from Nature, that deep valleys were formed at that time. This is (1) because the cavities, made larger by the force of subterranean fires, furnished room for greater downfalls; (2) because a return passage had to be opened for the waters into the deeper parts of the earth; P. 74. (3) because to-day, in places far from the sea are seen deep valleys filled with many marine deposits.

As for the fifth aspect, which revealed huge plains after the earth had again become dry, Nature proves that those plains existed, and Scripture does not gainsay it. For the rest, whether the entire sea presently receded, or whether, indeed, in the course of ages new chasms opening afforded opportunity for disclosing new regions, it is possible to determine nothing with certainty, since Scripture is silent, and the history of nations regarding the first ages after the deluge is doubtful in the view of the nations themselves, and thought to be full of myths. This, indeed, is certain, that a great amount of earth was carried down every year into the sea (as is easily clear to one who considers the size of rivers, and their long courses through inland regions, and the countless number of mountain streams, in short, all the sloping places of the earth), and that the earth thus carried down by rivers, and added day by day to the shore, left new lands suited for new habitations.

This is in fact confirmed by the belief of the ancients, in accordance with which they called whole regions the gifts of rivers[1] of like name, as also by the traditions of the Greeks,[2] since they relate that men, descending little by little from the mountains, inhabited places bordering on the sea that were sterile by reason of excessive moisture, but in course of time became fertile.

The sixth aspect of the earth is evident to the senses; herein the plains left by the waters, especially by reason of erosion, and P. 75. at times through the burning of fires, passed over into various channels, valleys, and steep places. And it is not to be wondered at that in the historians there is no account as to when any given change took place. For the history of the first ages after the deluge is confused and doubtful in secular writers; as the ages passed, moreover, they felt constrained to celebrate the deeds of distinguished men, not the wonders of Nature. Nevertheless the records, which ancient writers mention, of those who wrote the history of the changes which occurred in various places, we do not possess. But since the authors whose writings have been preserved report as marvels almost every year, earthquakes, fires bursting forth from the earth, overflowings of rivers and seas, it is easily apparent that in four thousand years[1] many and various changes have taken place.

---

[1] In Homer the river Nile (*Odyssey* IV. 477) is ὁ Αἴγυπτος and the country (*Odyssey* XVII. 448) is ἡ Αἴγυπτος. Herodotus (II. 5) calls Aegypt 'the gift' of the Nile, and Plato (*Timaeus*, 22 D) represents an Egyptian priest as saying to Solon: 'And from this calamity (*i.e.* periodic destruction) the Nile, which is our never-failing savior, saves and frees us.' Cf. Strabo, *Geography*, C. 36 (I. 2, 29).

[2] The tradition is explained at length in dialogue in Plato's *Laws*, 677–682 B:

'*Athenian*. Do the ancient traditions seem to you to contain any truth?

Kleinias. What traditions?

Ath. The traditions that many destructions of mankind were occasioned by deluges and diseases and many other things, as a result of which only a small remnant of the human race was left.

Kl. Every one believes all that.

Ath. Come then. Let us think that one of many such destructions was once occasioned by a deluge.

Kl. What are we to think about it?

Ath. That those who then escaped the destruction would only be some mountain shepherds, small sparks of the human race preserved on the mountain tops.

Kl. Clearly. . . .

Ath. After this they came together in greater numbers and increased the size of their cities, and turned to husbandry, first at the foot of the mountains. . . .

Ath. Homer also disclosed the form following the second, saying that the third arose thus. For he says, He founded Dardania, since holy Ilios had not yet been built on the plain, the city of mortal men, but they still dwelt at the foot of Ida with its many springs (*Iliad*, XX. 216–218). . . .

Ath. Now Ilios, we say, was built when men had come down from the heights into a large, fair plain, on a low hill watered by many rivers which descended from Ida.'

We may compare also Plato's *Timaeus*, 22 C; *Statesman*, 270; *Critias*, 109 D.

A summary of the passage in the *Laws* is given by Strabo in his *Geography*, C, 592, 593 (XIII. 1, 25):

'Plato conjectures that three forms of political commonwealth were established after the deluges. The first was upon the mountain tops, a simple and savage affair, composed of folk who feared the waters that still flooded the plains. The second was on the foothills, composed of folk who regained their courage little by little, since the plains were beginning to dry. The third was in the plains. One might mention a fourth and a fifth, and possibly more, but the last was on the sea-coast and in the islands, after all fear of a deluge had vanished.'

Far astray, therefore, do they wander, who criticize the many errors in the writings of the ancients, because they find there various things inconsistent with the geography of to-day. I should be unwilling to put credence in the mythical accounts of the ancients; but there are in them also many things to which I would not gainsay belief. For in those accounts I find many things of which the falsity rather than the truth seems doubtful to me. Such are the separation of the Mediterranean Sea[2] from the western ocean; the passage from the Mediterranean into the Red Sea; and the submersion of the island Atlantis.[3] The description of various places in the journeys of Bacchus, Triptolemus, Ulysses, Æneas, and of others, may be true, although
P. 76. it does not correspond with present day facts. Of the many changes which have taken place over the whole extent of Tuscany embraced between the Arno and the Tiber, I shall adduce evident proofs in the Dissertation itself; and although the time, in which the individual changes occurred, cannot be determined, I shall nevertheless adduce those arguments from the history of Italy, in order that no doubt may be left in the mind of anyone.

And this is the succinct, not to say disordered, account of the principal things which I had decided to set forth in the Dissertation, not only with greater clearness but also with greater fulness, adding a description of the places where I have observed each thing.

---

[1] In order to account for the evolution of earth features within the time limit imposed by his belief in Usher's chronology of creation, Steno is compelled to adopt a theory of violent catastrophes in nature.

[2] See p. 210, note 1.

[3] Plato, *Timaeus*, 25, C, D: 'But later, when violent earthquakes and deluges occurred, in a single day and night of misfortune, all your military power in a body sank into the earth, and in a like manner the island Atlantis sank and disappeared in the sea. For this reason the sea in that region is even now impassable and impenetrable because a shoal of mud (reading κάρτα βραχέος instead of βαθέος) forms a barrier which was caused by the sinking island.' Compare also *Critias*, 108 E, ff.

P. 77. Let Vincentius Viviani examine this work, and report whether there is anything in it which is contrary to the Catholic Faith or to good morals.

Vinc. de Bardis, *Vicar General of Florence.*

Most Illustrious and Most Reverend Sir:

Having seen the new and admirable *Prodromus* of the most distinguished Steno, the *Dissertation Concerning a Solid Naturally Contained Within a Solid*, or rather, if you prefer, of the whole of Physics, and having recognized in it a perfectly sincere manifestation of the Catholic faith and of good morals, as in the very candid author, I have indeed thought the same worthy of being entrusted to type on this, the thirtieth day of August, 1668.

Vincentius Viviani.

Let it be printed servatis servandis.

Vinc. de Bardis, *Vicar General of Florence.*

The seventh day of December, 1668.

Let Franciscus Redi, Consultor of the Holy Office at Florence, examine it and report.

Fr. Jacobus Tosini, *Vicar General of the Holy Office at Florence.*

Most Reverend Father:

The *Prodromus* of the very learned and expert Nicolaus Steno's *Dissertation Concerning a Solid Naturally Contained* P. 78. *Within a Solid*, adorned in the highest degree with sound and noble learning, I have seen, and I have judged it worthy of printing.

Franciscus Redi.

With the foregoing attestation let it be printed at Florence this day, the thirteenth of December, 1668.

Fr. Joseph Tamagninus, *Chancellor of the Holy Office at Florence.*

Gio. Federighi, *Senator and Auditor of the Holy Apostolic Chamber, and through him, Benedetto Gori.*

EXPLANATION OF THE FIGURES

Inasmuch as the brevity of my hurried writing has left not a few things insufficiently explained, especially where the treatment concerns angular bodies and the strata of the earth, in order to afford some sort of remedy for that defect, I have decided to add here the following figures, chosen from very many others.

[Plate IX]

The first thirteen figures, intended to illustrate the angular bodies of crystal, fall into two classes.

The first class contains seven varieties of a plane in which the axis of the crystal lies.

In figures 1, 2, and 3, the axes of the parts, of which the body of the crystal is composed, form a straight line; but there is an intermediate prism, which is lacking in Figure 1, appears rather short in 2, longer in 3.

In Figure 4, the axes of the parts which make up the body of the crystal do not form a straight line.

Figures 5 and 6 belong to the class of those which I could present in countless numbers to prove that in the plane of the axis both the number and the length of the sides are changed in various ways without changing the angles; that various cavities are left in the very middle of the crystal, and that various layers are formed. Figure 7 in the plane of its axis shows how both the number and the length of the sides are sometimes increased in various ways, sometimes diminished, from the new crystalline matter placed above the planes of the pyramids.

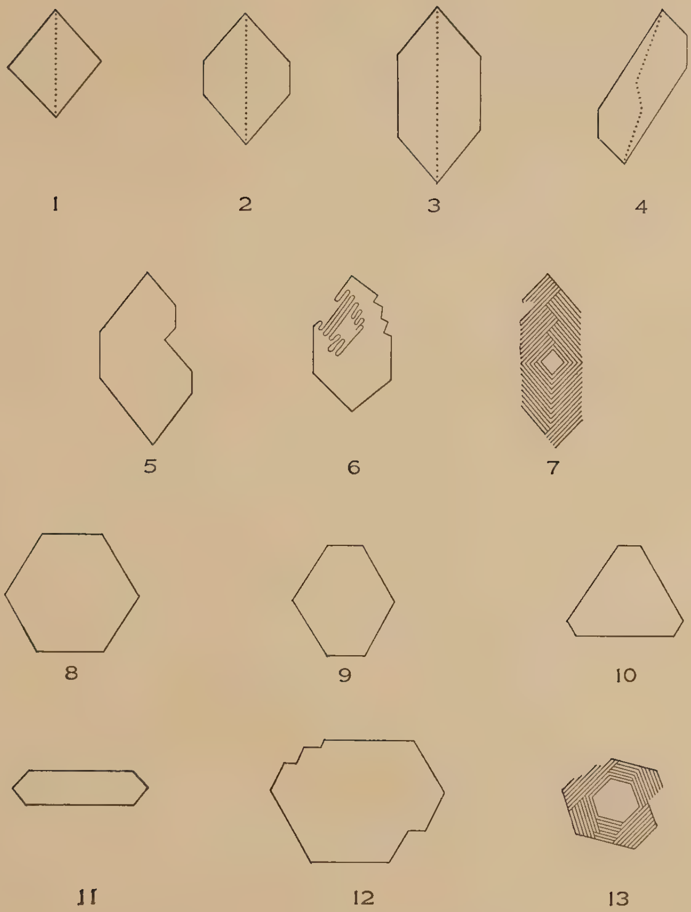
The second class contains six varieties of base of planes.[1]

In Figures 8, 9, 10, and 11, there are only six sides; with this difference, nevertheless, that in Figure 8 all the sides are equal, while

---

[1] The cross section parallel to the basal pinacoid.

Plate IX.


1
2
3
4
5
6
7
8
9
10
11
12
13

Steno's Figures 1–13, in Exact Size.

in Figures 9 and 11 not all, but only the opposite sides, are equal; in Figure 10, any given opposite sides are unequal.

In Figure 12 the plane of the base, which ought to be a hexagon, is bounded by twelve sides.

Figure 13 shows how sometimes the length and number of the sides are changed in various ways without changing the angles, on the plane of the base, while new crystalline matter is being placed upon the planes of the pyramids.

[Plate X]

The six following figures explain two different kinds of angular bodies of iron.

Figures 14, 15, 16 serve to illustrate those angular bodies of iron which are enclosed by twelve planes; Figure 14, in fact, shows all the twelve planes spread out into a single plane, six of these being triangular and brilliant, the remaining six pentagonal and striated. Figure 15 is the plane of the base of the same body. Figure 16 is the plane of the axis of the same body.

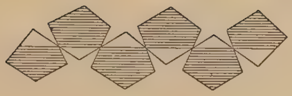
Figures 17, 18, and 19 serve to illustrate those angular bodies of iron which are bounded by thirty planes.

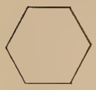
Figure 17 shows all the thirty planes spread out into a single plane; of these six planes are pentagonal and brilliant, twelve triangular and also brilliant, six triangular and striated, six oblong quadrilaterals and brilliant.

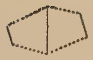
Figure 18 is the plane of the base of the same body.

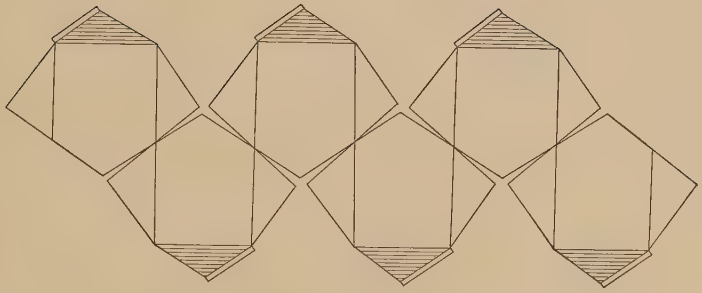
Figure 19 is the plane of the axis of the same body.

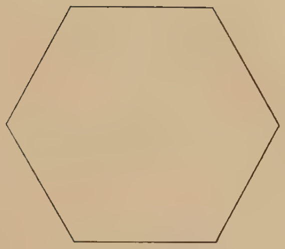
Plate X.

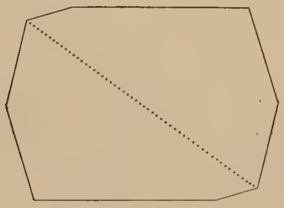

14


15


16


17


18


19

Steno's Figures 14–19, in Exact Size except 17.

[Plate XI]

The last six figures, while they show in what way we infer the six distinct aspects of Tuscany from its present appearance, at the same time serve for the readier comprehension of what we have said about the earth's strata. The dotted lines represent the sandy strata of the earth, so called from the predominant element, although various strata of clay and rock are mixed with them; the rest of the lines represent strata of rock, likewise named from the predominant element, although other strata of a softer substance are sometimes found among them. In the Dissertation itself I have explained the letters of the figures in the order in which the figures follow one another: here I shall briefly review the order of change.

Figure 25 shows the vertical section of Tuscany at the time when the rocky strata were still whole and parallel to the horizon.

Figure 24 shows the huge cavities eaten out by the force of fires or waters while the upper strata remained unbroken.

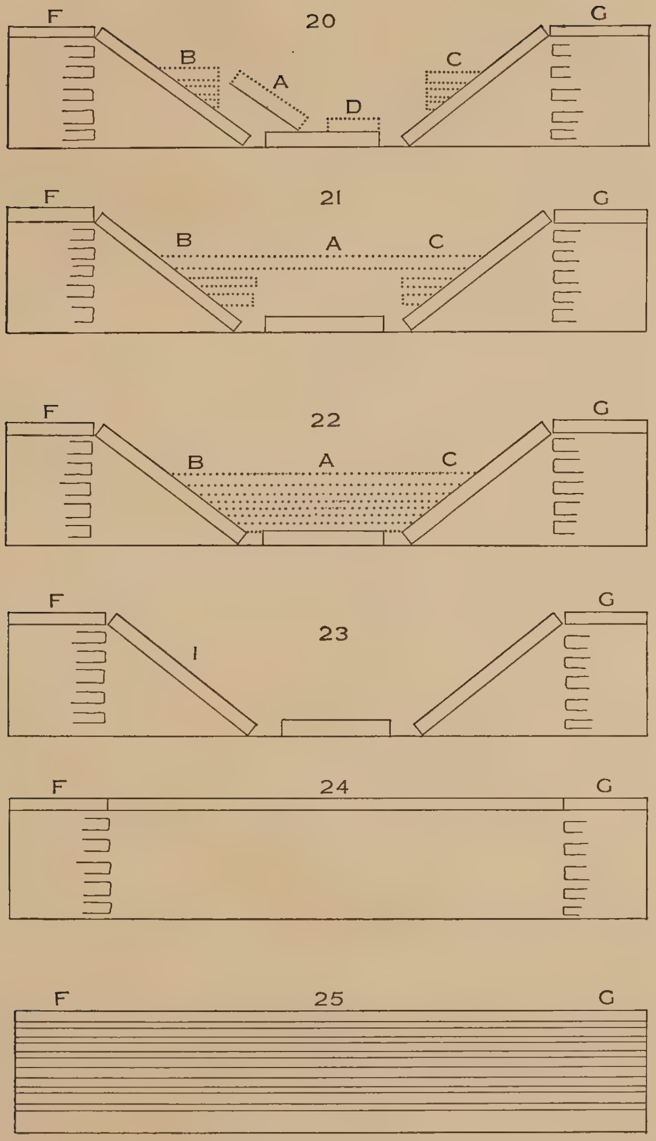
Figure 23 shows the mountains and valleys caused by the breaking of the upper strata.

Figure 22 shows new strata, made by the sea, in the valleys.

Figure 21 shows a portion of the lower strata in the new beds destroyed, while the upper strata remain unbroken.

Figure 20 shows the hills and valleys produced there by the breaking of the upper sandy strata.

Plate XI.


F
20
G
B
A
C
D
F
21
G
B
A
C
F
22
G
B
A
C
F
G
23
I
F
24
G
F
25
G

Steno's Figures 20–25, in Exact Size.

LIBRARY
LOS ANGELES COUNTY MUSEUM

INDEX

Abel, 211, n. 1; 224, n. 1.
Academics, 213, n. 2; 214.
Accademia del Cimento, founding of, 180; Steno member of, 209.
Acosta, de, 236, n. 2.
Accretions, 225; inorganic, 232, n. 2; to marcasites, 247.
Æneas, 269.
Æschylus, 205, n. 3.
Æsop, 252, n. 1.
Aëtites, 224, n. 1.
Africa, land bridge from, 174.
Agate, 224, 225.
Agent, as form or idea, 216; determining motion, 216; universal, 217.
Agricola, George, on glossopetræ, 211, n. 1; on mountain formation, 232, n. 2; on gnomes, 232, n. 2; on divining rod, 236, n. 1.
Air, in breathing, 221; Hippocrates' theory of, 223; explosion of, 231; in crystals, 238.
Albertus Magnus, on glossopetræ, 211, n. 1.
Alchemy, 249, n. 2.
Alimentary canal, 221.
All Souls' Day, 180, n. 3.
Alps, 234, n. 2.
Alum, 243; feathery, 225.
Amethysts, 225.
Amianthus, 224.
Ammon, marine deposits at temple of, 210, n. 1.
Ammonium chloride, 261, n. 2.
Amniotic fluid, 220.
Amsterdam, Steno's arrival in, 176, 183, 208, n. 1.
Anaximander, 251, n. 1.
Ancients, 267, 269; on fossils, 210 f.
Ancona, 256.
Angelis, de, on Steno's degree, 176, n. 1.
Angles, law of constancy of interfacial, 171; of crystals, 237, 248; of hematite, 245; of marcasites, 248.
Angular bodies, 220; with meaning of crystals, 225, 226, 244; of iron, 244 f.; place and production of, 244; planes of hematite, 245; striation of, 244 f.
Animals, 216; fluids in, 221; formation of, 221; parts of, 257; place of growth in, 220; shells of, 218.
Antimony, 218.
Apennines, 260, n. 1.
Aqua fortis, 243, n. 1; regia, 243, n. 1.
Aquaria, 253.
Arabian Gulf, 210, n. 1.
Arbuthnot, 266, n. 1.
Arezzo, fossils at, 259.
Aristotle, 251, n. 1.
Armenia, salt lakes in, 210, n. 1.
Arno, extinct animals in valley of, 174, 260, n. 1; 269.
Arnolfini, Lavinia Felice Cenanni, 180.
Artaxerxes, 210, n. 1.
Arteries, 222.
Ashes, in strata, 229, 232.
Atlantic, level of, 210, n. 1.
Atlantis, 269.
Atoms, 216.
Axes, of crystals, 237, 272 f.

Bacchus, 269.
Bacon, Francis, 181, n. 3; 196, 205, n. 3.
Balanoids, 256.
Banks, Sir Joseph, 197.
Bardis, de, 271.
Barlæus, Caspar, 232, n. 1.
Bartholin, Thomas, 175, 176 f., 207, n. 3; 211, n. 1.
Baudry, Paul, 205, n. 3.
Beaumont, Elie de, 201.
Bezoar, 224.
Bitumen, 229, 232, 261.
Bladder, 221.
Blaes, Gerard, 176, 177, 178.
Blondel, 181.
Blood, circulation of, 222.
Boccone, 260, n. 1.
Bodies, produced naturally, 215; not produced by earth and rocks, 218.
Bohemia, 232, n. 2.
Bologna, 186.
Bones, broken, 225; fibrous, 258; fossil taken for human, 258; enclosed in solids, 218.
Boni, 232, n. 2.
Borch, Ole, *see* Borrichius.
Borrichius, 175, 178.
Bossuet, 180.
Boyle, Robert, 198, 199, 199, n. 1; 200, n. 1; 232, n. 1; 236, n. 1; 241, n. 2; 243, n. 1; 249, n. 2.
Brazil, 232.

Breccia, 255, n. 1.
Bruno, Giordano, 169.
Burial, Chinese, 236.
Byssus, 255.
Byzantium, 210, n. 1.

Calculi, 221.
Callus, 225.
Canals, 222.
Candolle, 181, n. 3.
Capellini, Giovanni, 186.
Capillaries, 221, 222, 223.
Casserius, 176.
Cause, First, 264, 266.
Caverns, 234, 265, 267.
Caves, 234.
Cavities, in body, 222; in crystals, 238; in earth, 230; in rocks, 238, 256; in shells, 255.
Chalcedony, 224.
Chamberlain, 255, n. 1.
Charcoal, 229, 260.
Chemists, 216.
Chéreau, 175, n. 2; 176, n. 1.
Chiana, 260, n. 1.
Chinese, 236.
Chorion, 220.
Christian V, 183.
Cicero, 176, n. 4; 205, n. 3.
Cinnabar, 218.
Clanis, 260, n. 1.
Clays, 255, n. 1.
Cleavage, of selenites, 249, n. 1.
Cockles, 210, n. 1; 251.
Concretions, 226, n. 1; 255, n. 1.
Consani, Vincenzo, 185.
Copenhagen, 175, 176, 185.
Copper, 218, 225, 246.
Cosimo III, 182, 183, 185.
Cracks, in concretions, 255, n. 1; in strata, 231, 232.
Crania, 257.
Creation, strata at, 228; chronology of, 269.
Crete, 210, n. 1.
Cribration, 222, 223, 224.
Crusts, 228, 229.
Crystals, angles of, 237; cause of, 242; columnar form of, 171; faces of, 171; figures of, 272 ff.; formation of, 237 ff.; growth of, 171, 238 f.; hardening of, 218, 237; hues of, 240, 243; enclosed in solid, 218; law of interfacial angles of, 171; lustre of, 240; of mountains, 225; movement in, 242; of niter, 171, 219; nucleus of, 171, 172; orientation of molecules in, 171; parts of, 237; phantom, 171; planes of, 237, 239; prisms of, 237; pyramids of, 237; means quartz, 218, n. 1; 220, n. 1; 237, n. 1; in shells, 254; surface of, 239.
Crystalline matter, 238, 239, 240, 241.
Crystallography, 171.
Cubes, of marcasites, 247; truncated, 245.
Cuvier, 260, n. 1.
Cybotus, 232, n. 2.
Cynics, 213, n. 2; 214.

Damigeron, 224, n. 1.
Dardania, 268, n. 2.
Dati, Carlo, 180.
Davis, Robert, 198.
Deluge, Universal, 169, 174, 258, 264, 265, 266, 268.
Democritus, 205.
Demons, 232, n. 2.
Demosthenes, 224, n. 1.
Dendrites, 225, 262.
Denmark, 232, n. 2.
Dennis, 258, n. 2.
Depéret, 260, n. 1.
Deposits, alluvial, 263; marine, 172, 226.
Descartes, 170, 179, 228.
Devil-fishes, 257, n. 1.
Diamonds, 225, 246.
Diels, 251, n. 1.
Dissertation, 181, 208, 215, 248, 254, 269, 276.
Divination, 236.
Divining rod, 173, 236, n. 1.
Dog's head, dissection of, 177.
Dragon, 236.
Drebell, Cornelius, 249, n. 2.
Ducts, lymphatic, 222.

Eagle, 224, n. 1.
Eagle-fish, teeth of, 257.
Eagle-rays, 257, n. 1.
Eaglestone, 224, 224, n. 1.
Earth, productivity of, 212, 216, 217.
Earthquakes, 173, 232, n. 2; 235, 269.
Egypt, 210, n. 1; 268, n. 1.
Elba, 236, 244, n. 1; 262.
Elements, chemical, 216; four, 216.
Elephants, 174, 259.
Elixir, 249, n. 2.
Epicurus, 213, n. 2.
Erasistratus, 222, n. 2.
Eratosthenes, 210, n. 1.
Erosion, 268.
Ethiopian, hue of, 252.
Eutropius, 260, n. 1.
Euxine, outlet of, 210, n. 1.
Exhalations, 235.
Eyelids, 221.

Eyes, 221, 242.
Eysson, 177.

Fabronius, 232, n. 2.
Faces, of crystal, 239.
Fat, 225.
Femurs, 257, 259.
Federighi, 271.
Ferdinand II, collection of minerals, 246, n. 1; death of, 182; dedication to, 205; fossils found by, 260, n. 1; patron of Steno, 169, 179, 180, 207, 211, n. 1.
Fibres, of muscle, 222, 224, 225; of plants, 221, 225.
Fiesole, 259.
Figures, explanation of, 272 ff.
Filaments, of mineral, 224; in rocks, 256; in receiver, 242; of shells, 174, 250 f., 254.
Filings, of iron, 241.
Fire, agency in breaking up crystals, 243; agency in mountain formation, 232, n. 2; proof of in strata, 229; subterranean, 232, 266, 267.
Fish, 253, 257.
Fissures, containing crystals, 238; containing minerals, 218.
Floods, 211, 229, 258, 260, 269.
Florence, Geologists' Congress in honor of Steno, 186; fossils from Arezzo in, 260, n. 1; Steno's arrival in, 183; topography of, 173.
Flourens, 182, n. 3.
Fluid, aqueous, 219; common, 222, 223; defined, 222; differing from solid, 171; external, 220, 221: internal, 220, 221, 223; peculiar, 223; penetrating, 217; permeating, 214, 221; petrifying, 256.
Fontenelle, 208, n. 2.
Foramina, of body, 221.
Fossils, elephants, 260, n. 1; origin, 170, 173, 200; from sea, 208, 208, n. 2; 210, 211.
Frederik III, 181, 182, 208, n. 1.
Furnaces, 249, n. 2,
Fürstenberg, von, 184.

Galen, 222, n. 2.
Galileo, 247, n. 2.
Gases, subterranean, 231.
Generation, spontaneous, 251, n. 1.
Genesis, 263, n. 3; 264, n. 1, 2.
Germany, silver in, 246.
Gerra, 210, n. 1.
Giants, 258, n. 1.
Gibraltar, land passage at, 210, n. 1.
Glands, 207, 222.
Glass, distinguished from crystal, 171; formation of, 243.
Glossopetræ Melitenses, 211, 211, n. 1.
Gnomes, 232, n. 2.
God, 216.
Gold, 243, n. 1.
Golias, Jacob, 178.
Goniometer, 171.
Gori, 271.
Granites, 225.
Grafting, 215.
Grass, in strata, 228.
Greeks, 268.
Griffenfeldt, Count, 178, 183.
Growth, of crystals, 238; of mountains, 232.
Gypsum, 218, n. 1.

Hamburg, 184.
Hannibal, pack animals of, 174, 259, 260, n. 1.
Hannover, 182, n. 1.
Hardening, of crystals, 237; of solids, 218.
Harvey, 222, n. 4; 251, n. 1.
Heart, 207.
Heat, 232.
Heiberg, 255, n. 1.
Hellespont, 210, n. 1.
Hematite, 244, n. 1.
Heraclitus, 205, n. 3.
Herodotus, 268, n. 1.
Hesperian, sinking of, 198, n. 3.
Hills, formation of, 232, n. 2; 263.
Hippocrates, 223.
Hoar-frost, 261.
Holland, religious tolerance in, 178, 180.
Homer, 268, n. 1.
Hooke, Robert, 173, 197, 201.
Hoover, 232, n. 2; 236, n. 1.
Horizontality, of strata, 172, 230.
Horne, Van, 178.
Horns, 257.
Hues, of crystals, 243; of pearls, 252; of shells, 251, 254.
Hughes, 176, n. 1.
Humboldt, A. von, 182, n. 3.
Hungary, 232, n. 2.
Huxley, 182, n. 3; 251, n. 1.

Ice, making, 215.
Ida, 268, n. 2.
Idols, 236.
Iliad, 268, n. 2.
Ilios, 268, n. 2.
Incrustations, growth of, 220, 224; on solids, 226.
India, 246.
Innsbruck, 182.
Insects, 256.
Interstices, in body, 214.

Intestines, 221.
Iron, cubes of, 225; of Elba, 236; growth of, 236; n. 3; figures of crystals of, 273; filings of, 241, 242.
Irradiations, 237.
Italy, 269.

Jacobæus, Matthias, 178.
Johann Friedrich, Duke of Hannover, 183, 184.

Kidneys, 222, n. 2.
Kircher, Athanasius, 180, 182, n. 2; 234, n. 1; 236, n. 1.
Knorr, 211, n. 1.

Laboulbène, 224, n. 2.
Labyrinth, 206.
Lake Sirbonis, 210, n. 1.
Lamellibranch, 256, n. 1.
Lapilli, 231.
Lapis lazuli, 218.
Laws, of nature, 215.
Layers, deposition of, 226, 227; mineral layers in rivers, 228; of rock, 226.
Lefebure, 205, n. 3.
Leibnitz, 182, n. 1; 211, n. 1.
Lernæan Hydra, 206.
Leyden, 178.
Libya, connected with Europe, 210, n. 1.
Limonite, 236, n. 3.
Linnæus, 187.
Lithodomus, 256, n. 1.
Lithophagus, 256, n. 1.
Littré, 243, n. 1.
Liver, 222, n. 2.
Livy, 260, n. 1.
Load-stone, *see* Magnet.
Lucca, 180.
Lucullus, 253.
Lungs, 222, n. 2.
Lustre, in crystals, 240.
Lydia, 266.
Lymphatic ducts, 222.

Maar, 175, n. 1; 178, n. 1, 2; 179, n. 2; 182, n. 2; 188 ff., 195, 197, 198, n. 2, 3; 202, 206, n. 1; 207, n. 5; 211, n. 1; 222, n. 1; 222, n. 3; 223, n. 1; 224, n. 2; 226, n. 2; 232, n. 1, 2; 234, n. 1; 236, n. 1, 2; 246, n. 3; 255, n. 1.
Magalotti, Lorenzo, 180.
Magnet, 170; filings about, 241, 242; lines of force, 171.
Maillet, de, 169.
Major, Forsythe, 260, n. 1.
Malta, Bartholin's journey in, 211, n. 1; stones from, 211, n. 1; teeth from, 257.
Manni, 184.
Marbles, 225.
Marcasites, cubes of, 225; formation of crystals of, 247 ff.; hardening in, 218; enclosed in solid, 218; = pyrites, 218, n. 1.
Maria Flavia del Nero, 180.
Marine deposits, 172, 210.
Matiana, 210, n. 1.
Matrix, of crystals, 238.
Matter, constitution of, 216; surrounding mussels, 254.
Maurits, Count Jan, 232, n. 1.
Maximilian Heinrich, 184.
Maxims of Morality, 214.
Medici, Leopold de', 180, 209, n. 1.
Mediterranean, 210, n. 1; 265, 269.
Medulla, 225.
Melina, 256, n. 1.
Membrane, dividing, 222.
Menstruum, 243, 243, n. 1.
Mercury, 218, 243, n. 1; tree, 261, 262.
Metals, growth of, 232, n. 2.
Metamorphism, 255, n. 1.
Microscope, 256.
Mineral, replacements of, 173; repositories of, 235 ff.; veins of, 173.
Minerals, formation of, 261; enclosed in bodies, 218; origin of, 235 f.
Miners, 236.
Mines, 232, n. 2.
Molars, 257, n. 1.
Molecules, orientation of, 171.
Mollusks, bivalve, 256, 257, 258; petrified, 225, 226; produced by Nature, 217; shells of, 249 ff.; structure of, 173.
Monardes, 246, n. 3.
Montanari, 232, n. 2.
Moss, 262.
Motion, artificial, 215; of crystalline matter, 241; determination of, 214 f., 216; first cause of, 215; of particles in fluids, 249; principle of, perception of, 214.
Mountains, causes of, 173; chains of, 234; crests of, 234; crystals of, 219; under deluge, 264; growth of, 232, 232, n. 2; origin of, 231 ff.; overthrown, 234; overwhelmed, 263; of scripture, 265; types of, 173; upheaval of, 228.
Mount Casius, 210, n. 1.
Mouth, 221.
Movements, in crystal, 242; of fluid forming marcasites, 248.
Munster, 184.
Murano, 182.

Murchison, 234, n. 2.
Muscle, fibres, 222.
Muscles, Steno's work on, 208 ; substance of, 222.
Musgrave, George, 232, n. 1.
Mussels, 241 ; pearl-bearing, 255 ; substance in bivalve, 253.
Myliobatidæ, 257, n. 1.
Myths, 267, 269.
Mytilidæ, 256, n. 1.

Naples, 182, 253, n. 1; 256.
Natural Questions, solution of, 210, 213.
Nature, Laws of, 214 ; products of, 215, 216, 217 ; unknown, 217.
Needham, 176, n. 2.
Nepheline, 255, n. 1.
Nephiri, 255, 255, n. 1.
Nephrite, 255, n. 1.
Nile, 268, n. 1.
Nilsdatter, Anna, 178, n. 3 ; 207, n. 4.
Niter, crystals of, 219.
Nose, 221.

Odyssey, 180, 268, n. 1.
Œsophagus, 221.
Oldenburg, Henry, 197, 199 ff.
Onyx, 224.
Orange, 220.
Organs, excretory, 221.
Ostrea, 256, n. 1.
Ovum, 222.
Oysters, 210, n. 1; 241, 251, 255.

Palissy, Bernard, 208, n. 2; 224, n. 1.
Panchina, 258, n. 2.
Parenchymata, 222, 224.
Parotid duct, 176.
Particles, added to a solid, 226 f.; changeable, 216; growth of, 220, 221; motion of, 214, 216.
Passageways, for things issuing from the earth, 234 f.
Paulli, Jacob Henry. 177.
Paulli, Simon, 176.
Pearls, formation of, 252; imitation of, 252; in mussels, 255; structure of, 174.
Pectens, 256.
Pectin, 256. n. 1.
Pedersen, 175, 178, n. 3.
Peduncle, 220.
Pelusium, 210, n. 1.
Penetrating fluid, 241.
Pepys, Samuel, 197, n. 1.
Peripatetics, 213, n. 2; 214.
Peru, 236.
Phrygia, 210, n. 1.
Pine cones, in strata, 228.
Pinna, 255.
Pisa, 255, n. 1.
Piso, Willem, 232, n. 1.
Pitti Palace, 182, 246, n. 2.
Place, definition of term, 219; place of crystal formation, 237; of production, 217.
Plains, 267.
Planes, of crystals of hematite, 244, 245; of crystals of quartz, 237, 239; of diamonds, 246; of marcasites, 248.
Plants, anatomy of, 221; fibrous parts of, 225; fossil, 226, 260 f.; formation of, 221; enclosed in solid, 218; metallic, 225; place of, 219; roots of, 220.
Plato, 222, n. 4; 268, n. 2; 269, n. 3.
Pleistocene, 260, n. 1.
Plenkers, 175, 175, n. 1; 178, n. 1; 178, n. 3; 179, n. 2; 180 f.; 180, n. 2; 183, 184, 185, 188, 211, n. 1.
Pliny, 211, n. 1; 224, n. 1; 253, n. 1.
Plutarch, 253, n. 1.
Poisoner, 224, n. 1.
Polybius, 260, n. 1.
Pompeius, 253, n. 1.
Pontus, 210, n. 1.
Pope Innocent XI, 184.
Pores, 221, 223, 253.
Porta all' Arco, 258, n. 2.
Prague, 182.
Pregnancy, 224, n. 1.
Prism, 237.
Prodromus, bibliography of, 194 ff.; date of composition, 181; division of subjects, 170; geometrical form of, 170; science of, 169; scope of, 181 f.; use of word, 181, n. 3; translation of, 205 ff.
Production, of diamonds, 246; of hematite, 244; of substances, 216.
Propontis, 210, n. 1.
Pumice stone, in strata, 229.
Pyramids, of crystal, 237, 239; of hematite, 245.
Pyrites, 218, n. 1; 248, n. 1.

Quartz, 218, n. 1; chemical constitution of, 219, n. 1; = crystals, 220, n. 1; 237, n. 1; 245, n. 3.
Quicksand, 235.

Rabelais, 205, n. 3.
Rains, 229, 232, 267.
Ray, 232, n. 1; 257, n. 1.
Receiver, 242.
Redi, Francesco, 180, 181, 251, n. 1; 271.
Red Sea, passage to, 269.

Refraction, of light, 171; in glass, 243.
Replacements, 220, 225.
Repositories, of minerals, 235 f.
Reservoirs, 234, 266.
Retort, 242.
Rivers, agency in mountain formation, 232, n. 2; changed by earthquake, 235; deposits of, 267; gifts of, 268.
Rocks, 172.
Rome, 182, 229, 258, 259.
Rome de l'Isle, 171.
Roots, of plants, 220.
Rose, Valentine, 224, n. 1.
Rosnel, de, 236, n. 2; 246, n. 3.
Rush, in strata, 228.

Salisbury, 255, n. 1.
Salivary duct, 177.
Salt, deposit of, 228.
Salt-water lakes, 210, n. 1.
Sand, layers of, 228.
Sandstone, 258, n. 2.
San Lorenzo, 185, 186.
Santa Maria Nuova, 179, 180.
Sap, of earth, 232, n. 2.
Sardinia, 210, n. 1.
Savignani, Emilio, 180.
Scallop shells, 210, n. 1.
Scandinavia, 184.
Scapulæ, 259.
Schmidt, Waldmar, 186 f.
Schumacher, Peter, *see* Griffenfeldt.
Schwerin, 185.
Scripture, 263, 264, 265, 266, 267.
Sea, 173, 228, 265.
Sea-devils, 257, n. 1.
Secretions, 223, 226, n. 1.
Sediments, 172, 206, 220, 227, 229.
Selenites, 218, 218, n. 1; 249.
Seneca, 206, n. 2; 213, n. 2.
Septarium, 255, n. 1.
Serpentine, 255, n. 1.
Shark, teeth of, 206, 207, 211, 211, n. 1; 251, 257.
Sheep's head, dissection of, 177.
Shells, crystalline, 218; in earth, 253; helical, 256; of marine animals, 226; of mollusks, 173, 250 f.; petrified, 218; porous, 254.
Ships, in deposits, 228.
Sicily, 210, n. 1; 256.
Silver, 218, 246.
Skeletons, 260, n. 1.
Skin, 221.
Slipping, of strata, 231.
Solid, addition to, 220; contained within solid, 170, 208; differs from fluid, 214; dug from earth, 226; hardening of, 218; production of, 209, 218, 220, 224, 226.
Solon, 268, n. 1.
Soul, world, 216; agency of, 217.
Speculation, 170.
Spinoza, Baruch, 178, 184, n. 1.
Spleen, 222, n. 1.
Springs, 229, 235, 266.
Steno, life of, 175 ff.; writings of, 188 ff.; bibliography on, 202 f.
Stichman, Johannes, 207, n. 4.
Stiermark, 232, n. 2.
Stoics, 213, n. 2; 214.
Stokes, 225, n. 1.
Stomach, 221.
Stones, 235 f.
Storms, 229.
Strabo, 210, n. 1; 268, n. 1; 268, n. 2.
Strata, alterations of, 234; deposition of, 172, 227 ff.; fissures in, 173; horizontality of, 172; manner and place of production, 219; order of, 173; slipping of, 266; thrusts of, 173; in Tuscany, 262 ff., 273; variation in character of, 172.
Strato, 210, n. 1.
Striation, in crystals, 239, 240; in hematite, 244, 246; in marcasites, 247, 248.
Substance, crystalline, 225; place and manner of production, 170; tenuous, 216.
Sulphur, 232.
Sun, 216, 217.
Swammerdam, Jan, 178, 251, n. 1.
Sylvius, 177, 178, 179.

Talc, 249, 249, n. 2.
Tamagninus, 271.
Targioni, 260, n. 1.
Taygetus, 232, n. 2.
Teeth, of sharks, 211, 257; eagle-fish, 257; from Malta, 257.
Telliamed, *see* de Maillet, 169.
Tendons, 225.
Testacea, 251, 253, 256.
Theatrum Anatomicum, 183.
Thera, 232, n. 2.
Therasia, 232, n. 2.
Thévenot, 178, 179.
Thrusts, 231.
Tiber, 260, n. 1, 269.
Time, evolution of, 258.
Titian, 205, n. 3.
Titopolis, 184.
Tolerance, religious tolerance in Holland, 178.
Tongue stones, 211, n. 1.

Tools, of miners, 236.
Tosini, 271.
Tozzetti, 255, n. 1; 260, n, 1.
Trachea, 221.
Traditions, of early civilization, 268, n. 2.
Trasumene Lake, 259.
Trebia, 260, n. 1.
Trees, in strata, 228.
Trigautius, 236, n. 1.
Triptolemus, 269.
Truth, proverb of Truth in a well, 205, n. 3.
Tuff, 220.
Tuscany, geological changes in, 170, 209, 234, n. 2; 262 ff., 269, 276; fossils in, 260, n. 1; 265, 266; pearl-bearing mussels from, 255.
Tylor, 258, n. 1.

Ulysses, 269.
Umbilical vessels, 220.
Urethra, 221.
Usher, chronology of, 174, 266, n. 1; 269, n. 1.
Uterus, 221, 222.

Valleys, 232, n. 2; 265, 267.
Valmont-Bomare, 224, n. 2.
Vapors, subterranean, 229.
Vegetatio, 232, n. 2.
Veins, 222; in rocks, 225.
Vertebræ, of fishes, 257.
Vienna, 182.
Vinci, Leonardo da, 169, 173.
Virtus formativa, 211, n. 1.
Viscera, 225.
Vitriol, 243.
Viviani, Vincenzo, 169, 179, 180, 181.
Volcanoes, 172.
Volterra, shells in, 174; walls of, 258, 259.

Walchs, 211, n. 1.
Water, agency in mountain formation, 232, n. 2; in crystals, 238; deposits of turbid water, 219; issuing from earth, 234.
Weld, 197, n. 1.
Well, proverb of Truth in, 205, n. 3.
Wells, 235.
Wharton, 176, n. 5; 177.
White, 249, n. 2.
Wichfeld, 175, n. 1; 178, n. 1; 179, n. 2; 255, n. 1.
Willis, 179.
Wind, agency in mountain formation, 232, n. 2; breaking from mountain, 234 f.
Winslow, Jacques Bénigne, 178, n. 4.
Winter, 197.
Woodward, 266, n. 1.
Woodworth, 198.
Worms, 221, 255, 256.
Wren, 197, n. 1.

Xanthus, 210, n. 1.
Xerxes, 253, n. 1.

Zittel, von, 182, 204, 256, n. 1.

University of Michigan Studies

HUMANISTIC SERIES

General Editors: JOHN G. WINTER AND EUGENE S. McCARTNEY

Size, 22.7 × 15.2 cm. 8°. Bound in cloth

VOL. I. ROMAN HISTORICAL SOURCES AND INSTITUTIONS. Edited by Henry A. Sanders, University of Michigan. Pp. vii + 402. (*Out of print.*)

VOL. II. WORD FORMATION IN PROVENÇAL. By Edward L. Adams, University of Michigan. Pp. xvii + 607. $4.00. Postage extra.

VOL. III. LATIN PHILOLOGY. Edited by Clarence Linton Meader, University of Michigan. Pp. vii + 290. (*Out of print.*)

Parts Sold Separately in Paper Covers:

Part I. THE USE OF IDEM, IPSE, AND WORDS OF RELATED MEANING. By Clarence L. Meader. Pp. 1–112. $0.75.

Part II. A STUDY IN LATIN ABSTRACT SUBSTANTIVES. By Manson A. Stewart. Pp. 113–78. $0.40.

Part III. THE USE OF THE ADJECTIVE AS A SUBSTANTIVE IN THE DE RERUM NATURA OF LUCRETIUS. By Frederick T. Swan. Pp. 179–214. $0.40.

Part IV. AUTOBIOGRAPHIC ELEMENTS IN LATIN INSCRIPTIONS. By Henry H. Armstrong. Pp. 215–86. $0.40.

VOL. IV. ROMAN HISTORY AND MYTHOLOGY. Edited by Henry A. Sanders. Pp. viii + 427. (*Out of print.*)

Parts Sold Separately in Paper Covers:

Part I. STUDIES IN THE LIFE OF HELIOGABALUS. By Orma Fitch Butler, University of Michigan. Pp. 1–169. $1.25.

Part II. THE MYTH OF HERCULES AT ROME. By John G. Winter, University of Michigan. Pp. 171–273. $0.50.

Part III. ROMAN LAW STUDIES IN LIVY. By Alvin E. Evans. Pp. 275–354. $0.40.

Part IV. REMINISCENCES OF ENNIUS IN SILIUS ITALICUS. By Loura B. Woodruff. Pp. 355–424. $0.40.

VOL. V. SOURCES OF THE SYNOPTIC GOSPELS. By Rev. Dr. Carl S. Patton. Pp. xiii + 263. $1.30. Postage extra.

Size, 28 × 18.5 cm. 4to.

VOL. VI. ATHENIAN LEKYTHOI WITH OUTLINE DRAWING IN GLAZE VARNISH ON A WHITE GROUND. By Arthur Fairbanks, Director of the Museum of Fine Arts, Boston. With 15 plates, and 57 illustrations in the text. Pp. viii + 371. $4.00. Postage extra.

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

VOL. VII. ATHENIAN LEKYTHOI WITH OUTLINE DRAWING IN MATT COLOR ON A WHITE GROUND, AND AN APPENDIX: ADDITIONAL LEKYTHOI WITH OUTLINE IN GLAZE VARNISH ON A WHITE GROUND. By Arthur Fairbanks. With 41 plates. Pp. x + 275. $3.50. Postage extra.

VOL. VIII. THE OLD TESTAMENT MANUSCRIPTS IN THE FREER COLLECTION. By Henry A. Sanders, University of Michigan. With 9 plates showing pages of the Manuscripts in facsimile. Pp. viii + 357. $3.50. Postage extra.

Parts Sold Separately in Paper Covers:

Part I. THE WASHINGTON MANUSCRIPT OF DEUTERONOMY AND JOSHUA. With 3 folding plates. Pp. vi + 104. $1.25. Postage extra.

Part II. THE WASHINGTON MANUSCRIPT OF THE PSALMS. With 1 single plate and 5 folding plates. Pp. viii + 105–357. $2.00. Postage extra.

VOL. IX. THE NEW TESTAMENT MANUSCRIPTS IN THE FREER COLLECTION. By Henry A. Sanders, University of Michigan. With 8 plates showing pages of the Manuscripts in facsimile. Pp. x + 323. $3.50. Postage extra.

Parts Sold Separately in Paper Covers:

Part I. THE WASHINGTON MANUSCRIPT OF THE FOUR GOSPELS. With 5 plates. Pp. vii + 247. $2.00. Postage extra.

Part II. THE WASHINGTON MANUSCRIPT OF THE EPISTLES OF PAUL. With 3 plates. Pp. vii, 249–315. $1.25. Postage extra.

VOL. X. THE COPTIC MANUSCRIPTS IN THE FREER COLLECTION. By William H. Worrell, Hartford Seminary Foundation. With 12 plates. Pp. xxvi + 396. $4.75. Postage extra.

Parts Sold Separately in Paper Covers:

Part I. THE COPTIC PSALTER. The Coptic Text in the Sahidic Dialect, with an Introduction, and with 6 plates showing pages of the Manuscript and Fragments in Facsimile. Pp. xxvi + 112. $2.00. Postage extra.

Part II. A HOMILY ON THE ARCHANGEL GABRIEL BY CELESTINUS, ARCHBISHOP OF ROME, AND A HOMILY ON THE VIRGIN BY THEOPHILUS, ARCHBISHOP OF ALEXANDRIA, FROM MANUSCRIPT FRAGMENTS IN THE FREER COLLECTION AND THE BRITISH MUSEUM. The Coptic Text, with an Introduction and Translation, and with 6 plates showing pages of the Manuscripts in Facsimile. Pp. 113–396. $2.50. Postage extra.

VOL. XI. CONTRIBUTIONS TO THE HISTORY OF SCIENCE. By Louis C. Karpinski and John G. Winter. With 11 plates. Pp. xi + 283. $3.50. Postage extra.

Parts Sold Separately:

Part I. ROBERT OF CHESTER'S LATIN TRANSLATION OF THE ALGEBRA OF AL-KHOWARIZMI. With an Introduction, Critical Notes, and an English Version. By Louis C. Karpinski, University of Michigan. With 4 plates showing pages of manuscripts in facsimile, and 25 diagrams in the text. Pp. vii + 164. $2.00. Postage extra.

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

University of Michigan Studies — *Continued*

Part II. The Prodromus of Nicolaus Steno's Latin Dissertation on a Solid Body Enclosed by Process of Nature within a Solid. Translated into English by John G. Winter, University of Michigan, with a Foreword by William H. Hobbs. With 7 plates. Pp. vii + 169–283. $1.30. Postage extra.

Vol. XII. Studies in East Christian and Roman Art. By Charles R. Morey, Princeton University, and Walter Dennison. With 67 plates (10 colored) and 91 illustrations in the text. Pp. xiii + 175. $4.75. Postage extra.

Parts Sold Separately:

Part I. East Christian Paintings in the Freer Collection. By Charles R. Morey. With 13 plates (10 colored) and 34 illustrations in the text. Pp. xiii + 86. $2.50. Postage extra.

Part II. A Gold Treasure of the Late Roman Period from Egypt. By Walter Dennison. With 54 plates and 57 illustrations in the text. Pp. 89–175. $2.50. Postage extra.

Vol. XIII. Fragments from the Cairo Genizah in the Freer Collection. By Richard Gottheil, Columbia University, and William H. Worrell, University of Michigan. Text, with Translation, Notes and an Introduction. With 52 plates showing the different styles of writing in facsimile. Pp. xxxi + 273. $4.00. Postage extra.

Vol. XIV. Two Studies in Later Roman and Byzantine Administration. By Arthur E. R. Boak and James E. Dunlap, University of Michigan. Pp. x + 324. $2.25. Postage extra.

Parts Sold Separately in Paper Covers:

Part I. The Master of the Offices in the Later Roman and Byzantine Empires. By Arthur E. R. Boak. Pp. x + 160. Paper covers. $1.00.

Part II. The Office of the Grand Chamberlain in the Later Roman and Byzantine Empires. By James E. Dunlap. Pp. 161–324. $1.00.

Vol. XV. Greek Themes in Modern Musical Settings. By Albert A. Stanley, University of Michigan. With 10 plates. Pp. xxii + 385. $4.00. Postage extra.

Parts Sold Separately in Paper Covers:

Part I. Incidental Music to Percy Mackaye's Drama of Sappho and Phaon. Pp. 1–68. $.90.

Part II. Music to the Alcestis of Euripides with English Text. Pp. 71–120. $.80.

Part III. Music for the Iphigenia among the Taurians by Euripides, with Greek Text. Pp. 123–190. $.75.

Part IV. Two Fragments of Ancient Greek Music. Pp. 217–225. $.30.

Part V. Music to Cantica of the Menaechmi of Plautus. Pp. 229–263. $.50.

Part VI. Attis: A Symphonic Poem. Pp. 265–384. $1.00.

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

University of Michigan Studies — *Continued*

Vol. XVI. Nicomachus of Gerasa: Introduction to Arithmetic. Translated into English by Martin Luther D'Ooge, with Studies in Greek Arithmetic by Frank Egleston Robbins and Louis C. Karpinski. Pp. vii + 318. $3.50. Postage extra.

Vols. XVII, XVIII, XIX, XX. Royal Correspondence of the Assyrian Empire. Translated into English with a transliteration of the Text and a Commentary. By Leroy Waterman, University of Michigan. (*In press.*)

Vol. XXI. The Minor Prophets in the Freer Collection and the Berlin Fragment of Genesis. By Henry A. Sanders, University of Michigan, and Carl Schmidt, University of Berlin. With 7 plates. Pp. xii + 436. $3.50. Postage extra.

Vol. XXII. A Papyrus Codex of the Shepherd of Hermas. By Campbell Bonner, University of Michigan. (*In press.*)

Vol. XXIII. The Complete Commentary of Oecumenius on the Apocalypse: Now printed for the first time from Manuscripts at Messina, Rome, Salonika and Athos. By H. C. Hoskier. Pp. viii + 260. $4.00. Postage extra.

Vol. XXIV. Zenon Papyri in the University of Michigan Collection. By C. C. Edgar. (*In press.*)

FACSIMILES OF MANUSCRIPTS

Size, 40.5 × 35 cm.

Facsimile of the Washington Manuscript of Deuteronomy and Joshua in the Freer Collection. With an Introduction by Henry A. Sanders. Pp. x; 201 heliotype plates. The University of Michigan. Ann Arbor, Michigan, 1910.

Limited edition, distributed only to Libraries, under certain conditions. A list of Libraries containing this Facsimile is printed in *University of Michigan Studies, Humanistic Series*, Volume VIII, pp. 351–353.

Size, 34 × 26 cm.

Facsimile of the Washington Manuscript of the Four Gospels in the Freer Collection. With an Introduction by Henry A. Sanders. Pp. x; 372 heliotype plates and 2 colored plates. The University of Michigan. Ann Arbor, Michigan, 1912.

Limited edition, distributed only to Libraries, under certain conditions. A list of Libraries containing this Facsimile is printed in *University of Michigan Studies, Humanistic Series*, Volume IX, pp. 317–320.

Size, 30.5 × 40.6 cm.

Facsimile of the Washington Manuscript of the Minor Prophets in the Freer Collection and the Berlin Fragment of Genesis, with an Introduction by Henry A. Sanders. With 130 plates. The University of Michigan. Ann Arbor, Michigan, 1927.

Limited edition, distributed only to Libraries, under certain conditions. A list of Libraries containing this Facsimile is printed in *University of Michigan Studies, Humanistic Series*, Volume XXI, pp. 431–434.

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

SCIENTIFIC SERIES

Size, 28 × 18.5 cm. 4°. Bound in cloth

VOL. I. THE CIRCULATION AND SLEEP. By John F. Shepard, University of Michigan. Pp. ix + 83, with an Atlas of 63 plates, bound separately. Text and Atlas, $2.50. Postage extra.

VOL. II. STUDIES ON DIVERGENT SERIES AND SUMMABILITY. By Walter B. Ford, University of Michigan. Pp. xi + 194. $2.50. Postage extra.

Size, 23.5 × 15.5 cm.

VOL. III. THE GEOLOGY OF THE NETHERLANDS EAST INDIES. By H. A. Brouwer. With 18 plates and 17 text figures. Pp. xii + 160. $3.00. Postage extra.

VOL. IV. THE GLACIAL ANTICYCLONES: THE POLES OF THE ATMOSPHERIC CIRCULATION. By William Herbert Hobbs. With 3 plates and 53 figures. Pp. xxiv + 198. $2.75. Postage extra.

MEMOIRS OF THE UNIVERSITY OF MICHIGAN MUSEUMS

Size, 26 × 17 cm. 4°. Bound in cloth

VOL. I. THE WHIP SNAKES AND RACERS: GENERA MASTICOPHIS AND COLUBER. By A. I. Ortenburger, University of Oklahoma. With 36 plates and 64 text figures. Pp. xviii + 247. $6.00. Postage extra.

VOL. II. DESCRIPTION OF THE SKULL OF A NEW FORM OF PHYTOSAUR, WITH NOTES ON THE CHARACTERS OF DESCRIBED NORTH AMERICAN PHYTOSAURS. By E. C. Case. With 7 plates and 24 text figures. Pp. vi + 56. $2.00. Postage extra.

University of Michigan Publications

HUMANISTIC PAPERS

General Editor: EUGENE S. McCARTNEY.

Size, 22.7 × 15.2 cm. 8°. Bound in cloth

THE LIFE AND WORK OF GEORGE SYLVESTER MORRIS: A CHAPTER IN THE HISTORY OF AMERICAN THOUGHT IN THE NINETEENTH CENTURY. By Robert M. Wenley, University of Michigan. Pp. xv + 332. $1.50. Postage extra.

LATIN AND GREEK IN AMERICAN EDUCATION, WITH SYMPOSIA ON THE VALUE OF HUMANISTIC STUDIES, Revised Edition. Edited by Francis W. Kelsey. Pp. xiii + 360. $1.50. Postage extra.

Size, 18 × 12 cm. Bound in paper

THE MENAECHMI OF PLAUTUS. The Latin Text, with a Translation by Joseph H. Drake, University of Michigan. Pp. xi + 129. $.60. Postage extra.

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

University of Michigan Publications — *Continued*

LANGUAGE AND LITERATURE

Vol. I. Studies in Shakespeare, Milton and Donne. By Members of the English Department of the University of Michigan. Pp. viii + 232. Cloth. $2.50. Postage extra.

Vol. II. Elizabethan Proverb Lore in Lyly's 'Euphues' and in Pettie's 'Petite Pallace,' with Parallels from Shakespeare. By Morris P. Tilley. Pp. x + 461. Cloth. $3.50. Postage extra.

Vol. III. The Social Mode of Restoration Comedy. By Kathleen M. Lynch. Pp. x + 242. Cloth. $2.50. Postage extra.

Vol. IV. Stuart Politics in Chapman's 'Tragedy of Chabot.' By Norma D. Solve. Pp. x + 176. Cloth. $2.50. Postage extra.

Vol. V. El Libro del Cauallero Zifar: Part I, Text. By C. P. Wagner. With 9 facsimiles. Pp. xviii + 532. Cloth. Postage extra.

Vol. VI. El Libro del Cauallero Zifar: Part II, Commentary. By C. P. Wagner. (*In preparation.*)

Vol. VII. Strindberg's Dramatic Expressionism. By C. E. W. L. Dahlström. (*In press.*)

HISTORY AND POLITICAL SCIENCE

The first three volumes of this series were published as "Historical Studies" under the direction of the Department of History. Volumes IV and V were published without numbers.

Vol. I. A History of the President's Cabinet. By Mary Louise Hinsdale. Pp. ix + 355. Cloth. $2.00. Postage extra.

Vol. II. English Rule in Gascony, 1199–1259, with Special Reference to the Towns. By Frank Burr Marsh. Pp. xi + 178. Cloth. $1.25. Postage extra.

Vol. III. The Color Line in Ohio; A History of Race Prejudice in a Typical Northern State. By Frank Uriah Quillan. Pp. xvi + 178. Cloth. $1.50. Postage extra.

Vol. IV. The Senate and Treaties, 1789–1817. The Development of the Treaty-Making Functions of the United States Senate during Their Formative Period. By Ralston Hayden, University of Michigan. Pp. xvi + 237. Cloth. $1.50. Postage extra.

Vol. V. William Plumer's Memorandum of Proceedings in the United States Senate, 1803–1807. Edited by Everett Somerville Brown, University of Michigan. Pp. xi + 873. Cloth. $3.50. Postage extra.

Vol. VI. The Grain Supply of England during the Napoleonic Period. By W. F. Galpin, Syracuse University. Pp. xi + 305. Cloth. $3.00. Postage extra.

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

University of Michigan Publications — *Continued*

Vol. VII. Eighteenth Century Documents relating to the Royal Forests, the Sheriffs and Smuggling: Selected from the Shelburne Manuscripts in the William L. Clements Library. By Arthur Lyon Cross, University of Michigan. With 4 plates. Pp. x + 328. $3.00. Postage extra.

Vol. VIII. The Low Countries and the Hundred Years' War, 1326–1347. By Henry S. Lucas, University of Washington. Pp. xviii + 696. Cloth. $4.00. Postage extra.

CONTRIBUTIONS FROM THE MUSEUM OF PALEONTOLOGY

Vol. I. The Stratigraphy and Fauna of the Hackberry Stage of the Upper Devonian. By Carroll Lane Fenton and Mildred Adams Fenton. With 45 plates, 9 text figures and one map. Pp. xi + 260. Cloth. $2.75. Postage extra.

Vol. II. Consisting of 14 miscellaneous papers, published between July 10, 1924, and August 3, 1927. With 41 plates, 39 text figures and 1 map. Pages ix + 240. Cloth. $3.00. Postage extra.

Parts Sold Separately in Paper Covers:

No. 1. A Possible Explanation of Fenestration in the Primitive Reptilian Skull, with Notes on the Temporal Region of the Genus Dimetrodon, E. C. Case. Pp. 1–12, with five illustrations. $0.30.

No. 2. Occurrence of the Collingwood Formation in Michigan, by R. Ruedemann and G. M. Ehlers. Pp. 13–18. $0.15.

No. 3. Silurian Cephalopods of Northern Michigan, by Aug. F. Foerste. Pp. 19–86, with 17 plates. $1.00.

No. 4. A Specimen of *Stylemys nebrascensis* Leidy, with the Skull Preserved, by E. C. Case. Pages 87–91, with 7 text figures. Price, $0.20.

No. 5. Note on a New Species of the Eocene Crocodilian *Allognathosuchus A. wartheni*, by E. C. Case. Pages 93–97, with 1 plate and 1 text figure. Price, $0.20.

No. 6. Two New Crinoids from the Devonian of Michigan, by G. M. Ehlers. Pages 99–104, with 1 plate. Price, $0.20.

No. 7. New Brachiopods from the Warsaw Formation of Wayne County, Kentucky, by G. M. Ehlers and M. S. Chang. Pages 105–111, with 1 plate. Price, $0.20.

No. 8. The Richmond Formation of Michigan, by R. C. Hussey. Pages 113–187, with 11 plates, 12 text figures and 1 map. Price, $0.75.

No. 9. Devonian Cephalopods from Alpena in Michigan, by Aug. F. Foerste. Pages 189–208, with 5 plates. Price, $0.35.

No. 10. The Vertebral Column of *Coelophysis* Cope, by E. C. Case. Pages 209–222, with 1 plate and 9 text figures. Price, $0.25.

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

University of Michigan Publications — *Continued*

No. 11. A New Species of Trionychid Turtle, *Amyda nelsoni*, from the Eocene Beds of Southwestern Wyoming, by E. C. Case. Pages 223–226, with 1 plate and 3 text figures. Price, $0.20.

No. 12. A Complete Phytosaur Pelvis from the Triassic Beds of Western Texas, by E. C. Case. Pages 227–229, with 1 plate. Price, $0.20.

No. 13. Discovery of a Hamilton Fauna in Southeastern Michigan, by G. M. Ehlers and Mary E. Cooley. Pages 231–236. Price, $0.15.

No. 14. *Anisotrypa waynensis*, a New Bryozoan from the Warsaw Formation of Kentucky, by Charles F. Deiss, Jr. Pages 237–239, with 2 plates. Price, $0.20.

Vol. III, No. 1. Indications of a Cotylosaur and of a New Form of Fish from the Triassic Beds of Texas, with Remarks on the Shinarump Conglomerate, by E. C. Case. Pages 1–14, with 1 plate. Price, $0.25.

No. 2. Fossil Fishes from the Triassic of Texas, by Aldred S. Warthin, Jr. Pages 15–18, with 1 plate. Price, $0.20.

No. 3. Contributions to the Geology of Foxe Land, Baffin Island, by L. M. Gould, Aug. F. Foerste and R. C. Hussey. Pages 19–77, with 17 plates, 1 text figure, and 1 map. Price, $0.75.

No. 4. Cystoids from the Trenton Group of Michigan, by R. C. Hussey. Pages 77–79, with 1 plate. Price, $0.20.

No. 5. Description of a Nearly Complete Skeleton of *Ostodolepis brevispinatus* Williston, by E. C. Case. Pages 81–107, with 3 plates, and 12 text figures. Price, $0.35.

UNIVERSITY OF MICHIGAN COLLECTIONS

Catalogue of the Stearns Collection of Musical Instruments (Second edition). By Albert A. Stanley. With 40 plates. Pp. 276. $4.00.

PAPERS OF THE MICHIGAN ACADEMY OF SCIENCE, ARTS AND LETTERS

(containing Papers submitted at Annual Meetings)

Editors: EUGENE S. McCARTNEY AND PETER OKKELBERG

Size, 24.2 × 16.5 cm. 8°. Bound in cloth

Vol. I (1921). With 38 plates, 1 text figure and 5 maps. Pp. xi + 424. $2.00. Postage extra.

Vol. II (1922). With 11 plates and 7 text figures. Pp. xi + 226. $2.00. Bound in paper, $1.50. Postage extra.

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

University of Michigan Publications — *Continued*

VOL. III (1923). With 26 plates, 15 text figures and 3 maps. Pp. xii + 473. $3.00. Bound in paper, $2.25. Postage extra.

VOL. IV (1924), PART I. With 27 plates, 22 text figures and 3 maps. Pp. xii + 631. $3.00. Bound in paper, $2.25. Postage extra.

VOL. IV (1924), PART II. A KEY TO THE SNAKES OF THE UNITED STATES, CANADA AND LOWER CALIFORNIA. By Frank N. Blanchard. With 78 text figures. Pp. xiii + 65. Cloth. $1.75. Postage extra.

VOL. V (1925). With 27 plates, 26 text figures and 1 map. Pp. xii + 479. $3.00. Bound in paper, $2.25. Postage extra.

VOL. VI (1926). (This volume contains papers in botany only.) With 28 plates, 4 text figures and 3 maps. Pp. xii + 406. $3.00. Bound in paper, $2.25. Postage extra.

VOL. VII (1926). (This volume does not contain papers in botany.) With 28 plates, 17 text figures and 7 maps. Pp. xii + 435. $3.00. Bound in paper, $2.25. Postage extra.

VOL. VIII (1927). With 32 plates, 35 text figures and 2 maps. Pp. xiv + 456. $3.00. Bound in paper, $2.25. Postage extra.

VOL. IX (1928). (This volume contains papers in botany and forestry only.) With 99 plates and 29 text figures. Pp. xiv + 597. $4.00. Bound in paper, $2.25. Postage extra.

VOL. X (1928). (This volume does not contain papers in botany and forestry.) With 24 plates, 61 text figures and 13 maps. Pp. xvii + 620. $4.00. Bound in paper, $2.25. Postage extra.

VOLS. XI, XII (1929). (*In press.*)

Orders should be addressed to The Librarian, University of Michigan, Ann Arbor, Michigan.

NHMRL0015214

S0-DTF-978